A simple event weighting technique for optimizing the measurement of the forward-backward asymmetry of Drell-Yan dilepton pairs at hadron colliders

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We describe a simple technique for optimizing the extraction of the forward-backward asymmetry (A_{fb}) of Drell-Yan lepton pairs (e^+e^-, $\mu^+\mu^-$) produced in $\bar{p}p$ and pp collisions at hadron colliders. The method employs simple event weights which are functions of the rapidity and $cos\theta$ decay angle of the lepton pair. It yields the best estimate of the acceptance corrected parton level ($\bar{q}q$) forward backward asymmetry as a function of final state dilepton mass ($M_{\ell\ell}$). Typically, when compared to the simple count method, the technique reduces the statistical errors by 20% for $\bar{p}p$, and 40% for pp collisions, respectively. In $\bar{p}p$ and pp collisions with $M_{\ell\ell}$> 300 GeV/c^2 this new technique can be used to search for new high mass and large width Z' bosons which may be best detected through the observation of deviations from the Standard Model expectation for the forward-backward asymmetry. In pp collisions with $M_{\ell\ell}$< 300 $GeVc^2$, this technique can be used to provide additional constraints on the antiquark distributions in the proton.

1. Introduction

The Drell-Yan process in which $q\bar{q}$ annihilations form intermediate γ^* or Z (γ^*/Z) vector bosons decaying to lepton (e^+e^-, $\mu^+\mu^-$) pairs is particularly useful in searching for new interactions at large momentum transfers ($Q^2 = M^2_{\ell\ell}$, where $M_{\ell\ell}$ is the invariant mass of the lepton pair). In leading order (LO) approximation, the momentum fractions x_1, x_2 carried by the initial state quarks and antiquarks in the proton and antiproton/proton, respectively, are related to the rapidity y [1] of the γ^*/Z boson via the equation $x_{1,2} = (M_{\ell\ell}/\sqrt{s})e^{\pm y}$, where $\sqrt{s}$ is the center of mass energy. Dilepton pairs produced at large y originate from collisions in which one parton carries a large and the other a small momentum fraction x.

Drell-Yan lepton pairs which are produced in $q\bar{q}$ annihilations display a forward- backward asymmetry because of the interference between photon and Z boson exchange[2]. This forward-backward asymmetry would be modified by new resonances (e.g. additional heavier Z' bosons[2]) or new interactions at large mass scales.

Although the mass limits from Tevartron ($\bar{p}p$) experiments CDF [3] and and DØ [4] for a variety of Z' models are in the 1 TeV/c^2 range, the limits are much lower if the Z' width (typically $\Gamma_{Z'} \approx 0.01 \cdot M_{Z'}$) is increased to account for the possibility of additional decays modes to exotic fermions (which are predicted in E_6 models [5]), and/or supersymmetric particles. The limits are even lower if one includes the possibility of a more general model with enhanced couplings to the third generation.

Such a Z' (which has larger width e.g. $\Gamma_{Z'} = 0.1\, M_{Z'}$) would produce only a small signal in the dilepton mass spectrum because the total cross section is proportional to the square of the amplitude. However, the change in the forward - backward asymmetry which results from the interference with the standard model process is linearly proportional to the amplitude and would be observable as a change in the forward backward asymmetry. This change will occur around the mass of the Z' boson, and also in some

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mass range below and above the mass of the Z' boson[6]. This point is illustrated in Figure

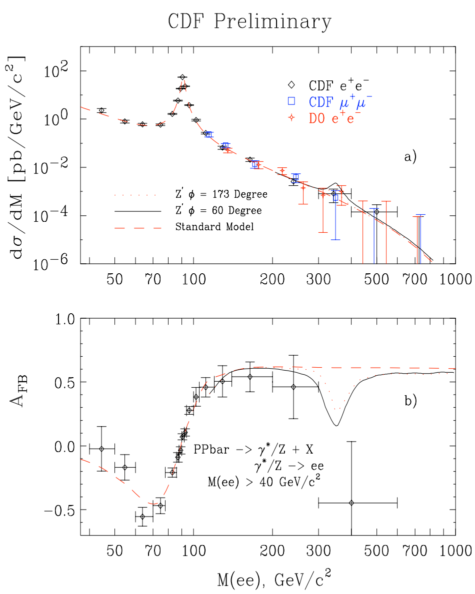

Figure 1. Early Tevatron data (120 pb^{-1}).

1 which is taken from Reference[6]. The figure compares early (120 pb^{-1}) high mass Drell-Yan $d\sigma/dM_{\ell}\ell$ data from CDF and DO, and early A_{FB} data from CDF to Standard Model theoretical predictions and (as an example) to a prediction with an extra E_6 boson with $M_{Z'} = 350\ GeV/c^2$ and $\Gamma_{Z'} = 0.1M_{Z'}$, for $\phi = 60^0$ (solid) and $\phi = 173^0$ (dotted). As can be seen for this case, the signal in A_{fb} is larger than the signal in $d\sigma/dM_{\ell}\ell$. Since such new particles or new interactions may be best detected through the observation of deviations from the Standard Model expectation for forward-backward asymmetry, it is useful to devise experimental technique to measure the forward-backward asymmetry (A_{fb}) with the best possible precision. Since fine mass bins ($\approx 25GeV/c^2$) are required, the number of events per bin at large $M_{\ell\ell}$ is small and the measurements are statistically limited.

In this communication we describe a simple technique for optimizing the extraction of the forward-backward asymmetry. The method works very well for both large and small statistical samples. The method employs simple event weights which are functions of the rapidity and $cos\theta$ decay angle of the lepton pair.

The new method yields the smallest statistical uncertainty in the measurement of the forward backward asymmetry as a function of $M_{\ell\ell}$. It can be directly applied to current $\bar{p}p$ data at the Fermilab Tevatron, as well as new data that will be collected in pp collisions at the Large Hadron Collider (LHC).

2. $q\bar{q}$ annihilations

The differential cross-section for the parton level process for $q\bar{q}$ annihilation can be written as

$$\frac{d\sigma}{d(cos\theta)} = A(1 + cos^2\theta + q(]\theta)) + Bcos\theta \quad (1)$$

$$q(\theta) = \frac{1}{2}C_1(M_{\ell\ell}, P_T, y)(1 - 3cos^2\theta) \quad (2)$$

where θ is the emission angle of the positive lepton relative to the quark momentum in the center of mass frame, and A and B are parameters that depend on the weak isospin and charge of the incoming fermions. The $q\bar{q}$ center of mass frame is well defined when the lepton pair has zero transverse momentum (P_T). For a non-zero transverse momentum of the dilepton pair, the $q\bar{q}$ center of mass frame is approximated by the Collins-Soper frame[7].

The term $q(\theta, M_{\ell}\ell, P_T, y)$ is a small QCD correction term[7] which is zero when the transverse momentum of the dilepton pair is zero. In general it is a function of the dilepton mass ($M_{\ell\ell}$), transverse momentum (P_T), and rapidity (the rapidity dependence is weak). The $q(\theta, M_{\ell\ell}P_T, y)$ term integrates to zero when the cross section is integrated over all $cos^2\theta$.

In equation 2, the parameter $C_1 = 0$ for P_T=0 and increases with transverse momentum. In our calculations we will use $C_1 = 0$ (e.g. P_T=0). In order to include the dependence on the $q(\theta, M_{\ell\ell}, P_T, y)$ term, we either parametrize C_1 from a Monte Carlo simulation such as PYTHIA[8], or account for it in the acceptance/modeling correction.

For example, using PYTHIA we find that in the production of Z bosons at the Tevatron $C_1 \approx 0.25$ for P_T=45GeV/c, and increases to $C_1 \approx 0.65$ for P_T=120 GeV/c. For entire sample of Z boson production at the Tevatron the average value of C_1 is about 0.063. In general QCD corrections to the B parameter are small. We can either assume that $C_1 = 0$, or or C_1 is equal to its average value, or parametrize the approximate linear dependence for $C1$ on P_T as function of dilepton mass. We can then rely on Monte Carlo acceptance/modeling corrections to account for the remaining higher order small QCD corrections to the angular distribution that has been assumed in the analysis.

The cross sections for forward ($for-$) events (σ_F) and backward ($back-$) events (σ_B) are given by

$$\begin{aligned} \sigma_f &= \int_0^1 \frac{d\sigma}{d(cos\theta)} d(cos\theta) \qquad (3)\\ &= A\left(1+\frac{1}{3}\right) + B\left(\frac{1}{2}\right) \\ \sigma_b &= \int_{-1}^0 \frac{d\sigma}{d(cos\theta)} d(cos\theta) \qquad (4)\\ &= A\left(1+\frac{1}{3}\right) - B\left(\frac{1}{2}\right) \end{aligned}$$

The electroweak interaction introduces the asymmetry (a linear dependence on $cos\theta$), which can be expressed as

$$A_{fb}^{total} = \frac{\sigma_f - \sigma_b}{\sigma_b + \sigma_b} = \frac{3B}{8A} \qquad (5)$$

For $\bar{p}p$ collisions (e.g. at the Tevatron), the direction of the quark is predominately in the proton direction, and the direction of the antiquark is predominately in the antiproton direction. Therefore, the forward backward asymmetry for $q\bar{q}$ processes is easy to measure in $\bar{p}p$ collisions.

If N_f is in number of events in the forward direction of the quark and N_b is the number of events in the backward direction of the quark we obtain the following expression for the total forward backward-asymmetry (A_{fb}^{total}) and its error (ΔA_{fb}^{total}):

$$\begin{aligned} [A_{fb}]^{total} &= \frac{N_f - N_b}{N_f + N_b} = \frac{N_f - N_b}{N} \qquad (6)\\ \frac{N_f}{N_b} &= \frac{1 - A_{fb}^{total}}{1 + A_{fb}^{total}} \\ N_f &= \frac{1 + A_{fb}^{total}}{2} N \\ N_b &= \frac{1 - A_{fb}^{total}}{2} N \\ \Delta A_{fb}^{total} &= \frac{2}{N}\left[\frac{N_f N_b}{N}\right]^{1/2} \\ \Delta A_{fb}^{total} &= \left[\frac{1 - (A_{fb(expected)}^{total})^2}{N}\right]^{1/2} \qquad (7) \end{aligned}$$

where we have used $\Delta N_f = (N_f)^{1/2}$ and $\Delta N_b = (N_b)^{1/2}$, and $N = N_f + N_b$. Since for Poisson statistics[10], the fractional error is $(1/N_{expected})^{1/2}$ and not $(1/N_{observed})^{1/2}$, we use $A_{fb(expected)}$ in equation 7. For $\bar{p}p$ collisions above the Z mass peak, $A_{fb(expected)}$=0.6. In this region, $\Delta A_{fb} = 0.800 \cdot (1/N)^{1/2}$.

Therefore, a measurement with 100 events yields a statistical error of 0.08. This level of precision is needed to observed the deviation from the Standard Model for the Z' example shown in figure 1. Later in this paper we show that a a reduction in the error (of about 20%) can be obtained by using the information in the angular distribution of the forward and backward events.

In the next section we discuss the measurement of the forward-backward asymmetry in pp collisions (e.g. at the Large Hadron Collider).

3. Misidentification of the quark direction in pp collisions

For $\bar{p}p$ collisions the direction of the quark is primarily along the direction of the proton. However, there is a a small probability for a misidentification ($misID$) that originates from the charge misidentification probability in the tracker. There is an additional small $misID$ that comes from the small fraction of events in which a sea antiquark in the proton interacts with a sea quark in the antiproton. This $misID$ is very small for large dilepton final state mass.

Although pp collisions are symmetric, there is still a forward backward asymmetry if the quark direction is defined to be the direction of motion of the Derll-Yan pair. This arise from the fact that on average, quarks carry a larger fraction of the momentum than antiquarks. However, there is a significant $misID$ that originates from the fraction of events for which the antiquark carries a larger fraction of the momentum than the quark. This $misID$ dilutes the observed asymmetry.

In most theoretical studies of the production of new Z' bosons in pp collisions at the LHC, this dilution is included in calculation for the prediction for the observed forward-backward asymmetry. Here, we show that we can obtain a higher sensitivity to new particle searches by correcting the data for the $misID$ fraction on an event by event basis.

At small rapidity the misidentification probability w_i is large (0.5 at $y = 0$). At large rapidity the misidentification probability w_i is small. We show below that by taking this information into account we can reduce the error on the extracted $q\bar{q}$ asymmetry.

4. Correcting for $misID$ in pp collisions

We illustrate this point for the case of a high statistics measurement of the forward backward asymmetry in a specific mass bin (e.g 300 GeV/c^2).

We first extract the $q\bar{q}$ asymmetry by correcting the measured asymmetry for the average $misID$ probability (this commonly used method is called the event count method).

We then show that we reduce the error on the extracted quark-antquark asymmetry by binning the data in ten rapidity bins and fitting for the weighted average of the extracted parton level asymmetries from all of the ten rapidity bins.

Then we derive an event weighting technique that is equivalent to the fit method in the high statistics case, but which can also be used in the limit of very small statistical samples.

We first define the $misID$ probability (w_i) for each y_i bin

$$x_{1(2)} = \frac{M_{\ell\ell}}{\sqrt{s}} \times e^{+(-)y_i} \tag{8}$$

$$w_i \approx \frac{\sum_{flavor} v_q \{q(x_2) \cdot \bar{q}(x_1)\}}{\sum_{flavor} v_q \{q(x_2) \cdot \bar{q}(x_1) + q(x_1) \cdot \bar{q}(x_2)\}} \tag{9}$$

Where $M_{\ell\ell}$ is the dilepton mass ($M_{\mu\mu}$ or M_{ee}), and y_i is the rapidity of the dilepton pair. Here $q(x)$ denotes the quark distributions ($u(x)$, $d(x)$, $s(x)$, $c(x)$, $b(x)$) and $\bar{q}(x)$ denotes the antiquark distributions ($\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$, $\bar{c}(x)$, $\bar{b}(x)$) for the various flavors in the nucleon. The parameter v_q denotes the Z/γ couplings of to each flavor (which a function of the dilepton mass). At large $M_{\ell\ell}$ the $\bar{u}(x)$ and $u(x)$ quark distributions dominate the expression for the dilution factor.

Note that the $misID$ in the forward-backward asymmetry should include the interference between photon and Z boson exchange for each quark flavor, which is a more complicated function of the couplings in general. In addition, the $misID$is affected by radiative emission of photons and detector resolution. Therefore, it is best to use a Monte Carlo generator (such as PYTHIA[8] or ZGRAD2[9]) to empirically determine the functional dependence of the dilution factor w_i as a function of $y_{\ell\ell,measured}$ and $M_{\ell\ell,measured}$.

$$w_i = f(y_{\ell\ell,measured}, M_{\ell\ell,measured}). \tag{10}$$

We now proceed to correct for the $misID$ and extract the $q\bar{q}$ forward-backward asymmetry for each one of the ten y_i rapidity bins. In the expressions below, $n_{f,i}$ and $n_{b,i}$ are the defined as the measured (i.e. diluted) number of forward events and backward events in each bin, and $N_{f,i}$ and $N_{b,i}$ are defined as the number of true forward and true backward events (for $q\bar{q}$ collisions) in the bin.

For a given y_i rapidity bin with a misidentification probability w_i the measured and true number of forward and backward events are related by the following expressions .

$$\begin{aligned} n_{f,i} &= N_{f,i}(1-w_i) + N_{b,i}(w_i) \qquad (11)\\ n_{b,i} &= N_{b,i}(1-w_i) + N_{f,i}(w_i)\\ N_{f,i} &= n_{f,i}(1-w_i)/L_i - n_{b,i}(w_i)/L_i\\ N_{b,i} &= n_{b,i}(1-w_i)/L_i - n_{f,i}(w_i)/L_i \end{aligned}$$

where $L_i = (1-2w_i)$ is defined as the dilution factor. For this y_i rapidity bin, the corrected parton level asymmetry is given by

$$\begin{aligned} A_{fb}^{total} &= \frac{N_{f,i} - N_{b,i}}{N_{f,i} + N_{b,i}} \qquad (12)\\ N_{f,i} - N_{b,i} &= n_{f,i}/L_i - n_{b,i}/L_i\\ N_{f,i} + N_{b,i} &= n_{f,i} + n_{b,i} = n_i \end{aligned}$$

which yields

$$A_{fb,i}^{total} = \frac{1}{L_i}\frac{n_{f,i} - n_{b,i}}{n_i} \qquad (13)$$

$$\left[\Delta A_{fb-i}^{total}\right] = \frac{1}{L_i}\frac{2}{n_i}\left[\frac{n_f n_{b,i}}{n_i}\right]^{1/2} \qquad (14)$$

where we have used $\Delta n_{f,i} = (n_{f,i})^{1/2}$ and $\Delta n_{b,i} = (n_{b,i})^{1/2}$.

We find that a measurement of the $q\bar{q}$ asymmetry in the case where there is $misID$ probability of w_i results in an increase of the error in the extracted parton level asymmetry by a factor of $1/L_i$ (which is equivalent to reducing the number of events by a factor of $L_i^2 = (1-2w_i)^2$).

If we want to combine different y_i bins together, we need to weight the events by the inverse of the square of the statistical error in each bin. This is achieved by multiplying the expressions for $N_{f,i}$ and $N_{b,i}$ by L_i^2. Since this factor appears both in the numerator and denominator of the expression for A_{fb-i}^{total}, it does not change the extracted value or error of the parton level asymmetry. However, when we combine y bins together using event weighting, this factor accounts for the difference in statistical errors between the y_i bins as follows.

$$\begin{aligned} k_{1,i} &= (1-w_i)(1-2w_i) \qquad (15)\\ k_{2,i} &= (w_i)(1-2w_i)\\ N_{total} &= \sum_{all-events}[1]\\ S_f &= \sum_{for-events} k_{1,i} - \sum_{back-events} k_{2,i}\\ [\Delta S_f]^2 &= \sum_{for-events} k_{1,i}^2 + \sum_{back-events} k_{2,i}^2\\ S_b &= \sum_{back-events} k_{1,i} - \sum_{for-events} k_{2,i}\\ [\Delta S_b]^2 &= \sum_{back-events} k_{1,i}^2 + \sum_{for-events} k_{2,i}^2\\ A_{fb}^{total} &= \frac{S_f - S_b}{S_f + Sb} \end{aligned}$$

Now $[\Delta S_f]$ and $[\Delta S_b]$ are correlated with each other in a complicated way. In order to simplify the calculation of the error, we combine terms to isolate sums which are for forward events, and sums which are for backward events, separately as follows:

$$\begin{aligned} k_{A,i} &= k_{1,i} - k_{2,i} = (1-2w_i)^2 \qquad (16)\\ k_{B,i} &= k_{1,i} + k_{2,i} = (1-2w_i)\\ N_{total} &= \sum_{all-events}[1]\\ A &= S_f + S_b = A_1 + A_2\\ B &= S_f - S_b = B_1 - B_2\\ A1 &= \sum_{forward-events} k_{A,i}\\ A2 &= \sum_{back-events} k_{A,i}\\ B1 &= \sum_{forward-events} k_{B,i} \end{aligned}$$

$$
\begin{aligned}
B2 &= \sum_{back-events} k_{B,i} \\
[\Delta A_1]^2 &= \sum_{forward-events} k^2_{A,i} \\
[\Delta A_2]^2 &= \sum_{back-events} k^2_{A,i} \\
[\Delta B_1]^2 &= \sum_{forward-events} k^2_{B,i} \\
[\Delta B_2]^2 &= \sum_{back-events} k^2_{B,i} \\
A^{total}_{fb} &= \frac{B}{A} = \frac{B_1 - B_2}{A_1 + A_2}
\end{aligned}
$$

Now ΔA_1 is 100% correlated with ΔB_1 and ΔA_2 is 100% correlated with ΔB_2 . We handle these correlations as follows.

$$
\begin{aligned}
\Delta A_1 &= \Delta B_1 \cdot \frac{A1}{B1} \qquad (17) \\
\Delta A_2 &= \Delta B_2 \cdot \frac{A2}{B2} \\
\left[\Delta A^{total}_{fb}\right]^2 &= \frac{1}{(A_1 + A_2)^4}\left[E_1^2 + E_2^2\right] \\
E_1^2 &= \frac{[\Delta B_1]^2}{B_1^2}(A2B1 + A1B2)^2 \\
E_1^2 &= \frac{[\Delta B_2]^2}{B_2^2}(A1B2 + B1A1)^2
\end{aligned}
$$

A specific numerical example is shown in Table 1. Here we show the case of a measurment of A_{fb} in ten bins of y for the range of y between 0 and 2.0. For this study we assume an asymmetry $A_{fb} = 0.6$ is measured with 1000 events (for $0 < y < 2$) in pp collisions at the LHC. We assume that the differential cross section is constant in y. The $misID$ probability is assumed to be $w(y) = 0.5(2 - y)/2$ which is a simple approximation to to the $misID$ values for a dilepton mass of 300 GeV/c^2 at the LHC. In each range in y we compare the error in the extracted $\bar{p}p$ asymmetry from a simple count of events; the error extracted from a least square fit to the values extracted from each y bin, and the error from the our event weighting formula. As expected, the error from the least square fit to the ten y bins is the same as the error from our weighting formula. The error using a simple count is about 20% larger than the error using the event weighting scheme. The last column shows the error for the case of $\bar{p}p$ collisions (with a $misID$=0).

5. Including information in the angular distribution in $\bar{p}p$ collisions

We now investigate how much can be gained by looking at the asymmetry in bins of $x_j = cos\theta_j$. We start with the case of $\bar{p}p$ and divide the sample into ten bins in $cos\theta_j$. The asymmetry as a function of x_j bin is:

$$
\begin{aligned}
A_{fb-j}(x_j) &= \frac{\sigma_f(x_j) - \sigma_b(y_j)}{\sigma_b(x_j) + \sigma_b(x_j)} \qquad (18) \\
A_{fb-j}(x_j) &= \frac{N_{f,j} - N_{b,j}}{N_{f,j} - N_{b,j}} \\
&= \frac{Bx_j}{A(1 + x_j^2 + q(M_{\ell\ell}, \theta, P_T, y)} \\
&= A^{total}_{fb-j}\left[\frac{8x_j}{3(1 + x_j^2 + q(\theta)}\right]
\end{aligned}
$$

At $x_j = cos\theta_j = 0$, the measured asymmetry $A_{fb-j}(0)$=0. At $x_j = cos\theta_j = 0.45$, the measured asymmetry $A_{fb-j}(0.45) = A^{total}_{FB}$. At $x_j = cos\theta_j = 1$, the asymmetry $A_{fb}(1) = (4/3)A^{total}_{fb}$. The measured asymmetry in each $x_j = cos\theta_j$ bin can be related to the total (integrated over all $cos\theta$) asymmetry and therefore provides an independent measurement A^{total}_{fb-j} of the total asymmetry.

$$
\begin{aligned}
A^{total}_{fb-j} &= \frac{3}{8} \cdot \frac{N_{f,j} - N_{b,j}}{N_{f,j} + N_{b,j}} \cdot \frac{1}{M_j} \qquad (19) \\
\Delta A^{total}_{fb-j} &= \frac{3}{8M_j}\frac{2}{N_{f,j} + N_{b,j}}\left[\frac{N_{f,j}N_{b,j}}{N_{f,j} + N_{b,j}}\right]^{1/2} \\
M_j &= \frac{x_j}{(1 + x_j^2 + q(M_{\ell\ell}, \theta, P_T, y))}
\end{aligned}
$$

where we have used $\Delta N_{f,j} = (N_{f,j})^{1/2}$ and $\Delta N_{b,j} = (N_{b,j})^{1/2}$.

The above expression shows that for case in which we have same number of events in each of the bins, the error in the extracted measurement of A^{total}_{fb-j} from the data in a specific $x_j = cos\theta_j$ bin

Table 1
Numerical example of measuring A_{fb} in ten bins of y for the range of y between 0 and 2.0. For this study we assume an asymmetry $A_{fb} = 0.6$ which is measured with 1000 events (for $0 < y < 2$) in pp collisions at the LHC. We assume that the differential cross section is constant in y. The $misID$ probability is assumed to be $w(y) = 0.5(2-y)/2$ which is a simple approximation to to the $misID$ values for a dilepton mass of 300 GeV/c^2. In each in range in y we compare the error in the extracted $\bar{q}q$ asymmetry from a simple count of events, the error using a least square fit to the values extracted from each y bin, and the error from the proposed event weighting formula. The error using a simple count is about 20% larger than the error using the the proposed event weighting scheme. The last column shows the error for the case of $\bar{p}p$ collisions (with a $misID$=0).

$y-range$ range	N bins	$Simple\ Count$ $Error\ pp$	$y\ bins\ fit$ $Error\ pp$	$Weights$ $Error\ pp$	$Improv.$ $Factor\ pp$	$\bar{p}p$ $Error$
0.-0.2	1	1.999	1.9991	1.9991	1.00	0.087
0-0.4	2	0.706	0.6301	0.6298	1.12	0.061
0-0.6	3	0.383	0.3350	0.3346	1.14	0.050
0-0.8	4	0.248	0.2145	0.2141	1.15	0.043
0-1.0	5	0.177	0.1512	0.1511	1.16	0.039
0-1.2	6	0.134	0.1136	0.1131	1.17	0.035
0-1.4	7	0.106	0.0886	0.0882	1.18	0.033
0-1.6	8	0.086	0.0711	0.0706	1.21	0.031
0-1.8	9	0.071	0.0582	0.0578	1.23	0.029
0-2.0	10	0.060	0.0483	0.0479	1.26	0.027

is equal to the error of the measured asymmetry in the bin divided by a factor $M_j = \frac{x_j}{(1+x_j^2+q(\theta))}$. This factor comes from the fact that the extracted total forward-backward asymmetry is more sensitive to events at large $x_j = cos\theta_j$.

We now convert the procedure to event weight technique. We define $A^{total}_{fb-j} = (3/8)(N_{A,j}/N_{B,j})$.

$$\begin{aligned} N_{f,j} &= N_{A,f}(1 + x_j^2 + q(\theta)) + x_j N_{B,f}(x_j) \\ N_{b,j} &= N_{A,f}(1 + x_j^2 + q(\theta)) - x_j N_{B,f}(x_j) \end{aligned} \tag{20}$$

From which we get:

$$\begin{aligned} N_{A,j} &= \frac{N_{f,j}}{2(1+x_j^2+q)} + \frac{N_{b,j}}{2(1+x_j^2+q)} \\ N_{B,j} &= \frac{N_{f,j}}{2x_j} - \frac{N_{b,j}}{2x_j} \end{aligned} \tag{21}$$

In order to properly weight events for different $cos\theta_j$ bins by the inverse of the square of the error for each bin we multiply the above expressions by

$$M_j^2 = \frac{x_j^2}{(1+x_j^2+q(M_{\ell\ell},\theta,P_T,y))^2} \tag{22}$$

and get :

$$\begin{aligned} z_{1,j} &= \frac{1}{2}\frac{x_j^2}{(1+x_j^2+q(\theta))^3} \\ z_{2,j} &= \frac{1}{2}\frac{x_j}{(1+x_j^2+q(\theta))^2} \\ A_1 &= N_{f,j}\cdot(z_{1,j}) \\ A_2 &= N_{b,j}\cdot(z_{1,j}) \\ B_1 &= N_{f,j}\cdot(z_{2,j}) \\ B_2 &= N_{b,j}\cdot(z_{2,j}) \\ [\Delta A_1]^2 &= N_{f,j}\cdot z_{1,j}^2 \\ [\Delta A_2]^2 &= N_{b,j}\cdot z_{1,j}^2 \\ [\Delta B_1]^2 &= N_{f,j}\cdot z_{2,j}^2 \\ [\Delta B_2]^2 &= N_{b,j}\cdot z_{2,j}^2 \end{aligned} \tag{23}$$

$$\begin{aligned} A_j &= N_{f,j}(z_{1,j}) + N_{b,j}(z_{1,j}) \\ &= A_1 + A_2 \end{aligned} \tag{24}$$

$$\begin{aligned} B_j &= N_{f,j}(z_{2,j}) - N_{b,j}(z_{2,j}) \\ &= B_1 - B_2 \end{aligned} \tag{25}$$

$$A^{total}_{fb-j} = \frac{3}{8}\frac{B_j}{A_j} = \frac{3}{8}\frac{B_1 - B_2}{A_1 + A_2}$$

Table 2
Proton-Antiproton collisions: Numerical example for an asymmetry $A_{fb} = 0.6$ measured with 1000 events. Here A_{fb}^{total} is measured in ten bins of $cos\theta$. In each $cos\theta_j$ bin we compare the error from the standard error formula, and the error from the event weighting formula. As expected, the two yield identical results. In addition, we show a comparison of the average A_{fb}^{total} of all ten $cos\theta_j$ bins calculated two different ways. The error in the average extracted from a least square fit to the 10 A_{fb-j}^{total} values (0.0196) is close to the error in the average determined from the weighted sum of all the events (0.0210). The error in A_{fb}^{total} from the weighted sum of all the events is 20% lower than the error of 0.0253 obtained from a simple count of all forward and backward events.

$x_j =$ $cos\theta_j$	n_f	n_b	$A_{fb-j}(x_j)$ *measured*	A_{fb-j}^{total} *extracted*	ΔA_{fb-j}^{total} $cos\theta_j$ *bin*	ΔA_{fb-j}^{total} *from weights*
0.05	41	34	0.080	0.60	0.864	0.864
0.15	47	29	0.235	0.60	0.284	0.284
0.25	55	25	0.376	0.60	0.165	0.165
0.35	63	21	0.499	0.60	0.114	0.114
0.45	72	18	0.599	0.60	0.085	0.085
0.55	82	16	0.676	0.60	0.066	0.066
0.65	92	14	0.731	0.60	0.054	0.054
0.75	104	14	0.768	0.60	0.045	0.046
0.85	116	14	0.790	0.60	0.041	0.041
0.95	128	14	0.799	0.60	0.038	0.028
all	800	200	0.6	0.60	0.0196	0.0210
all	800	200	0.6	0.60	simple-count	0.0253

Now ΔA_1 is 100% correlated with ΔB_1 and ΔA_2 is 100% correlated with ΔB_2 . We handle these correlations as follows.

$$\begin{aligned}
\Delta A_1 &= \Delta B_1 \cdot \frac{A1}{B1} \qquad (26)\\
\Delta A_2 &= \Delta B_2 \cdot \frac{A2}{B2}\\
\left[\Delta A_{fb-j}^{total}\right]^2 &= \left[\frac{3}{8}\right]^2 \frac{1}{(A_1+A_2)^4}\left[E_1^2+E_2^2\right]\\
E_1^2 &= \frac{\left[\Delta B_1\right]^2}{B_1^2}(A2B1+A1B2)^2\\
E_1^2 &= \frac{\left[\Delta B_2\right]^2}{B_2^2}(A1B2+B1A1)^2
\end{aligned}$$

Table 2 shows the results of a numerical example for an asymmetry $A_{fb} = 0.6$ measured with 1000 events (we assume q=0). Here A_{fb}^{total} is measured in ten bins of $cos\theta$. In each $cos\theta_j$ bin we compare the error from the standard error formula, and the error from the event weighting formula. As expected, the two yield identical.

In addition, we show a comparison of the average A_{fb}^{total} for the ten $cos\theta_j$ bins calculated in three different ways. The error in the average extracted from a least square fit to the 10 A_{fb-j}^{total} values (0.0196) is close to the error in the average determined from the weighted sum of all the events (0.0210). The error in A_{fb}^{total} from the weighted sum of all the events is 20% lower than the error of 0.0253 obtained from a simple count of all forward and backward events.

For the case of low statistics, we can use the event weighting technique to combine all the events at all value of $cos\theta$ (we do not need to bin the events in $cos\theta$). The following are the formulae to extract the best value and error from the entire range in $cos\theta$ using the event weighting technique :

$$\begin{aligned}
z_{1,j} &= \frac{1}{2}\frac{x_j^2}{(1+x_j^2+q(\theta,P_T,y)))^3} \qquad (27)\\
z_{2,j} &= \frac{1}{2}\frac{x_j}{(1+x_j^2+q(\theta,P_T,y)))^2}
\end{aligned}$$

Table 3
Proton-Antiproton collisions: Numerical example of measuring A_{fb} for different acceptance ranges in $cos\theta$. The example is for an asymmetry $A_{fb} = 0.6$ and a total of 1000 events (for all values of $cos\theta$). For each $cos\theta$ range we show the error in A_{fb} from a simple count, the error from the event weighting procedure, and the error from fitting bins in $cos\theta$ (which is very similar to event weighting). Also shown is the improvement factor in the error when the event weighting procedure is used (versus a simple count). For a typical range of $cos\theta$, using the simple weighting formula leads to more than 20% reduction in the error.

cosθ range	*N bins*	*Simple Count Error*	*cosθ bins fit Error*	*Event Weights Error*	*Improvement factor*
0-0.1	1	0.8642	0.8642	0.8642	1.000
0-0.2	2	0.3042	0.2796	0.2687	1.132
0-0.3	3	0.1644	0.1410	0.1403	1.172
0-0.4	4	0.1058	0.0884	0.0881	1.201
0-0.5	5	0.0749	0.0611	0.0611	1.225
0-0.6	6	0.0563	0.0459	0.0454	1.241
0-0.7	7	0.0442	0.0346	0.0354	1.247
0-0.8	8	0.0358	0.0277	0.0288	1.242
0-0.9	9	0.0298	0.0230	0.0242	1.227
0-1.0	10	0.0253	0.0196	0.0210	1.205

$$
\begin{aligned}
N_{total} &= \sum_{all-events} [1] \\
A_1 &= \sum_{forward-events} [z_{1,j}] \\
A_2 &= \sum_{back-events} [z_{1,j}] \\
B_1 &= \sum_{forward-events} [z_{2,j}] \\
B_2 &= \sum_{back-events} [z_{2,j}] \\
[\Delta A_1]^2 &= \sum_{forward-events} [z^2_{1,j}] \\
[\Delta A_2]^2 &= \sum_{back-events} [z^2_{1,j}] \\
[\Delta B_1]^2 &= \sum_{forward-events} [z^2_{2,j}] \\
[\Delta B_2]^2 &= \sum_{back-events} [z^2_{2,j}] \\
A &= A_1 + A_2 \\
B &= B_1 - B_2 \\
[A_{fb}]^{total} &= \frac{3}{8}\frac{B}{A} = \frac{B_1 - B_2}{A_1 + A_2}
\end{aligned}
$$

$$
\begin{aligned}
\Delta A_1 &= \Delta B_1 \cdot \frac{A1}{B1} \\
\Delta A_2 &= \Delta B_2 \cdot \frac{A2}{B2} \\
\left[\Delta A_{fb}^{total}\right]^2 &= \left[\frac{3}{8}\right]^2 \frac{1}{(A_1 + A_2)^4} \left[E_1^2 + E_2^2\right] \\
E_1^2 &= \frac{[\Delta B_1]^2}{B_1^2}(A2B1 + A1B2)^2 \\
E_1^2 &= \frac{[\Delta B_2]^2}{B_2^2}(A1B2 + B1A1)^2
\end{aligned}
$$

Table 3 shows a numerical example of measuring A_{fb} for different acceptance ranges in $cos\theta$ (we assume q=0). The example is for an asymmetry $A_{fb} = 0.6$ and an a sample of 1000 events (for $0 < cos\theta < 1$). For each range of acceptance in $cos\theta$ we show the error in A_{fb} from a simple count, the error from the event weighting procedure, and the error from fitting bins in $cos\theta$ (which is very similar to event weighting). Also shown is the improvement factor in the error when the event weighting procedure is used (versus a simple count). For a typical range of acceptance in $cos\theta$, using the simple weighting formula leads to more than 20% reduction in the error.

Note that when we use the angular distribution weights, the extracted A_{fb}^{total} is automatically corrected for the acceptance in $cos\theta$ (since the acceptance cancels to first order). However, $N_{total} = \sum_{all-events}[1]$ is equal to the observed number of events and is not corrected for acceptance. For N_{total}, an acceptance correction is needed for the determination of $d\sigma/dM_{\ell\ell}$

6. Combining $misID$ weighting and angular distribution weighting in pp collisions

In pp collisions each event is can be characterized by a $misID$ factor $w_i(y_i)$ which is related to the quark and antiquark distribution at its value of (y_i). In addition, each event has a measured value of $x_j = cos\theta_j$. The expressions for combining events with different $x_j = cos\theta_j$ and $misID$ w_i values are given by:

$$\begin{aligned}
k_{A,i} &= k_{1,i} - k_{2,i} = (1-2w_i)^2 \qquad (28)\\
k_{B,i} &= k_{1,i} + k_{2,i} = (1-2w_i)\\
N_{total} &= \sum_{all-events}[1]\\
z_{1,j} &= \frac{1}{2}\frac{x_j^2}{(1+x_j^2+q(\theta))^3}\\
z_{2,j} &= \frac{1}{2}\frac{x_j}{(1+x_j^2+q(\theta))^2}\\
A_1 &= \sum_{for-events}[z_{1,j}k_{A,j}]\\
A_2 &= \sum_{back-events}[z_{1,j}k_{A,j}]\\
B_1 &= \sum_{for-events}[z_{2,j}k_{B,j}]\\
B_2 &= \sum_{back-events}[z_{2,j}k_{B,j}]\\
[\Delta A_1]^2 &= \sum_{for-events}\left[z_{1,j}^2k_{A,j}^2\right]\\
[\Delta A_2]^2 &= \sum_{back-events}\left[z_{1,j}^2k_{A,j}^2\right]\\
[\Delta B_1]^2 &= \sum_{for-events}\left[z_{2,j}^2k_{B,j}^2\right]\\
[\Delta B_2]^2 &= \sum_{back-events}\left[z_{2,j}^2k_{B,j}^2\right]\\
A &= A_1 + A_2\\
B &= B_1 - B_2\\
[\Delta A_1] &= [\Delta B_1]\cdot\frac{A1}{B1}\\
[\Delta A_2] &= [\Delta B_2]\cdot\frac{A2}{B2}\\
A_{fb}^{total} &= \frac{3}{8}\frac{B}{A} = \frac{B_1 - B_2}{A_1 + A_2}\\
\left[\Delta A_{fb}^{total}\right]^2 &= \left[\frac{3}{8}\right]^2\frac{1}{(A_1+A_2)^4}\left[E_1^2 + E_2^2\right]\\
E_1^2 &= \frac{[\Delta B_1]^2}{B_1^2}(A2B1 + A1B2)^2\\
E_1^2 &= \frac{[\Delta B_2]^2}{B_2^2}(A1B2 + B1A1)^2
\end{aligned}$$

Table 4 shows a numerical example of the improvement in the errors that that can be realized by using the information for both the $misID$ (w_i) and $cos\theta_i$ on an event by event basis, versus the measurment of A_{fb} using a simple count of all events for all values of $cos\theta$ and y (within the experimental acceptance). We show the case for an asymmetry $A_{fb} = 0.6$ measured with 10^6 pp events (over all values of $cos\theta$ and $0 < y < 2$) for the case of a dilepton mass of 300 GeV/c^2 at the LHC. The two dimensional table shows the improvement factor in the the errors (over the simple count method) when we use event weighting in both $cos\theta$ and $MisID$ as a function of y. For a typical experimental acceptance in $cos\theta$ and y, the weighting formula leads to a 40% reduction in the error (over the error obtained from a simple count).

The following are the advantages for using the event weighting.

1. For $\bar{p}p$ collisions (for which event weighting can only be done in $cos\theta$) the error is typically reduced by a factor of 1.2.
2. For pp collisions ((for which event weighting can be done in both $cos\theta$ and y)t he error is reduced by a a factor of 1.4.
3. The method provides the acceptance corrected asymmetry without applying any ac-

Table 4
Proton-Proton Collisions: Numerical example of measuring A_{fb} integrating over various ranges of $cos\theta$ and various ranges of y for the case of a dilepton mass of 300 GeV/c^2 at the LHC. Here we assume that the asymmetry $A_{fb} = 0.6$ measured with 10^6 events for values of $cos\theta$ and $0 < y < 2$. Shown is the improvement factor in the the error (over the simple count method) when we use event weighting in both $cos\theta$ and $w = MisID$ as a function of y. For a typical range of acceptance in $cos\theta$ and y, the weighting formula leads to a 40% reduction in the error (over the error obtained from a simple count).

$y - range$	0-0.2	0-0.4	0-0.6	0-0.8	0-1.0	0-1.2	0-1.4	0-1.6	0-1.8	0-2.0
$cos\theta$ $range$										
0-0.1	1.00	1.12	1.14	1.15	1.16	1.17	1.18	1.21	1.23	1.26
0-0.2	1.13	1.27	1.29	1.30	1.31	1.32	1.34	1.37	1.39	1.43
0-0.3	1.17	1.31	1.34	1.35	1.36	1.37	1.38	1.42	1.44	1.48
0-0.4	1.20	1.35	1.37	1.38	1.39	1.41	1.42	1.45	1.48	1.51
0-0.5	1.23	1.37	1.40	1.41	1.42	1.43	1.45	1.48	1.51	1.54
0-0.6	1.24	1.39	1.41	1.43	1.44	1.45	1.46	1.50	1.53	1.56
0-0.7	1.25	1.40	1.42	1.43	1.45	1.46	1.47	1.51	1.53	1.57
0-0.8	1.24	1.39	1.42	1.43	1.44	1.45	1.47	1.50	1.53	1.56
0-0.9	1.23	1.37	1.40	1.41	1.42	1.44	1.45	1.49	1.51	1.55
0-1.0	1.21	1.35	1.37	1.39	1.40	1.41	1.42	1.46	1.48	1.52

ceptance corrections for missing coverage in $cos\theta$. The acceptance fully cancels to first order, and is not used in the extraction of the acceptance corrected asymmetry.

4. Only small corrections need to be made are detector resolution and radiative smearing effects. Most of these correction are already included if an empirical fit $w_i = f(y_{\ell\ell,measured}, M_{\ell\ell,measured}) is used.$.

5. Only small corrections need to be made for QCD corrections to the angular distribution. Most of these corrections are already included if we use an empirical fit for $q(\theta, M_{\ell\ell}, P_T, y)$.

6. Since the method does not use the acceptance to first order, the weighted sums for A1, A2, B1, B2, ΔB_1, and ΔB_2 from different run conditions, or different final state leptons (muon,electrons) or different experiments (e.g. Dzero and CDF or CMS and ATLAS) can be directly added to provide a combined result. This is important for mass bins at high mass for which there are only a few events in each detector.

7. Corrections factors and systematic uncertainties in the weighting procedure

For pp collisions, the acceptance for forward and backward events is equal because of symmetry. Therefore the functional dependence of the acceptance in $cos\theta$ fully cancels if the acceptance for positive and negative muons is the same. For $\bar{p}p$ collisions one needs to correct for a possible small difference in the detector performance between the proton and antiproton directions.

7.1. backgrounds

The main experimental background is QCD dijet events. The QCD jet background is measured by statistically separating isolated muons (or electrons) from muons (or electron like objects) in jets on the basis of the transverse energy profile distributions in the calorimeter [11] (e.g. isolation energy variables).

In general QCD processes are mediated via spin 1 gluon exchange and therefore have the same angular distribution as Drell Yan events. If such is the case, the $factional$ QCD background is the same at all values of $cos\theta$ and is the same for positive and negative values of $cos\theta$. Therefore, in the expressions for the asymmetry, the QCD background cancels in the numerator. Corrections for this background only increases the level of the denominator. Therefore, a single multiplicative factor equal to $1/(1-f)$ (where f is the fraction of QCD background events) can be used to correct the extracted A_{fb} for QCD background. The same multiplicative correction factor for QCD background can be use for all three methods (simple count, fit to bins in $cos\theta$ and event weighting technique). Possible deviations from this assumption can be included in the systematic error.

Another background originates from electroweak (EW) processes (WW, WZ, W+jets, and $\tau^+\tau^-$) and $t\bar{t}$). This background is generally estimated from a Monte Carlo simulation. If the event weighting technique is used to extract the forward-backward asymmetry, then the event weighting technique can be used on Monte Carlo samples for the Drell-Yan signal and the electroweak background processes to determine the shift in A_{fb} from EW backgrounds. Note that the contribution from from $\tau^+\tau^-$ events is very small (and at high mass A_{fb} for $\tau^+\tau^-$ is similar to A_{fb} for $\mu\mu$ and e^+e^- events).

7.2. Systematic and PDF errors

As is generally the case in particle physics experiments, the procedure needs to be tested on Monte Carlo simulated data to determine the size of any pulls from the previously listed systematic effects. We note that this process to determine biases and pulls needs to be done for any procedure that is used to extract the forward-backward asymmetry for $q\bar{q}$ processes from data. Our procedure is designed to minimize these biases, but they still need to be determined from a full scale Monte Carlo simulation.

In *pp* collisions at the LHC, there are uncertainties from antiquark distribution functions that affect the $misID$ probabilities (which are determined using a Monte Carlo simulations)

For dilepton mass below 300 GeV/c^2, the Standard Model Parameters are already very well constrained by e^+e^- data from LEP, and A_{fb} data from $\bar{p}p$ collisions at the Tevatron.

Therefore, for the case of *pp* collisions at the LHC, requiring the asymmetry data below a dilepton mass of 300 GeV/c^2 (at the LHC) to agree with the Standard Model expectation yields additional constraints on the parton distribution functions for the antiquarks in the proton. We do this by binning the data in bins of y (reversing equation 13) and using $L_i = 1 - 2w_i$ as follows:

$$\begin{aligned}
A_{fb}^{measured}(w_i) &= \frac{n_{f,i} - n_{b,i}}{n_i} \qquad (29)\\
\Delta A_{fb}^{measured}(w_i) &= \frac{2}{n_i}\left[\frac{n_f n_{b,i}}{n_i}\right]^{1/2}\\
w_i^{measured} &= 0.5\left[1 - \frac{A_{fb}^{measured}}{2A_{fb}^{SM}}\right]\\
\Delta w_i^{measured} &= \frac{\Delta A_{fb}^{measured}}{4A_{fb}^{SM}}\\
w_i &\approx \frac{R_i}{1+R_i}\\
R_i &= \frac{\bar{q}(x_1)/q(x_1)}{\bar{q}(x_2)/q(x_2)}\\
R_i^{measured} &\approx \frac{w_i^{measured}}{1 - w_i^{measured}}\\
\Delta R_i^{measured} &\approx \frac{\Delta w_i^{measured}}{(1 - w_i^{measured})^2}
\end{aligned}$$

For example, if we have 100 events for dilepton mass of 300 GeV/c^2 in a $y_i = 1.1$ bin we estimate $w_i = 0.226$. For these values, $A_{fb,i}^{SM}$ =0.6, $A_{fb,i}^{measured}$=0.33, $\Delta A_{fb,i}^{measured}$=0.094. This provides a measurement of $w_i = 0.226$ with an error $\Delta w_i = 0.0236$ and an extracted value of $R_i = 0.290$ with an error $\Delta R_i = 0.039$. In order to improve the error by a factor of 1.2 we should use the event weighting technique in $cos\theta$ (equations 23 - 26) to evaluate $A_{fb}^{measured}$ and $\Delta A_{fb}^{measured}$ (instead of the event counting method expressions given in equation 29). With $cos\theta$ weighting and 100 events we find that $R_i = \frac{\bar{q}(x_1)/q(x_1)}{\bar{q}(x_2)/q(x_2)}$ of 0.29 can be measured with an error $\Delta R_i = 0.033$ (about 10%).

8. Summary

We have shown that a simple event weighting technique can be used to reduce the statistical error on the extracted $\bar{q}q$ forward-backward asymmetry from Drell-Yan events in $\bar{p}p$ and pp collisions. In addition to reducing the statistical error, the event weighting technique is not sensitive to details of the experimental acceptance.

In pp collisions at the LHC. the asymmetry data for a dilepton mass below 300 GeV/c^2 can be used to provide additional constraints on the antiquark fractions in the nucleon.

The asymmetry data for a dilepton mass above 300 GeV/c^2 (for both pp and $\bar{p}p$ collisions) can be used to search for new Z' bosons.

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