

Week 8: Stellar winds

So far, we have been discussing stars as though they have constant masses throughout their lifetimes. On the other hand, toward the end of the discussion of what happens for post-main sequence evolution of stars, we realized that stars which have masses larger than $1.4 M_{\odot}$ end their lifetimes in some kind of catastrophic state, undergoing supernova explosions, or at least undergoing formation of neutron stars. At the same time, in star clusters where the main sequence stars are well above $2M_{\odot}$, we see young white dwarfs in many cases. Also, the supernova rate that we see in the Universe isn't nearly as high as what we would expect based on a naive interpretation of the picture we looked at last week.

Something is clearly incorrect in our picture. The answer turns out to be that stars lose mass over their lifetimes. Massive stars tend to lose a lot of their total mass, and to lose mass even on the main sequence. Lower mass stars tend to lose much smaller fractions of their total masses, and to lose mass primarily when they have evolved into giants.

Observational evidence for mass loss

Now, instead of just invoking mass loss as a convenient way to explain the discrepancies between our simple picture of stellar evolution and the observations of, for example, white dwarfs in young-ish star clusters, we would like to see direct evidence that stars lose mass. One of the most prominent and earliest examples of evidence for stellar mass loss is that some stars show what are called P Cygni profiles in their spectra. P Cygni is a particular variable star, and often astronomers classify variable stars by their prototype object.

A P Cygni profile is a line profile where the same atomic transition is seen as a blueshifted absorption line and a redshifted emission line. This can be explained in terms of a stellar wind as follows:

1. The star is losing mass, in the form of a wind that moves radially outward from the surface of the star.
2. The gas in the outflow absorbs light from the star's continuum. In some cases, this absorption will take the form of an atomic transition. The excited states will decay, and re-emit light at the same wavelength.
3. The absorption will be of light moving outward, away from the star. The emission will be in a random direction.
4. Therefore, when the light is moving directly from the star toward the observer, there will be an absorption line. Every other direction will show only an emission line, because the absorbed light wasn't coming toward the observer anyway, but some fraction of the re-emitted light will come toward the observer.
5. If there's a stellar wind, the absorbed light will preferentially be blue shifted, since the wind in that direction will be moving toward the observer. The wind in all the other directions will be, on average, redshifted, but also spread over a broader range of wavelengths.

Very detailed theoretical modelling of the line profile can tell us something about the wind's velocity and density structure and give the mass loss rate. We won't learn how to go about the actual mechanics of doing that in this course.

The range of stellar mass loss rates turns out to be quite large. In some cases, like for the very most luminous stars, the mass loss rate can be $10^{-4}M_{\odot}$ per year. Since these stages of stellar evolution can last of order millions of years, stars can lose ten or more solar masses of material through stellar winds. On the other hand, the Sun is a more typical star in terms of stellar mass loss rate, and its mass loss rate is about 2×10^{-14} solar masses per year – the mass loss from solar-type stars has essentially no impact on their *main sequence* evolution – but as we will see later, mass loss after the main sequence can start to become much more important.

Non-steady state models

To deal with stellar winds, we have to make some fundamental changes to how we calculate things for stellar models, relative to what we were doing before. Up until now, we have assumed that the continuity equation is the same as conservation of mass. Now, we will have to deal with the cases where mass is not conserved in the star. It is, of course, still conserved in the Universe. One of the key consequences of this is that we won't be able to integrate over mass to get out quantities for the star, but we will instead have to integrate over volume. We will use the same notation we have before in the course, where d differentials are with respect to space/mass, and δ differentials are with respect to time. A key difference now will be that the time derivatives won't all be expected to vanish once we sort everything out. So, over δt , we expect $\delta(\rho dV) = dV\delta\rho$ to be the net mass flow.

We define \mathcal{J} to be the mass flux, and set it equal to $4\pi r^2\rho v$, where v is the flow velocity. We can see in a straightforward manner that \mathcal{J} will have units of mass per unit time.

Then:

$$dV\delta\rho = \mathcal{J}(V)\delta t - \mathcal{J}(V + dV)\delta t. \quad (1)$$

The equation above essentially states that the change in the density in a volume element in a unit of time is equal to the mass flux over a unit in time into that volume element minus the mass flux out of the volume element in the same unit of time. We can differentiate with respect to time and volume and get a new continuity equation:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\mathcal{J}}{\partial V} = 0. \quad (2)$$

For steady state flows, the local density remains constant, even as the mass moves around, so the first term of the continuity equation vanishes, and that implies that \mathcal{J} does not change as a function of radius, so we can state that:

$$\mathcal{J} = 4\pi r^2\rho v = \dot{M}, \quad (3)$$

and that \dot{M} is a constant.

Now, since we have net flows, we need to consider both the time and space derivatives of other properties of the gas. For an arbitrary quantity f , we can write down:

$$\dot{f} = \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} \quad (4)$$

From much earlier in the course, we had derived:

$$\ddot{r} \Delta m = -\frac{Gm \Delta m}{r^2} - \frac{\partial P}{\partial r} \frac{\Delta m}{\rho}, \quad (5)$$

as the “equation of motion” for a star – the description of how the a spherical shell of material would move around. We can rewrite \ddot{r} as \dot{v} . For a steady flow, $\frac{\partial v}{\partial t} = 0$, so $\dot{v} = v \frac{dv}{dr}$, so we can express the velocity derivative as:

$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM}{r^2}. \quad (6)$$

This equation can still be thought of as an expression of conservation of momentum – it’s just that now it’s conservation of momentum per unit mass.

Energy conservation

We can now look at how to treat energy conservation in this context.

$$\dot{u} + P \left(\frac{\dot{1}}{\rho} \right) = q - \frac{\partial F}{\partial m} \quad (7)$$

gives the general form of conservation of energy that we considered in Chapter 2. This has two quantities with time derivatives, so we will have to consider them separately.

$$\dot{u} = \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} \quad (8)$$

expresses the internal energy term, and

$$P \left(\frac{\dot{1}}{\rho} \right) = P \left[\frac{\partial}{\partial t} \left(\frac{1}{\rho} \right) + v \frac{\partial}{\partial r} \left(\frac{1}{\rho} \right) \right] \quad (9)$$

Next, in the regions where a wind is being ejected from a star, it is safe to assume that there is no nuclear energy generation. Also, we have previously considered the flux gradient with respect to mass; we must re-write this in terms of radius as $\frac{1}{4\pi r^2 \rho} \frac{\partial F}{\partial r}$. We then get:

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} + P \left[\frac{\partial}{\partial t} \left(\frac{1}{\rho} \right) + v \frac{\partial}{\partial r} \left(\frac{1}{\rho} \right) \right] = -\frac{1}{4\pi r^2 \rho} \frac{\partial F}{\partial r}, \quad (10)$$

and for a steady flow, the explicit time derivatives will go to zero, so that:

$$v \frac{\partial u}{\partial r} + P v \frac{\partial}{\partial r} \left(\frac{1}{\rho} \right) = -\frac{1}{4\pi r^2 \rho} \frac{\partial F}{\partial r}. \quad (11)$$

Developing simple relations

Now, if we take the momentum equation and multiply through by the velocity (which gives it units of energy) we can then add that equation to the energy equation above. Then we apply three simple algebra tricks – setting $v \frac{dv}{dr} = \frac{d}{dr} \left(\frac{1}{2} v^2 \right)$; $\frac{GM}{r^2} = -\frac{d}{dr} \left(\frac{GM}{r} \right)$, and $\frac{1}{\rho} \frac{dP}{dr} + P \frac{d}{dr} \left(\frac{1}{\rho} \right) = \frac{d}{dr} \left(\frac{P}{\rho} \right)$.

Then with a little more rearrangement, we can get:

$$\dot{M} \left(\frac{1}{2} v^2 + u + \frac{P}{\rho} - \frac{GM}{r} \right) + F = \text{constant} \quad (12)$$

This is all to say that we conserve all the different kinds of energy flow in the wind – bulk kinetic energy flow, internal energy flow, rate of work done against ambient gas pressure, rate of work done against gravity, and rate of photon energy flow.

Summary of qualitative ideas

So some key ideas to take away from all the mathematics here are that (1) energy can be conserved even in a fluid flow, where the properties of the fluid are changing with radius – it’s just a bit more complicated to account for everything than it is for a static star and (2) we can look now at some of the tradeoffs among the different forms of power that can be in the wind to understand some of their properties.

Energetics of wind: where does the speed come from?

One of the things we want to think about is how the winds are accelerated. Observationally, a few things tend to be true most of the time: the outer parts of the stellar wind are faster than the inner parts; the winds are “overpressured” relative to the interstellar medium; and the winds are hot. There are exceptions to these statements, but they are usually true.

The finding that the winds are accelerating as they move outwards might be a bit of a surprise to many students. The two most obvious sources of force, gravity and external pressure, should work to slow the winds down, not to speed them up. The internal energy will not be useful for accelerating the wind, even if it can be reduced, because there isn’t net momentum available in random motions. The answer then, is that the radiation pressure from the star exerts a force on the gas in the wind and that force is stronger than the forces slowing down the winds, at least until the wind gets far from the star.

Let’s do a quick energetic calculation to see if we can pump bulk kinetic energy into a stellar wind in a manner that will power its wind. A relatively extreme case for having a strong wind is that of a $30 M_{\odot}$ star with $\dot{M} = 10^{-6} M_{\odot}/\text{yr}$ and $v = 1000 \text{ km/sec}$.

First, let’s look at the radiative luminosity of the star. For massive stars, $L \propto M^3$, so let’s take the luminosity to be about $30^3 L_{\odot} = 2.7 \times 10^4 L_{\odot}$, or about 10^{31} W .

Next, let’s look at the kinetic power of the wind. $M_{\odot} = 2 \times 10^{30} \text{ kg}$, so $\dot{M} = 2 \times 10^{24} \text{ kg/yr}$, or about $7 \times 10^{16} \text{ kg/sec}$. Then $\frac{1}{2} \dot{M} v^2$ will be about $4 \times 10^{28} \text{ W}$ – less than 1% of the radiative power. We can turn a small amount of the light from the star into a stellar wind that qualifies as one of the most

powerful kinetic outflows in the Universe. We should also remember here that since the light from the star will mostly pass through its Galaxy out into the larger Universe, and the wind will interact with other gas in the Galaxy and blow out bubbles that the consequences of the wind are quite important even though they represent a small fraction of the total power.

Let's also look at the total mass loss here: The nuclear timescale for the star will be about $(30^{-2})(10^{10})$ years, or 10 million years. Over that time, we can expect the star to lose about 1/3 of its mass.

Phenomenology of how winds are accelerated

So, now let us go a bit beyond just energetic arguments and figure out how the winds are really accelerated. It is often assumed (and often found to be a reasonably good approximation) that the winds from a star have a uniform temperature. That temperature is surprisingly high compared to the temperature of the outer part of the star. Temperature is directly related to the sound speed in a gas, $v_s = \frac{RT}{\mu}$, where R here is the gas constant, not the radius. This speed will also be essentially the same as the escape speed from the place where the wind is launched, which will be near the stellar surface. For main sequence stars, this will be several hundred kilometers per second.

For some stars, almost no particles get to this speed – it corresponds to a temperature of about a million degrees! In the Sun, there would be essentially no gas at this speed, if not for magnetic heating in the Sun's corona – and even then, the solar wind isn't very important for understanding the evolution of the Sun.

For other stars, winds can strongly influence the evolution of the star, and take away a large fraction of the stellar mass. How can they work?!?

The answer is radiation pressure.

The Eddington limit, which we derived several chapters ago, can be expressed as:

$$\kappa L < 4\pi cGM \tag{13}$$

where the quantities are given for the star as a whole.

We need to remember here that in a lot of cases, the opacity can be much larger than just the electron scattering opacity. In particular, it is generally thought that most *strong* stellar winds are driven by radiation pressure on metal lines, unless the stars are so highly ionized that there are essentially no metals left. Since metal lines absorb photons rather than merely scattering them, a large fraction of the momentum of the absorbed photon can be transferred to the atom.

Let's look at the rough scaling of the stellar wind's properties with other things. Suppose that the wind absorbs a fraction ϕ' of the photon momentum and the wind travels at v_{esc} , the escape velocity of the star. Then:

$$\dot{M}\delta tv_{esc} = \phi' \frac{L}{c} \delta t. \tag{14}$$

That is, the momentum taken on by the stellar wind per unit time to accelerate

it up to v_{esc} is equal to the momentum absorbed by the stellar wind from the photons.

Now, we can also set $v_{esc} = \sqrt{\frac{2GM}{R}}$, and $\phi = \phi'/2$, and get:

$$\dot{M} = \left(\phi \frac{v_{esc}}{c} \right) \frac{LR}{GM} \quad (15)$$

The term inside parenthesis cannot be estimated accurately in a simple way, because as the wind absorbs light, it can become less ionized, and it is generally estimated from observations instead.