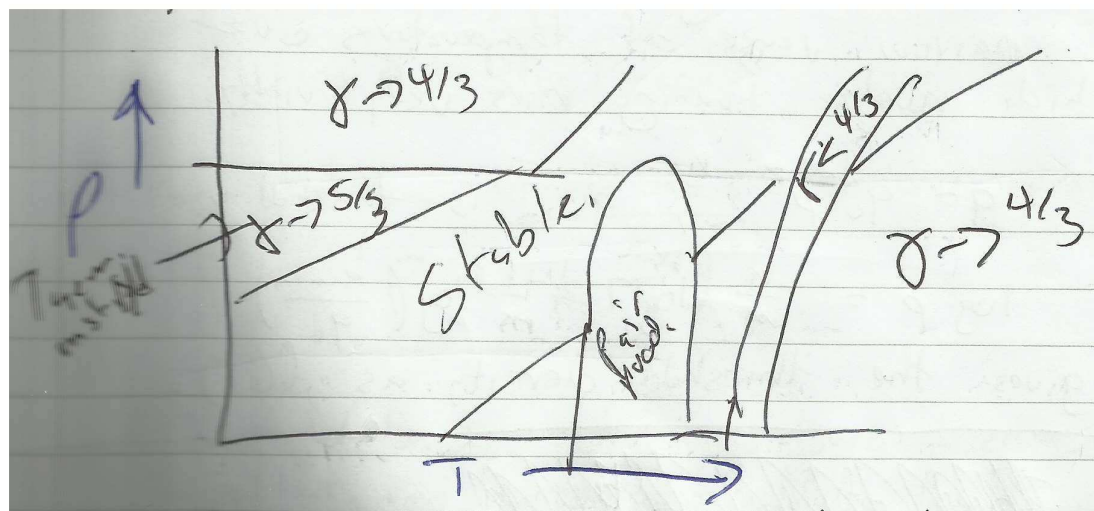


Week 7: Overview of Stellar Evolution

So far, we have focused mostly on the structure of stars – what a star looks like as a snapshot in time. In this week, we will look at the evolution of stars in a moderately quantitative way, focusing on the behavior of a single point at the center of the star. We can do this for two reasons: (1) stellar evolution is almost always governed by the nuclear burning, since the nuclear timescale is the longest of the key timescales (2) nuclear burning is dominated by what happens in the center of the star, so we can deal with inside-out evolution quite effectively.



We can set up a plane in terms of temperature and density, the two most important parameters (chemical composition being the other one that matters). We then break the plane up into regions, based on the equation of state. In the upper left, we get relativistic degenerate stars, below them, non-relativistic degenerate stars, in the middle, ideal gas, and on the right radiation pressure domination – take a look at the hand-drawn figure I’ve included here. We will also have to deal with some “special” regions, where pair production or photodisintegration of iron can affect the equation of state dramatically.

The boundary for non-relativistic degenerate and relativistic degenerate gas will just be a characteristic density. The boundary between non-relativistic degenerate gas and ideal gas can be found by setting the pressures equal – $P_{non-rel,deg} = K_1 \rho^{5/3}$, and $P = K_0 \rho T$ for ideal gas. Then, with some algebra, we find that $\log \rho = 1.5 \log T + const$ will give the boundary between non-relativistic degenerate gas and ideal gas. Similar arguments can be used to find that the relativistic degenerate gas and radiation pressure dominated gas regimes will be delineated from the ideal gas by lines with $\log \rho = 3 \log T + const$, obviously with a larger constant for the degenerate gas case than the radiation pressure domination case.

Next, we can plot the zones for nuclear burning. With the exception of the $p - p$ chain, these will mostly be nearly vertical lines in the diagram, since, except for the case of the $p - p$ chain, the nuclear burning rates are much more sensitive to temperature than to density – if we have a nuclear reaction rate law:

$$q = q_0 \rho^m T^n, \quad (1)$$

then the line in the diagram that we will draw will have slope $-\frac{n}{m}$.

Now, we can look at where in the diagram stars will become unstable. In terms of dynamical instability, this happens in the regions where γ approaches or drops below $4/3$ – the relativistic degenerate stars, the radiation pressure dominated stars, and the regions where there are changes in particle numbers due to iron photodisintegration or pair production.¹

Next, we can consider how stars will evolve through this diagram. A few weeks ago, we learned that $P_c \propto M^{2/3} \rho_c^{4/3}$ for all polytropic models, with the constant of proportionality changing very little as a function of polytropic index. It turns out that even for non-polytropes, this equation is a good approximation, so long as the star is in hydrostatic equilibrium.

Next, we can add in an equation of state. If we choose an ideal gas law, we end up with $\rho_c \propto T_c^3 M^{-2}$ – so for a given stellar mass, the star will follow a track parallel to the unstable branch tracks, as long as the mass of the star stays fixed, and the gas remains ideal, and all that happens is that the central temperature changes. The central temperature *will* change as the star evolves.

¹There is also thermal instability for any nuclear burning in degenerate gases, whether relativistic or non-relativistic, but if this happens in a surface layer only, then the star as a whole will not become unstable, so we won’t pay too much attention to this effect any more. You should know about it because classical novae and Type I X-ray bursts do interesting things, but they don’t affect global stability of white dwarfs and neutron stars.

When non-relativistic degeneracy sets in, the central density will go toward a constant, but the star will still be able to cool – it will thus follow a horizontal track in the $\rho - T$ diagram.

Now, we can consider how a star evolves in this diagram. It will start somewhere in the lower left, as a cool, low density gas cloud. It will collapse and heat, and eventually reach its “proper” track, the line with slope 3 that it will follow. It moves up that track until fusion starts. This basically halts the evolution of the star – it is maintained in a stable hydrostatic equilibrium by fusion support.

For low mass stars, the track will intersect with where the $p - p$ chain is located, while for high mass stars, with the CNO cycle. For extremely low mass objects, even the $p - p$ chain will not start. These will be either brown dwarfs or planets.

As H is burned, the chemical composition changes will lead to a very small amount of contraction, and hence movement to the upper right. The reason is that if there is less hydrogen available in the star for fusion, then the rate of fusion decreases. Then, there is less power being pumped into the star to prevent contraction, and the equilibrium configuration leads to the core of the star contracting a bit.

Eventually, the H will be exhausted, and real contraction will take place. For the lowest mass stars, this will lead to degeneracy and the star will become a helium white dwarf.²

For higher mass stars, helium fusion will start to take place. The process will continue to repeat with higher mass nuclei until the star either becomes degenerate and stops contracting, or enters into one of the unstable regions of the diagram.

We can follow the situation here: stars with M less than the Chandrasekhar mass will end their lives as white dwarfs, supported by degeneracy pressure. Stars more massive than the Chandrasekhar mass will run into either the iron photodisintegration track or the pair instability track. Stars at exactly the Chandrasekhar mass will collapse. Now, things aren’t really that simple, because we have to deal with some stars that have dramatic mass loss over their lifetimes, but it *is* true that the behavior will be described in this manner if we look at the final masses of the stars, rather than just the initial masses. It is also *mostly* true that the total amount of mass loss will be a monotonic function of the initial mass of the star, so that knowing the initial mass will allow us to predict the final mass fairly accurately.³

Theory of the main sequence

²Such low mass stars take longer than the current age of the universe to finish their nuclear burning, so we see them only in unusual cases where effects of binary stellar evolution have caused the star to evolve differently than single stars can evolve.

³For high mass stars, the thin shell instability we discussed earlier in the course may lead to some narrow mass ranges having some unusually strong mass loss – much stronger than for stars just above or below the same mass – but for the most part, there is a simple initial mass-final mass relation for stars, and the thin shell instability effects are not well understood as they are seen only in some computer simulations right now, and the observational data are not good enough to test the ideas yet.

Now, we can begin to try to develop some scaling relations for main sequence stars that go a bit beyond what we got from the pure polytrope approach before, but which are still based on relatively simple physics rather than complicated numerical simulations, and which thus can help provide more intuition about how stars work.

Observationally, we have tracks in the color-magnitude diagram, which imply that the main sequence follows a nice functional form between temperature and luminosity. This relation is nearly a power law:

$$\log L = \alpha \log T_{eff} + const, \quad (2)$$

and $L \propto M^\nu$, is also a good approximation.

So, let's show that this results from basic stellar theory.

We'll start with the basic equations of stellar structure and assume constant opacity and ignore radiation pressure.

Taking the basic equations of stellar structure:

$$\frac{dP}{dm} = -\frac{gm}{4\pi r^4} \quad (3)$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \quad (4)$$

$$\frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{F}{(4\pi r^2)^2} \quad (5)$$

$$\frac{dF}{dm} = q_0 \rho T^n \quad (6)$$

$$P = \frac{R_{gas}}{\mu} \rho T \quad (7)$$

Now, we can get crude solutions to the equations using dimensional analysis, and get characteristic numbers for each quantity. We let $x = \frac{m}{M}$, $r = f_1(x)R_*$, $P = f_2(x)P_*$, $\rho = f_3(x)\rho_*$, $T = f_4(x)T_*$ and $F = f_5(x)F_*$.

We can then add a few basic additional equations – $\rho_X = M/R_*^3$, $T_* = \frac{\mu P_*}{R_{gas} \rho_*}$, $F_* = \frac{ac}{\kappa} \frac{T_*^4 R_*^4}{M}$, and $F_* = q_0 \rho_* T_*^n M$. We put the rest of the physics off to the side in a set of differential equations in terms of $f_i(x)$, which we won't bother to solve.

We can then solve for scaling relations without making full models of stellar structure. We get:

$$T_* = \frac{\mu G M}{R_{gas} R_*} \quad (8)$$

which gives the *central* temperature;

$$F_* = \frac{ac}{\kappa} \left(\frac{\mu G}{R_{gas}} \right)^4 M^3, \quad (9)$$

which gives the same $L \propto M^3$ as Eddington's standard model, but by different arguments.

We can now look for a mass-radius relationship, which will depend on the nuclear burning process:

$$r_X \propto M^{\frac{n-1}{n+3}} \quad (10)$$

which for $n = 4$ (the $p - p$ chain) gives $R_* \propto M^{(3/7)}$, and for $n = 16$, gives a 15/19 exponent.

Now we can develop a luminosity temperature relationship and try to see what the H-R diagram should look like.

We start with $L = 4\pi R^2 \sigma T_{eff}^4$, where T_{eff} is now at the surface, and hence not clearly related directly to T_* .

Then, $L^{1-\frac{2(n-1)}{n+3}} \propto T_{eff}^4$.

Running through the algebra, we expect $L \propto T_{eff}^{5.6}$ for $p - p$ chain reactions and $L \propto T_{eff}^{8.4}$ for CNO cycle chain reactions.

We get that the $L - T$ relation is steeper on the upper main sequence than the lower main sequence, in agreement with observations.

Another thing we can do is to try to sort out a minimum stellar mass, below which a clump of gas turns into a brown dwarf rather than a star. The key here is that the central temperature of the star must be at least 4 million K in order for hydrogen fusion to take place. Now, $T \propto M/R \propto M^{4/(n+3)}$, with the exponent being 4/7 for the $p - p$ chain. Scaling from the Sun which has $T = 1.5 \times 10^7$ at its center, we find that $M_{min} = (4/15)^{7/4} M_\odot = 0.1 M_\odot$.

Now, we have made one "poor-ish" approximation here that serious affects low mass stars – that the opacity is constant. For small stars, Kramers opacity will dominate over electron scattering. If we work through the algebra carefully, we will end up with $L \propto M^5$ for such stars, if the $p - p$ chain operates in them.

Structure of stars at late times

At late times, we will see a lot of shell burning, which will mean that we will follow slightly different tracks than the ones based on core burning only.

As a red giant, for example, the core has no burning at all, but there may, or may not, depending on the mass, be shell burning of hydrogen. The core will have to contract. This will release gravitational energy – but the total energy of the star will not change, since this will happen much more slowly than a dynamical timescale, so the total energy released will be a small fraction of the total energy of the star. The envelope must then expand to deal with this release of energy from the core. Further, we can consider as an illustrative example that if a particle moves in to half its original radius, another one must move all the way out to infinity to balance the change in energy – relatively modest changes in the core size can thus lead to dramatic changes in the envelope size.

Soon, the core burning will start, and halt the collapse, and allow a new equilibrium to be established.

We will soon get to a more detailed treatment of these late stages of stellar evolution.

Now, let's think about what's missing here. From examinations of star clusters, we are fairly sure that stars of about 8-10 solar masses or less end their

lives as white dwarfs – but this is well above the Chandrasekhar limit. This is one cue that mass loss – and dramatic mass loss at that – is taking place. Another is the existence of $0.4 M_{\odot}$ white dwarfs. We know that stars so light should still be on the main sequence.

A few other key things are missing here – neutrino cooling in massive stars can dramatically affect the tracks followed at the ends of massive stars lifetimes.

What's more serious that's missing is a consideration of timescales and real observational appearances of stars. Dealing with these requires real, careful theoretical modelling that goes beyond the scope of a first course in stellar evolution. We must keep this in mind – it is the reason that most of the rest of the course will be qualitative.