Week 4: Nuclear physics relevant to stars

So, in week 2, we did a bit of "formal" nuclear physics – just setting out the reaction rates in terms of cross sections, but not worrying about what nuclear reactions actually take place. Here, we'll go beyond the formal stuff, and get into the nitty gritty. We won't cover details about why particular nuclei have higher or lower binding energies than others – you can learn that in a nuclear physics course. We'll just assume that between experiment and theory, the nuclear physicists have sorted out what happens, and see how it plays out in stars.

Let's write out an abstract nuclear reaction:

$$I(A_i, Z_i) + J(A_j, Z_j) \rightleftharpoons K(A_k, Z_k) + L(A_l, Z_l). \quad (1)$$

The energy released from this reaction, Q_{ijk} is $(M_i + M_j - M_k - M_l)c^2$. This neglects the masses of any light particles produced – but if neutrons turn into protons, new electrons are needed to have the same number of electrons and protons, and if protons turn into neutrons, the positron produced will annihilate quickly with an electron, releasing the energy as gamma-rays.

We can then rewrite the energy released:

$$Q_{ijk} = [(M_i - A_i m_H) + (M_j - A_j m_H) - (M_k - A_k m_H) - (M_l - A_l m_H)]c^2 - (A_i + A_j - A_k - A_l) m_H c^2 \quad (2)$$

with the last term naught due to conservation of baryon number.

This then motivates the definition of the mass excess: $\Delta M(I) \equiv (M_i - A_i m_H)c^2$.. Some texts will define the mass excess relative to actual proton masses, and some relative to $\frac{1}{12}{}^{12}$C. The mass excess is actually a measure of the nuclear binding energy for a particular isotope.

Now, let's look at the rate of nuclear reactions. The rate per volume per time is $n_i n_j R_{ijk}$, where R_{ijk} comes from the cross section (with a factor of 1/2 thrown in if $i = j$, to avoid double counting reactions). The energy release is $n_i n_j R_{ijk} Q_{ijk}$. The rate of energy release per unit mass is:

$$q = \frac{\rho}{m_H^2} \sum_{ijk} \frac{1}{1+\delta_{ij}} \frac{X_i}{A_i} R_{ijk} Q_{ijk}. \quad (3)$$

This represents *all* the energy released. However, some will go into neutrinos, and neutrinos will normally escape the star without interacting. Thus: $q_{net} = q_{nuc} - q_\nu$.

Neutrino cooling

Under specific and interesting, but rare circumstances, neutrino energy transport can dominate over photon energy transport, leading to extremely rapid cooling. This happens for very young neutron stars, and perhaps other types of hot neutron stars. One of the keys to understand is that while photon-photon interactions, and high energy photon electron interactions normally produce electron-positron pairs, any process which can produce e^-/e^+ pairs can also produce neutrino/anti-neutrino pairs. The cross section for this is small, but in some cases, there may be large numbers of photons and e^-/e^+ present, and there may be quasi-equilibrium reactions in which the two species are exchanged. The small fraction of reactions that then produce $\nu\bar{\nu}$ can take away substantial amounts of energy. These processes are only important at very high temperature and density.

Now, Q_{ijk} is a measure of the change in total binding energy. Nuclei can emit β^- of β^+ to change into more stable nuclei. They can also have nuclear excited states analogous to the excited states of atoms.

There's a general trend of increasing binding energy up to ^{56}Fe, and decreases for larger nuclei. The curve of binding energy versus baryon number is much steeper when going toward iron from H than toward iron from heavier elements. There are "jagged" parts of the curve as well. For example, more energy is released when going from H to He than from e.g. He to C. This explains why stars emit most of their radiation on the main sequence.

Now, on to actual nuclear reactions. The rate can be described as a product of a "typical" velocity and the cross-section at that velocity, but it's more properly an integral of the cross-section as a function of velocity, time velocity, over velocity.

It's instructive to look at the expected rate based on comparing the typical energy in a Maxwellian with the Coloumb barrier for getting to the size scale of the nucleus of the atom (once the lighter nucleon penetrates into the nucleus of the heavier atom, the strong nuclear force takes over). It turns out that the nuclear reaction rate should be extremely slow. This was known in the 1920s, and was one of the key reasons why nuclear energy generation wasn't accepted. The key breakthrough was made by George Gamow in 1928, and then appled to stars by Atkinson & Houtermans in 1929 – it was quantum tunnelling, which dramatically increases the probability that a charged particle will penetrate the nucleus of another atom.

The penetration probability is:

$$exp\left(\frac{-\pi Z_i Z_j e^2}{\epsilon_0 h v}\right), \tag{4}$$

so the cross-section is something like:

$$exp\left(\frac{-\pi Z_i Z_j e^2}{\epsilon_0 h v}\right) exp\left(\frac{m_g v^2}{2kT}\right). \tag{5}$$

To get the interaction rate, one must integrate over v, and the end result is:

$$\zeta v \propto (kT)^{-2/3} exp\left[-\frac{3}{2}\left(\frac{\pi Z_i Z_j e^2}{\epsilon_0 h}\right)^{2/3} \frac{m_g}{kT}^{1/3}\right]. \tag{6}$$

This just gives a cartoon description of what's really going on, so mostly, it's just useful for understanding the basic idea that fusion of heavier nuclei requires higher temperatures. There can be strong deviations from this relation if there is a nuclear resonance. A famous example will be discussed shortly.

Now, we can define a characteristic timescale to deplete a species: $\tau_i \equiv (n_j R_{ijk})^{-1}$. If the reaction between i and j is the dominant reaction supplying the luminosity of the star, then τ_i should be close to the nuclear evolution timescale of the star. Next, we note that nuclear reaction rates are often parameterized by power laws: $q \propto q_0 \rho T^n$, where n is determined by comparison with the actual rates. They aren't really well described by power laws over the full range of parameter space, of course.

1 p-p chain

The dominant nuclear burning process in low mass stars in the proton-proton chain. The net reaction is that four protons combine to form ^{4}He and release two positrons, but this obviously cannot happen directly. For quite some time, it was unclear how the chain would proceeed. Eventually, Hans Bethe (1939) found that the weak nuclear force allowed $p + p$ to produce deutrerium plus a positrion and a neutrino, starting the chain. The full chain is quite time consuming to type in, and is accurately described on Wikipedia, so please feel free to look there if you don't have a book.

The three key branches are one which goes through 2 ^{3}He called p-p I, one which goes through ^{7}Li, called p-p II, and one which goes through ^{8}B called p-p III. The latter is energetically unimportant, but very important to the solar neutrino problem because it produces high energy neutrinos. The branching ratios for the three p-p chains depend on temperature, density and chemical composition.

The energy release is $Q_{p-p} = 4\Delta M(^1\mathrm{H})$ - $\Delta M(^4\mathrm{He})$, which is 26.73 MeV. Under typical conditions, about 26 MeV goes to heating the star and a bit less than 1 MeV goes into the neutrinos, but the Boron decay releases 7.2 MeV neutrinos.

The rate is determined by the slowest part of the chain, which is the first part, and has a characteristic timescale of 10^{10} years for a solar-type star. $q_{p-p} \propto \rho T^4$.

2 CNO cycle

The CNO cycle is composed of two "bi-cycles" (note that these are not bicycles ☺). Some references break this down further into three or more cycles. The essence of this is that carbon, oxygen and nitrogen fuse with protons, but over the full course of the cycle, the only *net* reaction is the fusion of hydrogen into helium plus positrons and neutrinos. In one of the pathways, there is a brief production of fluorine, which quickly decays.

A few important qualitative trends exist for the CNO cycle. At no point is the total number of atoms in CNO(+F) changed. At high temperatures, the radioactive decays can act as the bottlenecks, while at low temperatures, the fusion of nitrogen will usually be the bottleneck. As a result, the CNO cycle usually results in nitrogen enhancement, even if nominally, going all the way through the cycle leads to no change in the relative C,N and O.

Because the CNO cycle requires overcoming larger potential barriers than the p-p chain, it has a stronger temperature dependence: $q_{CNO} \propto \rho T^{16}$. This means it tends to be the dominant fusion process in massive stars, while the p-p chain dominates in low mass stars.

3 Helium burning

Helium burns via a process called the triple α reaction. Most of the elements between helium and carbon are unstable, or at least not energetically favorable, and this is especially true of ^{8}Be, which is what He fuses to. The exponential decay time for ^{8}Be is only 2.6×10^{-16} seconds. This is shorter than the mean free time for α-particles at 10^8K, so ^{8}Be can fuse with another α to make ^{12}C, but the rate is expected to be quite low.

On the other hand, carbon and α particles fuse relatively quickly to make oxygen. If the production rate of carbon were what is "expected", based on just the standard tunneling probabilities, then one would expect there not to be much carbon around. Fred Hoyle noticed this, and predicted that there must be an excited state of carbon at 7.65 MeV, so that a resonance exists and the helium + beryllium reaction rate is dramatically enhanced. He insisted that experimental nuclear physicists look for this resonance solely on the basis of his intuition and astrophysical evidence. They looked, and he was correct.

So:

$$Q_{3\alpha} = 3\Delta M(^4He) - \Delta M(^{12}C) = 7.3MeV \tag{7}$$

The energy release per nucleon is thus about 10% of that for hydrogen fusion – this is not surprising, since on the asymptotic giant branch, where stars are fueled by He fusion, the luminosities are about 10 times those for main sequence stars of the same mass, and the lifetimes are about 100 times shorter.

Another key issue is that the nuclear power generation rate scales with ρ^2, since first the beryllium must be produced, and then the carbon, in quick succession, so $q_{3\alpha} \propto \rho^2 T^{40}$.

The next reaction is fusion of carbon into oxygen, after which things become more complicated. Not all the carbon will be fused into oxygen before the star becomes dense enough and hot enough that carbon-carbon fusion stars. At slightly higher temperatures (10^9 versus 5×10^8K), oxygen-oxygen fusion starts. Carbon fusion produces a combination of magnesium, sodium, neon, and, oxygen, while oxygen fusion produces sulfur, silicon, phosphorous, and magnesium. If the star is massive enough, the main product will be silicon.

4 Silicon burning

In principle, it seems like the fusion of two ^{28}Si nuclei into ^{56}Fe, the stablest nucleus should happen. It doesn't; the Coloumb barrier is too strong. Before that happens, the photon field in the star becomes so strong that silicon is photodisintegrated, as can be some other elements. Any reaction that produces a gamma-ray can be run in reverse to allow photodisintegration. Silicon photodisintegration happens at 3×10^9K, breaking Si up into relatively light particles. These can fused with the surviving Si nuclei to produce the elements heavier than Si. The gradual build-up of the elements heavier than Si is much easier to accomplish than Si-Si fusion.

The end result is that the star starts to approach "nuclear statistical equilibrium", in which all the different quantum states of all different isotopes will exist in proportion to their Boltzmann factors. Often the end result will be more of the iron peak elements (iron, cobalt and nickel) than actual statistical equilibrium predicts, because the reactions which destroy those elements run very slowly for temperatures less than about 7×10^9K, and silicon burning is generally happening only when the star has a relatively short amount of time left to live.

5 Heavy elements and other relatively rare elements

Neutron capture is a key process for producing certain elements, especially those heavier than iron. If free neutrons can be produced, then they will quickly be captured by nuclei. Configurations with roughly equal numbers of protons and neutrons are energetically favored (although for highly charged nuclei, somewhat higher numbers of neutrons are preferred). As a result, β^- decays are expected as nuclei capture neutrons – typically about one decay for every two neutrons captured. Exactly what proton/neutron ratio results depends on the rate of neutron capture. Slow neutron capture produces the $s-$process elements – those where β decays happen much faster than the capture of additional neutrons. The $r-$process elements are more neutron rich, as they are formed in scenarios with large numbers of free neutrons, so that the neutron capture rate is faster than the β decay rate – in particular, the $r-$process elements are mostly produced near the surface of a neutron star.

6 Pair production

At very high temperatures, pair production can be a significant process in stars. Photons can interact with particles, or with one another, to produce electron-positron pairs. This is largely only an important process for kT greater than 1.022 MeV (the rest mass of a pair). At $T \sim 5 \times 10^9$K, the positron to electron ratio can be significant. This can affect, e.g. the ratio of ion pressure to electron pressure.

7 Fe photodisintegration

$^{56}\mathrm{Fe} \rightarrow 13^4\mathrm{He} + 4n$ can take place. It absorbs 124 MeV of energy. For temperatures of more than 7×10^9K, helium starts to be more abundant than iron again! We can thus see that we start fusion around 10^6 K, and destroy its results a bit below 10^{10}K, and, not surprsingly, these tend to define the temperature range in the cores of stars.