

Some notes on the notes: when I present a hand-drawn figure in lectures, please don't expect that to be reproduced in the notes – I'll expect you to sketch those and take some skeletal notes of your own in class. Where I include graphics, they are usually taken from the web, and if I shown an image on the screen in class, I generally will put that image in here, since the main reason I'll show images is that they are too tough to draw on the board.¹ The notes will be to help guide you, and should not be taken as a transcript of what was said in class. They are not a replacement for reading the Prialnik book.

Week 1: Motivations for studying stars, and fundamental measurements of stars

The key goals for this course will be to help you to understand the physics that governs the structure and evolution of stars, to place stars into a broader context in astronomy, and to help you to be aware of some of the modern problems in stellar astrophysics. The emphasis will be on understanding the physics of stellar structure and evolution.

Some key reasons why stars are important:

- Production of the chemical elements – the Big Bang produces most of the net hydrogen, helium and lithium in the Universe. By “net” I mean that, for example, hydrogen fuses to helium in stars, but the helium often fuses later into heavier elements, and the helium content of the Universe. Everything else is produced in stars (or in supernovae, which are the last phase of the lives of massive stars)
- Production of most of the light in the Universe – if you want to understand where the light comes from you must understand stars. Accretion disks around supermassive black holes produce a non-trivial minority of the light in the universe, but stars definitely dominate the total.
- Tracers of the Universe – much of cosmology involves finding tracers of the distance scale of the Universe. These tracers are almost always stars of some kind, or stellar explosions of some kind, although there are some exceptions.
- Dead stars – black holes and neutron stars – are sites for studying extreme physics. Black holes are one of the few ways in the Universe to understand strong field general relativity. Neutron stars give important probes of general relativity, of the physics of dense matter (relevant for understanding particle physics), and of the physics of extremely large magnetic fields.
- Feedback of energy into the interstellar medium – stellar winds and supernovae can help heat and move gas around in galaxies and affect the evolution of galaxies.

¹I will be negligent in providing citations for these images in some cases. These notes are not formal academic writing, so don't take a lack of citation here as a license not to cite figures yourself. I really should be citing these, too.

- Nuclear physics – nuclear reactions fuel stars. Thus we can sometimes hope to learn about nuclear physics based on what happens in stars. A classic example is the “Hoyle state” of carbon-12. Ordinarily, it should be hard to produce carbon. However, there’s a resonance for an excited state of carbon-12 – an alpha particle plus a beryllium nucleus have almost exactly the same energy as this excited state, making the cross-section for the reaction much larger than for the reaction that produces the ground-state of carbon-12. Fred Hoyle predicted this on the basis of the fact that carbon exists in the universe, so there must be some way to crank up the cross-section for making it. It was only later verified experimentally.
- Must understand star formation to understand planet formation. Also, though, it’s important to understand stars in detail to find planets around the stars. The first extrasolar planets were met with worries, because the radial velocity variations could have been stellar pulsations instead.
- Most importantly, we astronomers, as pure scientists, should be interested in anything that happens in the sky, and should want to try to be able to explain it.

Fundamental measurements of stars

Key properties of stars:

Mass, radius, luminosity, temperature, chemical composition (sometimes consider all elements separately, sometimes lump all metals into metallicity, sometimes consider the α elements and the iron-peak elements as separate groups of metals), rotation rate, magnetic field, age, multiplicity

These quantities pretty much define a star. There can be some exceptions for the cases of stars which have undergone mass transfer. Also, in the very late stages of evolution of stars, there can be unusual chemical composition gradients due to convection and pulsations that can cause mixing to take place in ways that are far different from what happens on the main sequence. But, for the most part, if you know the values for all the numbers listed above, you know pretty much everything about a star. Now let’s look at how we go about measuring these things.

I’ll focus on the *model-independent* ways to get at these quantities. These are the ones used to build up a picture of stellar phenomenology (e.g. the Hertzsprung-Russell diagram, and an understanding e.g. of what mass stars have at what place on it). Once a picture of how stars behave has been built up from the data, model dependent techniques can be developed. These can benefit from indirect means for measuring quantities which can be easier to make than the direct measurements, but they cannot be used from the beginning to build up a picture.

Mass measurements

Mass measurements come from Kepler’s laws in binaries. We covered this quite extensively in Intro to Astronomy, and you should have also seen Kepler’s

Laws in classical mechanics as well. We'll refresh our memory about applying Kepler's laws on the problem sheet.

Radius measurements

Eclipsing binaries are a good place to build up measurements of radii of stars. The durations of the ingresses of the eclipses and of the eclipses themselves tell you about the radii of the stars. You also need to know what the orbital solutions are for the stars in the binary, too, to convert a time into a distance travelled.

For certain other types of stars, there are other direct radius measurement techniques. For the very nearest and largest stars, one can use optical or infrared interferometry to image the stars directly.

For pulsating stars, there is a technique called the Baade-Wesselink method. We didn't talk about it in lecture, but it's interesting. It's also one of these ideas that is quite straightforward to understand, but which must have taken a great deal of cleverness to have been thought of in the first place. The basic idea is as follows: (1) the stellar radius is changing (2) repeated measurements of the Doppler shift of the star will give the velocity of the envelope of the star (3) one can subtract off the mean velocity over the pulsation cycle, and then one gets the velocity of expansion or contraction (4) one can integrate the velocity of expansion between the time of minimum radius of the star and maximum radius of the star, giving the difference between the minimum radius and the maximum radius (5) by looking at the flux divided by σT^4 at minimum radius versus maximum radius, one can get the *ratio* of the minimum to the maximum radius (6) with the difference between the minimum and maximum radius and the ratio of the radii, one can solve for both radii. This thus turns into a technique which can give a model independent radius for the stars. It also, then, provides a calibration for the use of pulsating stars as distance indicators.

Luminosities of stars

Luminosity is straightforward to measure, if you know the distance to a star. $L = 4\pi d^2 F$, where L is the luminosity, d is the distance, and F is the flux of the star. The flux must be corrected for extinction from the interstellar medium. One must also be careful to get the bolometric corrections right, if one doesn't measure the flux across a bandpass which includes all of the star's light.

Rotation rates

There are a few ways to get rotation rates of stars. One must worry, of course, about whether the star is rotating differentially. Generally, what will be easiest to measure is the rotation rate of the surface of the star near the equator. We know that the Sun has a different rotation rate near the poles than near the equator (although not by a large factor). We also have good reason to suspect, from helioseismology, that the core of the Sun rotates at a different rate than the surface does.

Star spots are a good tracer of a star's rotation rate, although not all stars have spots. If a star does have spots, they will darken the star by enough that, with good data, the brightness of the star will vary periodically on the rotation period. If you want the rotational velocity of the star, rather than the rotational period, you still need to get the star's radius.

Stars with radiative, rather than convective envelopes, and old stars, even

with convective envelopes, tend to have few or no starspots. Also, the Sun has a magnetic cycle over which the number of starspots changes dramatically, and other stars probably do as well, but only about 25 stars other than the Sun have had their magnetic cycles measured. For stars without spots, other techniques must be used.

The most widely applicable is to measure the rotational broadening of the star's absorption lines, caused by the Doppler effect. However, this gives only the radial component of the rotational velocity. The inclination angle, which must be measured to solve for the actual rotational velocity, is almost impossible to measure. One can thus get the rotational velocities in a statistical sense for a group of stars, but getting the individual stars rotational velocities is rarely possible using this method.

Ages of stars

Ages of stars are extremely important to try to measure, but quite hard to measure.

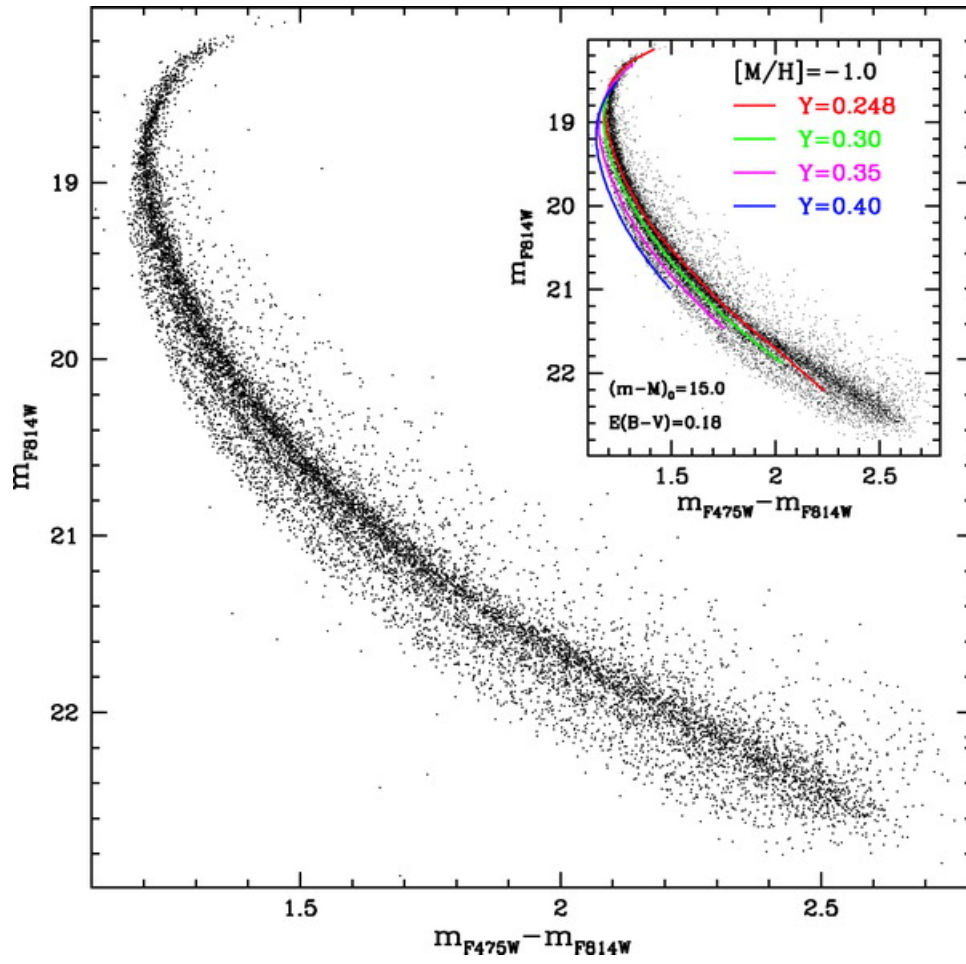
In star clusters, one can assume the stars are all the same age, and use the main sequence turn off to get at the age of the star cluster. This is the most commonly used way to get at the ages of stars, but it's highly model dependent. It relies on assumptions about the evolutionary timescales of stars which come from stellar evolution models. It also relies on the assumption that the stars really are all the same age. One of the most exciting developments in stellar physics recently is the discovery that the stars in a cluster are *not* all the same age – the discovery of multiple main sequences in certain globular clusters – see figure .

Neither of these assumptions is too bad – the multiple main sequences are exciting because they tell us that something in the standard textbooks for the past century is wrong – but the ages of the different sequences are all within a few percent. Stellar physics is now very well calibrated, so that's also not a big concern. The real problem with using star clusters is that most stars are not in clusters – or at least not in clusters large enough to measure a turnoff age to high precision (e.g. in a cluster of 50 stars, there might be quite a large range of ages possible between the turnoff age for the most massive main sequence star and that for the least massive evolved star).

As a result, a new technique is needed. About 5 years ago, the concept of gyrochronology was developed – using stellar rotation rates to measure the ages of stars. It was well known since the 1960s that the rotation periods of stars of a given mass scale as the square root of the stellar age. This result comes from calibrating using clusters. What wasn't well developed was how to set the constant of proportionality, nor how precise the ages from the method would be, but these details have now been recently worked out.

The general thought for why this technique works is that stars slow down their rotation due to a process called magnetic braking. The stellar wind is blown through the stellar magnetic field, meaning that the magnetic field is sweeping through a sea of charged particles. It is not well understood why this leads to the square root law, but empirically, it does.

Magnetic fields of stars



Magnetic fields of stars are measured using Zeeman splitting, and some sophisticated variations on Zeeman splitting. Very few measurements exist. We won't discuss stellar magnetic fields much in this course, but I thought you should just be aware of the word so you know what to look up if you ever want to read about stellar magnetic field variations.

Building up a picture

So, after one knows a bit about stellar distances, one can make a diagram of luminosity versus some measure of temperature (e.g. colors) of stars. This diagram is the Hertzsprung-Russell diagram. It can be used to group stars into temperature classes and luminosity classes, and this was realized early on - the spectra of stars at different places in the diagram look different, while the spectra of stars near one another look very similar.

Getting rough measurements of the temperature from the colors is straightforward. The luminosity must then obviously be related to the radius. So a good question is how the radius affects the spectrum.

It turns out that for a given color, the mass range is quite small compared to the radius range. The radius is thus related closely to the surface gravity and hence the surface pressure of the star.

Radius of the star II

Changing the pressure in a stellar atmosphere can have two broad classes of effects. One is that at high pressure, atoms and molecules are more likely to re-combine. This is basic thermodynamics - when the density is large, the Boltzmann factor becomes more important, and the extra entropy associated with having free particles becomes less important. Small radius, high pressure cool stars thus have more molecules. This can be seen strikingly in figure .

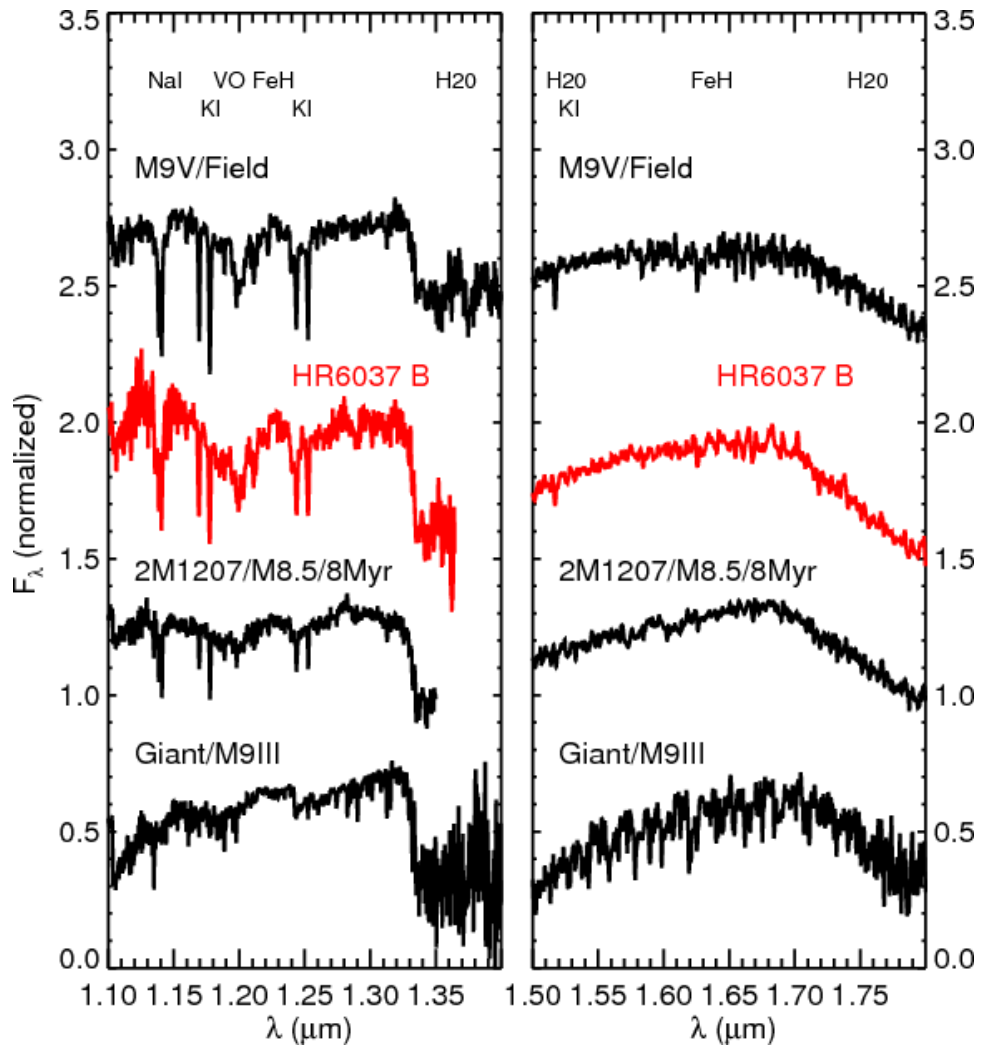
The other important issue is pressure broadening of lines. In most classes of stars, the Stark effect can be quite important. The wavelengths expected for atomic transitions are normally computed in a vacuum, and measured at low density. At high pressure, there will typically be some free electrons and some ions near the atoms undergoing atomic transitions. The electrons in the atom will feel forces not just from their "host" nucleus, but from the free electrons and ions. This will lead to a perturbation in what the energy levels are in the atom, broadening the line (since the perturbations will be by different amounts and with different signs for different atoms).

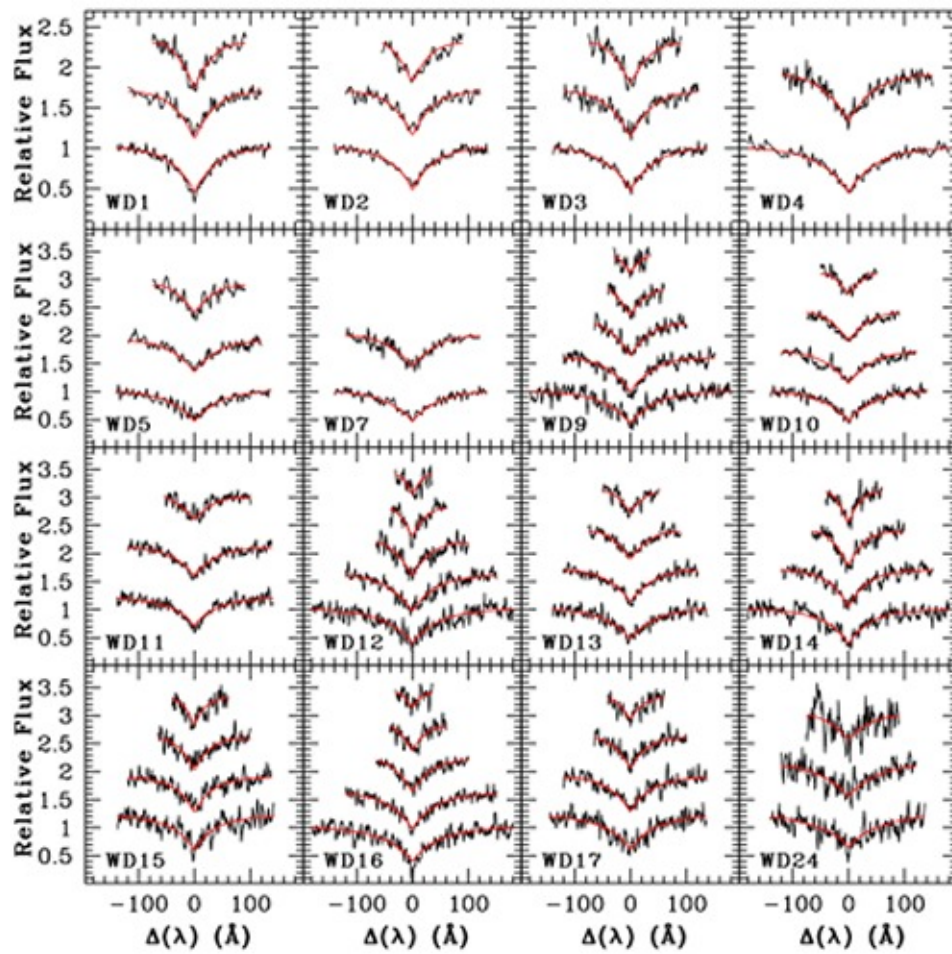
Pressure broadening is usually the most significant source of broadening in stars, but its strength varies depending on the specific atomic transition. Other sources of broadening include thermal doppler broadening, rotational doppler broadening, and turbulent doppler broadening. The Heisenberg uncertainty principle also contributes to the width of lines.

The effects of pressure broadening can be seen by comparing the Balmer lines of a white dwarf with those from a solar-like star in figure . The white dwarf lines are much broader, reflecting the much higher surface gravity of the white dwarf.

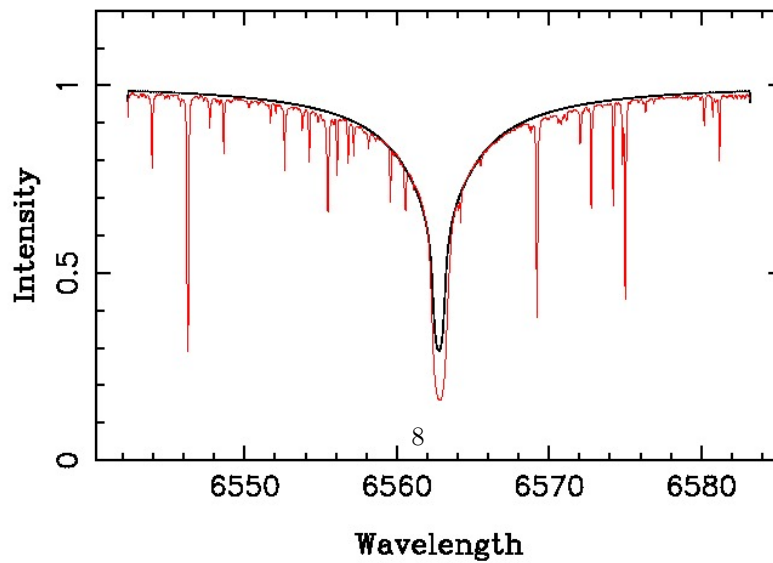
Temperatures of stars

First, we have to consider how to define the temperature of a star, since different parts of the star will have different temperatures. Theorists prefer using





Stehle (Stark) + BPO (neutrals)



a measure called the effective temperature, defined from $L_{bol} = 4\pi R_*^2 \sigma T_{eff}^4$. This has the advantage of being more or less what comes out of theoretical stellar interior models, but has the disadvantage of requiring knowledge of both the star's radius and its bolometric luminosity, both of which are hard to measure.

In the first year course, we talked about how to get temperatures by assuming a star is a blackbody, and measuring the peak wavelength, then using Wien's displacement law, $\lambda_{max}T = 2.9\text{mm K}$. This temperature is called the *color temperature*. The main problems with this are that stars aren't perfect blackbodies, and that many stars are reddened, and the reddening corrections aren't that easy to make just from the colors of the star, so color temperature measurements are more subject to inaccuracies than temperature measurements based on spectral lines – but they are a lot easier to make, since spectra are a lot harder to get than colors.

The most robust observational method for getting temperatures is by looking at the strengths of certain absorption lines. These can be converted into temperature measurements by using the Saha equation. Meghnad Saha was the first modern astrophysicist of real significance who came from outside Europe or North America, so this result has some cultural interest as well.

The Saha equation is:

$$\frac{n_{i+1}n_e}{n_i} = \left(\frac{2}{\Lambda}\right) \left(\frac{g_{i+1}}{g_i}\right) \exp\left(\frac{\Delta E}{k_B T}\right) \quad (1)$$

where n is the density of a species, g is the number of quantum states for a species, e stands for electrons, $i + 1$ stands for ions with charge $i + 1$, i stands for ions with charge i (or atoms for $i = 0$), ΔE is the change in energy between the free and bound state, k_B is Boltzmann's constant, T is the temperature, and Λ is the De Broglie wavelength of the electrons.

Saha derived this equation from equations in physical chemistry used to understand equilibrium concentrations of chemicals. There are a few implicit assumptions. One is that the stellar atmosphere is in local thermodynamic equilibrium. Another is that the density is small enough that charge screening by free electrons is not important. These are usually, but not always, reasonably well satisfied in stellar atmospheres. One can also see that one needs to get an idea of the pressure before trying to measure the temperature, so that one knows how to scale the number densities.

Chemical compositions

These will generally be the last things to measure. The reason is that you might be able to measure the number of atoms in a particular quantum state from the strength of an absorption line, but to figure out the fraction of the atoms of that element in that quantum state, you need to know the temperature and the pressure.

In star clusters, one can look at the color of the main sequence stars or the red giants stars, or of the integrated light and make a model dependent measurement of the “chemical composition” of the cluster, under the assumption that the stars all have the same composition. Like ages, this is a good, but imperfect

assumption when examined in detail. Additionally, one cannot, from colors alone, make a good estimate of the relative abundances of different metals.

What we really measure

It is useful to think about what we measure when we look at a star's spectrum. Basically, what we measure is the blackbody curve's flux for the temperature of the layer of the star from which the optical depth is unity. When there is a low opacity at a wavelength, then we see deeper into the star, where the star is hotter, and we see more flux. A consequence of this is that the relation between the equivalent width of an absorption line and the column density of atoms in that species is not always going to be linear. Another consequence is something called limb darkening – that the “edges” of a star's disk projected on the sky are cooler and fainter than the center of the star's disk projected on the sky.