

## Computational physics

### Assignment: Computing hyper-volumes

A first, simple Monte Carlo exercise that is often given to students is to compute the area of a circle using random numbers. For that purpose, what you do is to take two random numbers, each ranging from  $-1$  to  $+1$ , and then determine whether the sum of their squares is less than 1. The area of the square from which you have drawn the numbers will be 4, so then the area of the circle will be  $4P$ , where  $P$  is the probability that the numbers lie within the circle.<sup>1</sup>

It is straightforward to see how to extend this technique to an arbitrary number of dimensions, to calculate the volume of a sphere, or the higher dimensional equivalents of the volume in more dimensions.

#### *Baseline assignment*

1. Write a Monte Carlo program that computes the volume of a regular sphere. Show that you get a number that approaches  $4\pi/3$ .
2. Make a plot of the error in this estimate versus the number of random numbers drawn. Plot the error versus the square root of the number of steps, and show that they are proportional to one another.

#### *Challenge problems*

1. Extend the program to work in  $n$  dimensions, and compute the volume analog for  $n$  going up to 12.
2. Compare with the analytic formulae for what these volumes should be:

$$\begin{aligned} V_k &= \frac{\pi^{k/2}}{(k/2)!} \text{for even } k \\ V_k &= 2 \frac{(2\pi)^{(k-1)/2}}{(k!!)} \text{for odd } k \end{aligned} \tag{1}$$

Note that the “double factorial” expression – the two exclamation marks – refers to the product of all the odd integers up to that point. There are a variety of ways to compute that value. One is to use recursive function calls, one is to look up the meaning of the  $\Gamma$  function, and one is to use a “brute force” approach.

3. Again, anything else related to Monte Carlo integration that you find interesting will be accepted as a challenge problem.

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<sup>1</sup>You could also do this with random numbers from 0 to 1, and then the area of the circle would be four times the probability, still, since you would have looked at only one quarter of the circle.