Extended projects for Computational Physics

The purpose of these extended projects is to get you to develop some skill at working with more than one computational technique at the same time. I am intentionally leaving the requirements a little bit open-ended because I want to see you doing some exploration of what you can learn by using the same program with different sets of conditions, or very similar programs with small modifications. Physics is more fun when you explore!

If you wish to do a project on something different, please feel free to discuss it with me. I don't have any objections to this, but I wanted to put my time and energy into describing one project carefully, rather than giving you a menu of poorly-thought out projects.

Expectations for your reports

For the final projects, your reports must be much more involved than they have been for your past projects. I expect to see at least a few pages of text, with figures embedded in the text to illustrate what you have learned from doing the computational exercises. Write your report the way a section in a textbook would look if it were discussing the same topic, and then also make a technical appendix where you include your code.

Project: harmonic oscillators

To get started on this project, you may find the Fourier transform program useful, and the pendulum program useful. For the discussion here, I'm drawing pretty heavily on Wikipedia, after having verified from my freshman physics textbook that the information there is all correct. Wikipedia has nice notation, and is easy for you all to find if you wish to get a bit more context than what I'm giving you here.

You will have to modify the pendulum program to deal with an ideal spring, where:

$$F = -kx \tag{1}$$

instead of a force that scales with the sine of the angular displacement.

(1) So, first, run the program in the basic case of the simple harmonic oscillator, and see what happens. Take the Fourier transform of the output data to ensure that the frequency you get out of the system is the one you were expecting to get. Hint: it should be $\omega_0 = \sqrt{k/m}$.

(2) At this point you need to do at least one other thing with the program to merit a passing grade on the assignment. If you do more than one of them you can expect to get a better grade.

(a) Damped harmonic oscillators

In many cases, there will be some appreciable form of friction on a harmonic oscillator, and the frictional force will be proportional to the velocity of motion. Air resistance, and internal friction are two main sources of this.

We then get an additional force term, so that the force law can be defined as:

$$F = -kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}. \tag{2}$$

A variable ζ (zeta) can be defined to be $\zeta = \left(\frac{c}{2\sqrt{mk}}\right)$, and the differential equation can be re-written as:

$$\frac{d^2}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = 0 \tag{3}$$

Include the damping term. Discuss the qualitative behavior of the system for $\zeta > 1$, $\zeta = 1$, and $\zeta < 1$. Show also, using the Fourier transform program, how the frequency of the oscillations depends on ζ for $\zeta < 1$.

(b) Non-ideal springs

A non-ideal spring is one where F is not equal to kx, but rather is some other function of x. Let's keep this simple, and make the force law a polynomial. Next, let's remember that we don't want any forces that always act in the same direction, so let's keep only odd-powered terms. Finally let's not overthink things right from the start, so let's just add in a term with some constant times x^3.

Now, we can have "hard springs" and "soft springs". Hard springs are springs which restore themselves even faster when stretched out a lot, and soft springs are those which restore themselves more weakly when stretched a lot.

Set up force laws with hard and soft springs. Run the program for different sets of initial displacement. Plot the results. Also, take Fourier tranforms of the data you produce and see if you can find a trend in terms of the frequency of oscillation versus the displacement for the two different cases.

One other thing you might wish to try if you really want a challenge is the "bilinear oscillator". This has $F = -k_1 x$ for positive x and $F = -k_2 x$ for negative x. It could represent something like a cracked beam supporting part of a machine. In this simple case, it's a reasonably well-behaved system, and it's actually no harder to do stuff with it. But if you get to the step of driving and damping this oscillator in step (c), it becomes remarkably complicated for how simple the equations are.

(c) Driven, damped harmonic oscillators

You may wish to go back to the plain old simple $F = -kx$ force law for this part, so that you aren't dealing with too many difficult things all at the same time.

For a driven, damped harmonic oscillator, we add one more term to the equation – a driving force. Now,

$$F_d(t) - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}, \tag{4}$$

where $F_d(t)$ is the driving force. We normally presume that the driving force will be sinusoidal, at least for early-stage coursework.

Then,

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = \frac{1}{m}F_0\sin(\omega t) \tag{5}$$

This will have two components to the solution. One is a "transient" solution which depends on the initial conditions, and we tend to ignore that. The other is the steady state solution, which will be an oscillation:

$$x(t) = \frac{F_0}{m Z_m \omega} \sin(\omega t + \phi), \tag{6}$$

with

$$Z_m = \sqrt{(2\omega_0 \zeta)^2 + \frac{1}{\omega^2}(\omega_0^2 - \omega^2)^2}, \tag{7}$$

and

$$\phi = \arctan\left(\frac{2\omega\omega_0\zeta}{\omega^2 - \omega_0^2}\right) \tag{8}$$

A "resonance" can occur when the damping is weak and the driving force is at very close to the resonant frequency, $\omega_r = \omega_0\sqrt{1 - 2\zeta^2}$.

For this part, there are a lot of things you can try – but the most interesting is probably to compute the results of using several different driving frequencies, and show that the resonance develops where it should if and only if $\zeta < \sqrt{2}/2$.

(d) Anything else you find interesting! Remember, of course, that you can always poke into new things, if something sparks your interest!