Chapter 34  Electromagnetic Waves

34.1 Displacement Current and the General Form of Ampere's Law

Review Ampere's Law: \[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]

The line integral of \( \mathbf{B} \cdot d\mathbf{s} \) around any closed path equals \( \mu_0 I \). \( I \) is the total steady current passing through any surface bounded by the closed path.

Problem with capacitors:
Current pass through \( S_1 = I \)
Current pass through \( S_2 = 0 \)
This contradicts the Ampere's Law!

Reason: The current is discontinuous in a capacitor.

Solution: Maxwell added a "displacement current" in the Ampere's Law. Which makes the generalized Ampere's law valid in all cases.

Displacement Current:
\[ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} \]
\( \Phi_E \) is the flux of the electric field. \( \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} \)

Generalized Ampere's law:
\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

- Displacement current is zero, if current is continuous and steady. Because electric field flux is constant.

- As the capacitor is being charged (or discharged), no actual current passing through the plates. But the changing electric field between the plates may be thought of as a sort of current which is equivalent to the current in a wire.

Example: Displacement current in a capacitor.

The electric flux through \( S_2 \):
\[ \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA \]
The electric field:
\[ E = \frac{Q}{\varepsilon_0 A} \]
Therefore,
\[ \Phi_E = EA = \frac{Q}{\varepsilon_0} \]
The displacement current:
\[ I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} \]

\( I_d \) is identical to the current \( I \) through \( S_1 \)

Magnetic field are produced both by conduction current and by changing electric fields.
34.2 Maxwell's Equations and Hertz’s Discoveries

Maxwell's Equations

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \]  
(Gauss's law)

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]  
(Gauss's law in magnetism)

\[ \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \]  
(Faraday's law)

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]  
(Ampere-Maxwell law)

Force on a charged particle:

\[ \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \]  
(Lorentz force)

**Maxwell’s equations, together with the Lorentz force law give a complete description of all classical electromagnetic interactions.**

- Maxwell introduced the concept of displacement current: a time varying electric field produces a magnetic field, just as a time-varying magnetic field produces an electric field.
- Maxwell's equations also predicted the existence of electromagnetic (EM) waves that propagate through space with the speed of light. Light is a form of electromagnetic radiation.

- Heinrich Hertz first generated and detected electromagnetic waves.

\[ \omega = \frac{1}{\sqrt{LC}} \]

speed = \(3 \times 10^8\) m/s

34.3 Plane Electromagnetic Waves

- Review the properties of sinusoidal waves.

General equation:

\[ \frac{\partial^2 f}{\partial^2 x} = \left( \frac{1}{v^2} \right) \frac{\partial^2 f}{\partial^2 t} \]

Wave function:

\[ y = A \cos(kx - \omega t) \]

A: amplitude

k: angular wave number

\[ k = \frac{2\pi}{\lambda} \]
ω: angular frequency

\[ \omega = 2\pi f = \frac{2\pi}{T} \]

Speed of wave:

\[ v = \frac{\lambda}{T} = \lambda f \]

From Maxwell's 3rd and 4th equations (see textbook), it can be shown:

\[ \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \text{and} \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \]

These are wave equations. The speed of the EM wave

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792 \times 10^8 \text{ m/s} \]

\[ \frac{E}{B} = \frac{E_{\max} \cos(kx - \omega t)}{B_{\max} \cos(kx - \omega t)} \]

- From the relation (see textbook):

\[ \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \]

We have

\[ -kE_{\max} \sin(kx - \omega t) = -\omega B_{\max} \sin(kx - \omega t) \]

\[ kE_{\max} = \omega B_{\max} \quad \text{or} \quad \frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = \frac{2\pi f}{2\pi / \lambda} = f \lambda = c \]

Thus,

\[ \frac{E_{\max}}{B_{\max}} = \frac{E}{B} = c \]

34.3 Energy Carried by Electromagnetic Waves

- Electromagnetic waves carry energy.

- The rate of flow of energy crossing a unit area:

\[ \text{Poynting vector} \quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

Units: J/s·m² = W/m²

- Magnitude of \( \vec{S} \):

\[ S = \frac{EB}{\mu_0} \]

Using \( B = E/c \),

\[ S = \frac{E(t)^2}{\mu_0 c} = \frac{c}{\mu_0} B(t)^2 \]

Note that \( S \) is a function of time also.
• Wave intensity $I$: the time average of $S$ over one or more cycles.

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c^2} = \frac{c}{2\mu_0} B_{max}^2 = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{\mu_0 c} = \frac{c}{\mu_0} B_{rms}^2$$

• Energy densities:

$$u_B = \frac{B^2}{2\mu_0}, \quad \text{(magnetic field)}$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2, \quad \text{(electric field)}$$

Using $B = \frac{E}{c}$,

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0 \varepsilon_0 E^2}{2\mu_0} = \frac{\varepsilon_0 E^2}{2} = u_E$$

Thus, \[ u_B = u_E = \frac{B^2}{2\mu_0} = \frac{\varepsilon_0 E^2}{2} \]

For an EM wave, the instantaneous magnetic-field energy density = the instantaneous electric-field energy density

Total instantaneous energy density $u$: \[ u = u_B + u_E = \frac{B^2}{\mu_0} = \varepsilon_0 E^2 \]

• $I = S_{av} = cu_{av}$

The intensity of an EM wave = the average energy density $\times$ the speed of light.

\[ I = \frac{P(\text{source})}{4\pi r^2}, \quad \text{also } I = \frac{E_{max}^2}{2\mu_0 c} \]

Solving for $E_{max}$ gives:

$$E_{max} = \sqrt{\frac{\mu_0 c P(\text{source})}{2\pi r^2}} = \sqrt{\frac{(4\pi \times 10^{-7} n / A^2)(3 \times 10^8 m/s)(800 W)}{2\pi (3.5 m)^2}} = 62.6 V/m$$

$$B_{max} = \frac{E_{max}}{c} = \frac{62.6 V/m}{3 \times 10^8 m/s} = 2.09 \times 10^{-7} T$$

(b) Calculate the average energy density 3.5m from the source.

$$u_{av} = \frac{\varepsilon_0 E_{max}^2}{2} = 8.85 \times 10^{-12} C^2/Nm^2 (62.6 V/m)^2 / 2 = 1.73 \times 10^{-8} J/m^3$$

Example: A point source of EM radiation has an average power output of 800W.

(a) Calculate $E_{max}$ and $B_{max}$ at 3.5 m from the source.
34.5 Momentum and Radiation Pressure

- EM waves have linear momentum.
- A pressure is exerted on a surface when an EM wave strike it.

Total momentum delivered to a surface:
\[ p = \frac{U}{c} \] (complete absorption, blackbody)
\[ p = \frac{2U}{c} \] (complete reflection, mirror)
where \( U \) is the total energy of incident wave during time \( t \).

Total radiation pressure (force/area):
\[ P = \frac{S}{c} \] (complete absorption, blackbody)
\[ P = \frac{2S}{c} \] (complete reflection, mirror)

Example: Solar Energy: The sun delivers about 1000 W/m² of EM flux to the earth surface.

(a) Calculate the total power incident on a roof of dimensions 8 m × 20 m.
The Poynting vector \( S = 1000 \text{ W/m}^2 \)
Assume the sunlight is incident normal to the roof:
Power = \( SA = (1000 \text{ W/m}^2)(8\text{ m} \times 20\text{ m}) \)
\[ = 1.6 \times 10^5 \text{ W} = 160 \text{ KW} \]

(b) Calculate the radiation pressure and radiation force on the roof.
Assuming the roof is a perfect absorber.

Pressure:
\[ P = \frac{S}{c} = \frac{1000 \text{ W/m}^2}{3 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ N/m}^2 \]
Force:
\[ F = PA = (3.33 \times 10^{-6} \text{ N/m}^2)(160\text{ m}^2) = 5.33 \times 10^{-4} \text{ N} \]

(c) How much solar energy is incident on the roof in 1 h?
Energy = Power × time
\[ = (1.6 \times 10^5 \text{ W})(3600\text{s}) \]
\[ = 5.76 \times 10^8 \text{ J} \]

Example: A long, straight wire of resistance \( R \), radius \( a \), and length \( l \) carries a constant current \( I \). Calculate the Poynting vector on the surface of the wire.

The electric field:
\[ E = \frac{V}{l} = \frac{IR}{l} \]
The magnetic field on the surface:
\[ B = \frac{\mu_0 I}{2\pi a} \]
The Poynting vector \( \hat{S} \) is directed radially inward:
\[ \hat{S} = \frac{EB}{\mu_0} = \frac{1}{\mu_0} \left( \frac{RI}{l} \right) \frac{\mu_0 I}{2\pi a} = \frac{I^2 R}{2\pi a} = \frac{I^2 R}{A} \]
or \[ \text{AS} = I^2 R \]

- The rate at which EM energy flows into the wire, \( SA \), is the rate of energy dissipated as joule heat, \( I^2 R \).