Chapter 19

DC Electrical Circuits
Topics in Chapter 19

- EMF and Terminal Voltage
- Resistors in Series and Parallel
- Kirchhoff’s Rules
- EMFs in Series and Parallel; Charging a Battery
- Circuits with Capacitors in Series and in Parallel
- RC Circuits – Resistor and Capacitor in Series
EMF and Terminal Voltage

Electric circuit needs battery or generator to produce current – these are called sources of \textit{emf}.

A Battery is a nearly constant voltage source, but does have a small internal resistance, which reduces the actual voltage from the ideal \textit{emf}:

\[ V_{ab} = \mathcal{E} - Ir \]
EMF and Terminal Voltage

This resistance behaves as though it were in series with the emf.
Resistors in Series and in Parallel

A series connection has a single path from the battery, through each circuit element in turn, then back to the battery.
Resistors in Series

The current through each resistor is identical; but the voltage across each depends on its resistance. The sum of the voltage drops across the resistors equals the battery voltage.

\[ V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \]

\[ = IR_{eq} \quad \text{equivalent resistance} \]

\[ R_{eq} = R_1 + R_2 + R_3 \]
Resistors in Parallel

A parallel connection splits the current; the voltage across each resistor is the same:
Resistors in Parallel

The total current is the sum of the currents flowing through each resistor.

The Voltage across each is the same.

\[
I = I_1 + I_2 + I_3
\]

\[
\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}
\]

\[
\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]
Kirchhoff’s Rules

Some circuits cannot be broken down into series and parallel connections. These situations are a bit more complicated but follow certain rules.
Kirchhoff’s Rules

The Junction rule: The sum of currents entering a junction equals the sum of the currents leaving it.

Junction Eq for pt. a) gives

\[ I_3 = I_1 + I_2 \]

Other pts. give either the same equation, or are trivial.

Get \( n-1 \) different Eq for \( n \) junctions
Kirchhoff’s Rules

Loop rule: The sum of the changes in potential (or $\Delta V$) around a closed loop is zero.

\[ R_{eq} = 400 + 290 = 690 \ \Omega \]

\[ I = \frac{V}{R_{eq}} = \frac{12}{690} = 0.0174 \ \text{A} \]

\[ V_{ab} = IR_1 = 0.0174 \times 400 = 6.96 \ \text{V} \]

\[ V_{bc} = IR_2 = 0.0174 \times 290 = 5.04 \ \text{V} \]

The Loop Rule gives the Eq.

\[ V - IR_1 - IR_2 = 0 \]
Kirchhoff’s Rules

Problem Solving: Use of Kirchhoff’s Rules

2. Label each current (you can guess directions).

3. Identify unknowns.

4. Apply junction and loop rules; you need as many independent equations as there are unknowns.

5. Solve the equations, being careful with signs.

6. If the solution gives a current as ‘minus’, this simply means your direction guess was wrong.
EMFs in Series

EMFs in series in the **same** direction: the total voltage is the sum of the separate voltages

![Diagram of EMFs in Series](image)
EMFs in Series: Charging a Battery

EMFs in series, opposite direction: total voltage is the difference, but the lower-voltage battery gets charged as current flows.
EMFs in Parallel

Connecting EMFs in parallel is sensible only if the voltages are the same; this arrangement can often produce more current than a single emf.
Capacitors in parallel will have the same voltage across each one:

Total charge adds, and

\[ C_{eq} = C_1 + C_2 + C_3 \]
Capacitors in Series

Capacitors in series each have the same charge on each capacitor. Voltages add: \( V_{ab} = V_1 + V_2 + V_3 \)
Capacitors in Series

In this case, the reciprocals of the capacitances add to give the reciprocal of the equivalent capacitance:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
RC Circuits: Resistor and Capacitor in Series

When the switch is closed, the capacitor will begin to charge. When fully charged $Q_0 = C \mathcal{E}$ and $I = 0$.
RC Circuits: Resistor and Capacitor in Series

The voltage across the capacitor increases with time as an exponential approach to the applied emf.

\[ V_C = \mathcal{E}(1 - e^{-t/RC}) \]

Charge follows a similar curve since \( Q = CV \)

\[ Q = Q_0(1 - e^{-t/RC}) \]

These curves have a characteristic time constant \( \tau = RC \)
RC Circuits – Discharge of a Capacitor

If an isolated charged capacitor is connected across a resistor, it discharges with the same time constant.

\[ Q = Q_0 e^{-t/RC} \]

\( e^{-1} \sim 3/8 \) is a good approximation to get \( \tau \) quickly.

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Summary of Chapter 19

• A source of emf transforms energy from some other form to electrical energy

• A battery is a source of emf in series with an internal resistance

For resistors

in Series

\[ R_{eq} = R_1 + R_2 + R_3 \]

or in parallel

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]
Summary of Chapter 19

Ohm’s Law can be applied to simple resistance networks using $R_{eq}$. For more complicated circuits it is necessary to apply a more complicated set of rules to the analysis of current flow.

- **Kirchhoff’s rules:**
  1. sum of currents entering a junction equals sum of currents leaving it
  2. total potential difference around closed loop is zero
Summary of Chapter 19

- Capacitors in parallel:

\[ C_{eq} = C_1 + C_2 + C_3 \]

- Capacitors in series:

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]

- RC circuit has a characteristic time constant:

\[ \tau = RC \]