PHYS-3301

Lecture 4

Sep. 3, 2024

Chapter. 3
Wave & Particles I

EM-"Waves" behaving like "Particles"

Outline:

- Blackbody Radiation (Plank; 1900; 1918*)
- The Photoelectric Effect (Einstein; 1905; 1921*)
- The Production of X-Rays (Rontgen;1901; 1901*)
- **The Compton Effect (Compton; 1927; 1927*)**
- **Pair Production (Anderson; 1932; 1936*)**
- **Is It a Wave or a Particle? → Duality?**

The Compton effect
(Arthur Compton 1927)

1. According to special relativity, an object with zero mass should have momentum related to its energy by

$$E = pc$$

2. Classical electromagnetic wave theory shows that electromagnetic waves do carry momentum, although for a diffuse wave, we speak of momentum "density." It is related to the energy density by

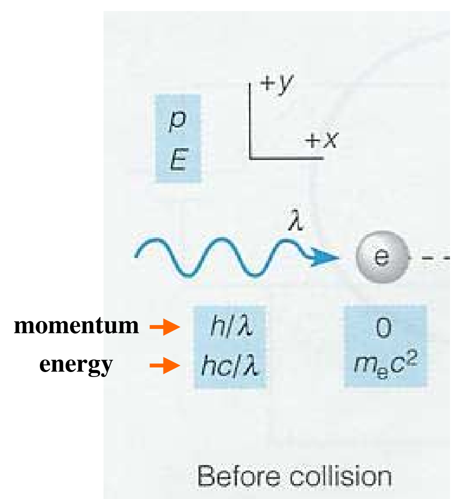
$$\frac{\text{energy}}{\text{volume}} = \frac{\text{momentum}}{\text{volume}} \times c$$

Is this true? – Compton provided the 1st experimental evidence!!

Hypothesis: $$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

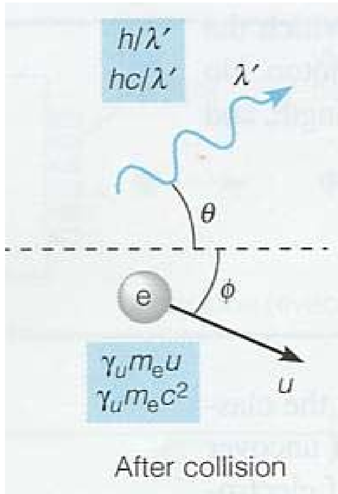
Experiment ?

Momentum & Energy when a photon strike a free electron



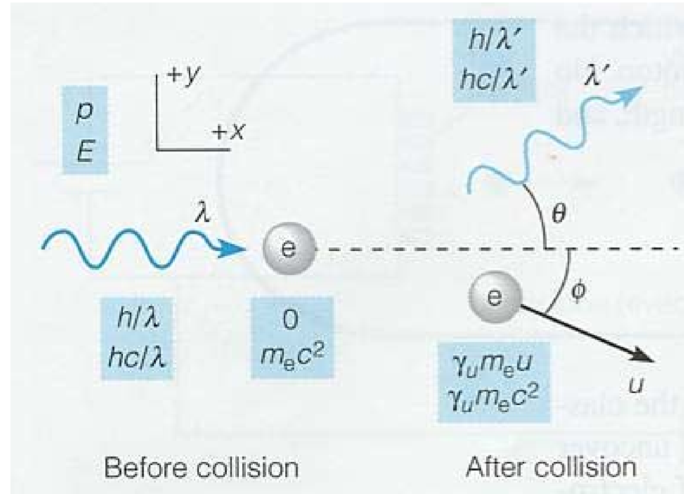
Before Collision: A photon approaches an electron at rest

Momentum & Energy when a photon strike a free electron

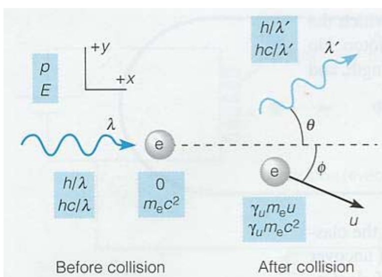


After Collision: The electron scatters at speed *u*, angle ϕ. A photon of wavelength λ' scatters at angle θ

Momentum & Energy when a photon strike a free electron



Energy and Momentum Conservation



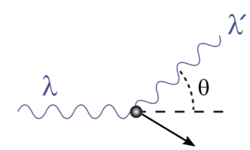
Momentum conserved:

x-component: $$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma_u m_e u \cos\phi$$

y-component: $$0 = \frac{h}{\lambda'}\sin\theta - \gamma_u m_e u \sin\phi$$

Energy conserved:

$$h\frac{c}{\lambda} + m_e c^2 = h\frac{c}{\lambda'} + \gamma_u m_e c^2$$



The conservation of energy E merely equates the sum of energies before and after scattering.

$$E_\gamma + E_e = E_{\gamma'} + E_{e'}.$$

Compton postulated that photons carry momentum;[3] thus from the conservation of momentum, the momenta of the particles should be similarly related by

$$\mathbf{p}_\gamma = \mathbf{p}_{\gamma'} + \mathbf{p}_{e'},$$

in which (p_e) is omitted on the assumption it is effectively zero.

The photon energies are related to the frequencies by

$$E_\gamma = hf$$
$$E_{\gamma'} = hf'$$

where h is Planck's constant.

Before the scattering event, the electron is treated as sufficiently close to being at rest that its total energy consists entirely of the mass-energy equivalence of its rest mass m_e:

$$E_e = m_e c^2.$$

After scattering, the possibility that the electron might be accelerated to a significant fraction of the speed of light, requires that its total energy be represented using the relativistic energy–momentum relation:

$$E_{e'} = \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}.$$

Substituting these quantities into the expression for the conservation of energy gives,

$$hf + m_e c^2 = hf' + \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}.$$

This expression can be used to find the magnitude of the momentum of the scattered electron,

$$p_{e'}^2 c^2 = (hf - hf' + m_e c^2)^2 - m_e^2 c^4. \qquad (1)$$

Equation (1) relates the various energies associated with the collision. The electron's momentum change includes a relativistic change in the mass of the electron so it is not simply related to the change in energy in the manner that occurs in classical physics. The change in the momentum of the photon is also not simply related to the difference in energy but involves a change in direction.

Solving the conservation of momentum expression for the scattered electron's momentum gives,

$$\mathbf{p}_{e'} = \mathbf{p}_\gamma - \mathbf{p}_{\gamma'}.$$

Then by making use of the scalar product,

$$p_{e'}^2 = \mathbf{p}_{e'} \cdot \mathbf{p}_{e'} = (\mathbf{p}_\gamma - \mathbf{p}_{\gamma'}) \cdot (\mathbf{p}_\gamma - \mathbf{p}_{\gamma'})$$
$$= p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'} \cos\theta.$$

Anticipating that $p_\gamma c$ is replaceable with hf, multiply both sides by c^2:

$$p_{e'}^2 c^2 = p_\gamma^2 c^2 + p_{\gamma'}^2 c^2 - 2c^2 p_\gamma p_{\gamma'} \cos\theta.$$

After replacing the photon momentum terms with hf/c, we get a second expression for the magnitude of the momentum of the scattered electron:

$$p_{e'}^2 c^2 = (hf)^2 + (hf')^2 - 2(hf)(hf')\cos\theta. \qquad (2)$$

Equating both expressions for this momentum gives

$$(hf - hf' + m_e c^2)^2 - m_e^2 c^4 = (hf)^2 + (hf')^2 - 2h^2 ff' \cos\theta,$$

which after evaluating the square and then canceling and rearranging terms gives

$$2hfm_e c^2 - 2hf'm_e c^2 = 2h^2 ff'(1 - \cos\theta).$$

Then dividing both sides by $2hff'm_e c$ yields

$$\frac{c}{f'} - \frac{c}{f} = \frac{h}{m_e c}(1 - \cos\theta).$$

Finally,[4] since $f\lambda = f'\lambda' = c$,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

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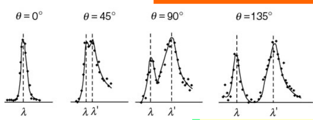
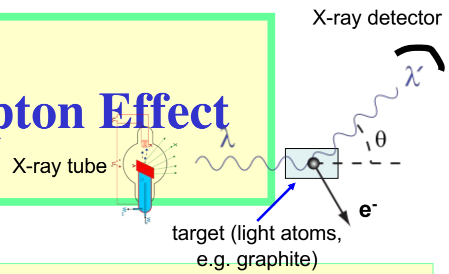


The Compton Effect



Photons carry momentum like particles

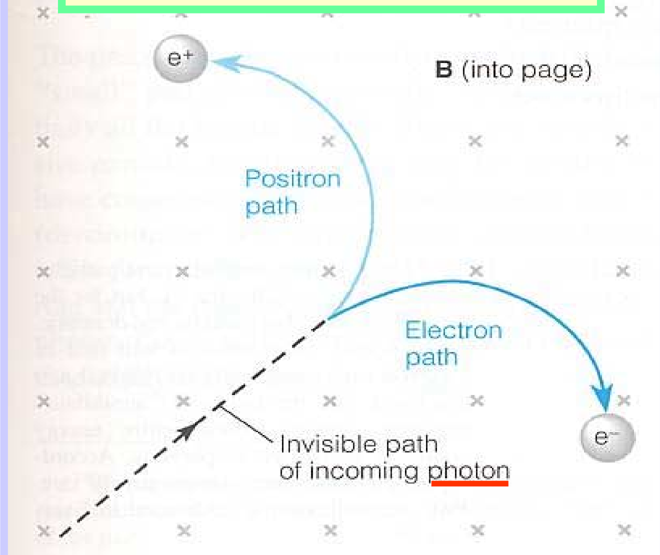
and scatter individually with other particles

indeed, the wavelength shift is independent of the target material and the initial photon wavelength.

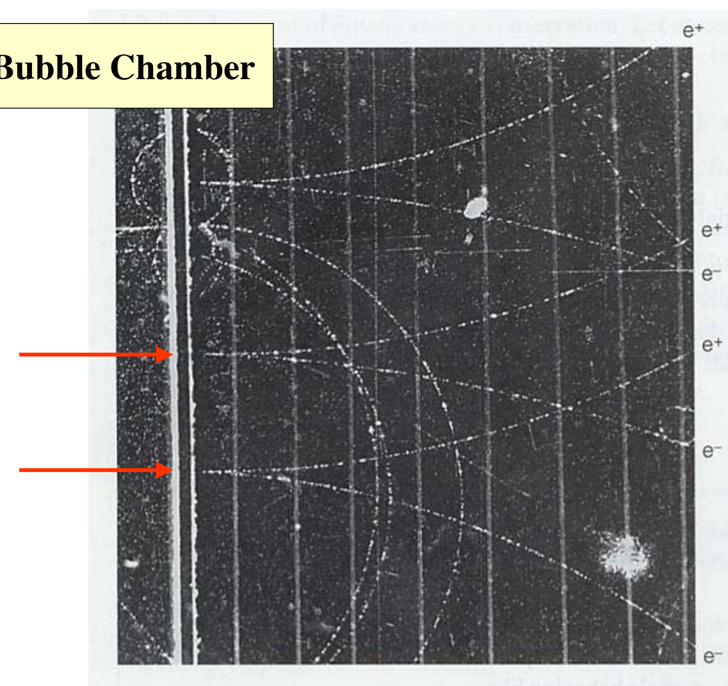
The photoelectric effect and the Compton effect are two important ways in which EM radiation interacts as a particle with matter.

We now discuss a third!

Particle-Antiparticle Pair Creation



Bubble Chamber



Q:
Calculate the energy and wavelength of the least-energetic photon capable of producing an electron-positron pair.
[Hint] Photon E goes to the massive particles as internal energy + KE. The least energetic one must still create the particles but would leave them no KE….

Example 2.5
Calculate the energy and wavelength of the least-energetic photon capable of producing an electron-positron pair.

Solution

The energy in the photon becomes the energy of the massive particles, internal/mass energy plus any kinetic energy. The minimum energy required is that which is barely able to produce the pair, with *no* kinetic energy. In this case, the photon energy equals just the internal energy of the pair:

$$2 \cdot m_e c^2 = 2(9.11 \times 10^{-31}\ \text{kg})(3 \times 10^8\ \text{m/s})^2$$
$$= 1.64 \times 10^{-13}\ \text{J} \quad (\cong 1\ \text{MeV})$$

Thus,

$$h\frac{c}{\lambda} = 2m_e c^2 \quad \Rightarrow \quad \lambda = \frac{hc}{2m_e c^2}$$
$$= \frac{(6.63 \times 10^{-34}\ \text{J·s})(3 \times 10^8\ \text{m/s})}{1.64 \times 10^{-13}\ \text{J}}$$
$$= 1.21 \times 10^{-12}\ \text{m}$$

Electromagnetic Waves
behaving like
Particles **"PHOTONS"**
(Chapter 2)

Black Body Radiation
The Photoelectric Effect
The Production of X-Rays ⟺ **PHOTONS** $E = hf$

The Compton Effect ⟺ **PHOTONS** $p = hf/c = h/\lambda$

Particle-Antiparticle Pair Production

Photon properties:

$$E = hf \qquad (2\text{-}1) \qquad\qquad p = \frac{h}{\lambda}$$

Photoelectric effect:

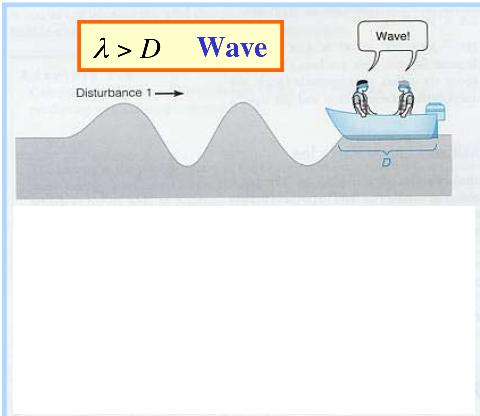
$$KE_{max} = hf - \phi \qquad (2\text{-}2)$$

Compton effect:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Is It a Wave or a Particles?
Duality

Ch.3 E&M-Waves behaving like Particles
Ch.4 Particles behaving like Waves



Is It a Wave or a Particles?
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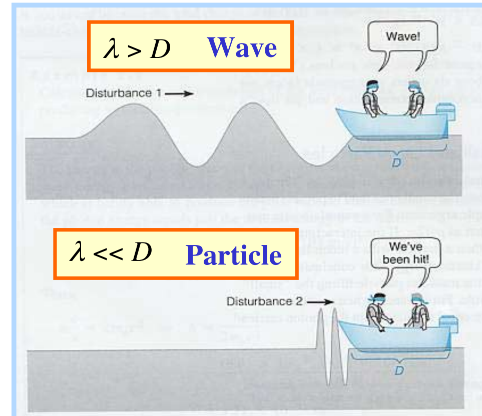
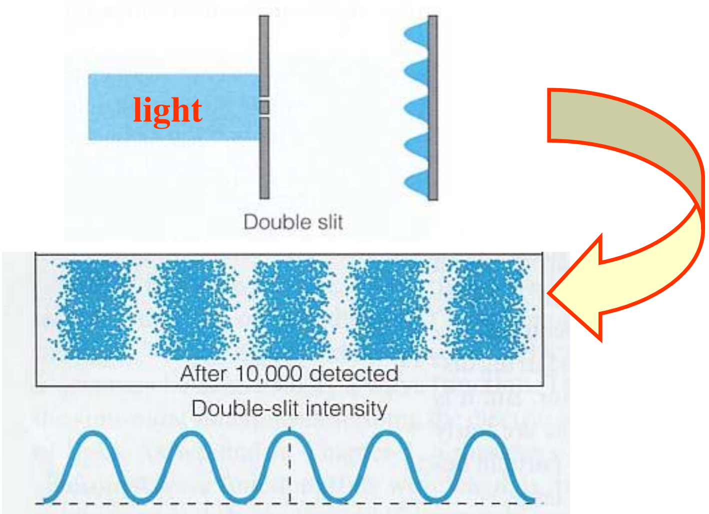
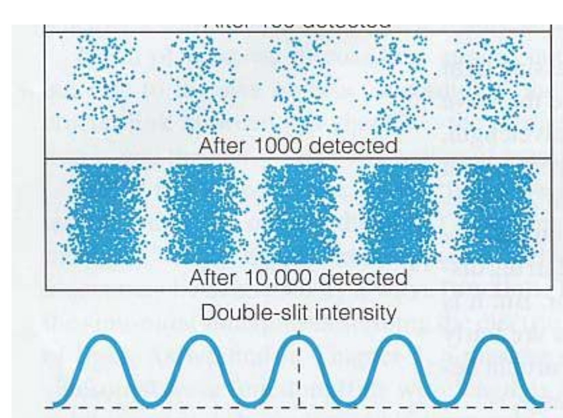
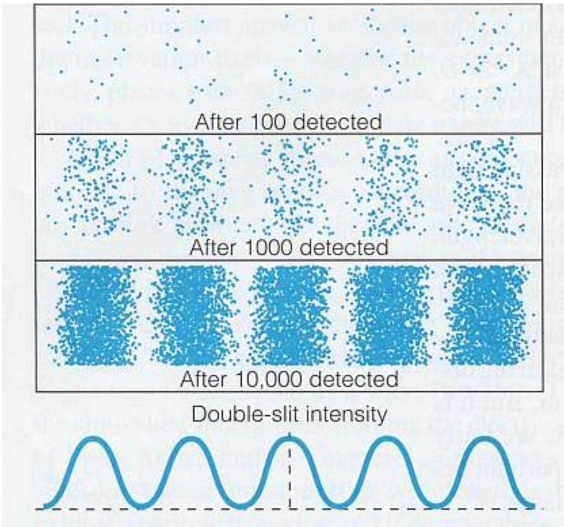
Ch.3 EM-Waves behaving like Particles
Ch.4 Particles behaving like Waves

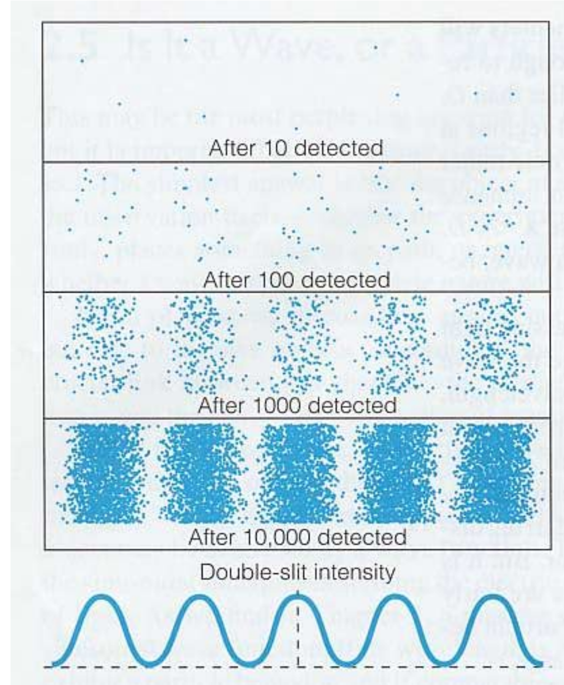


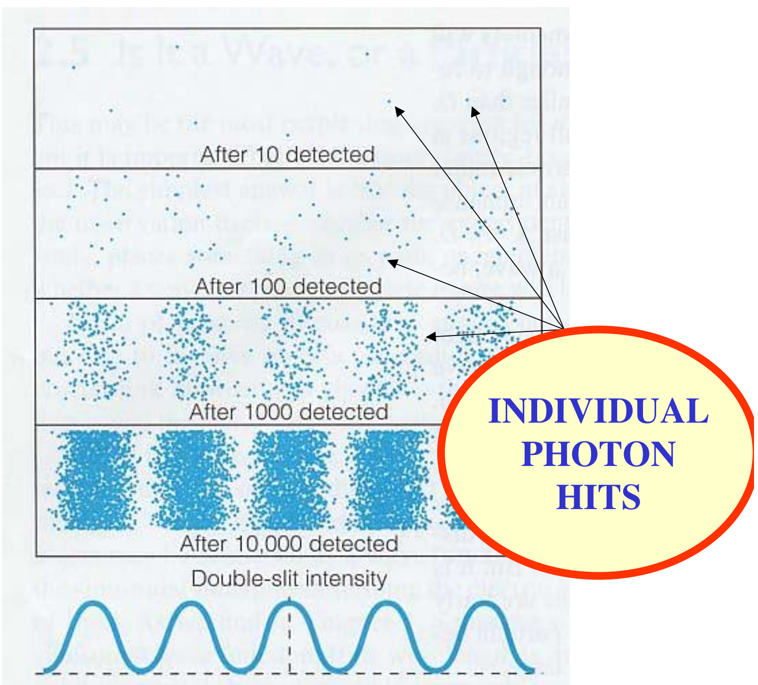
Double-slit Diffraction Experiment

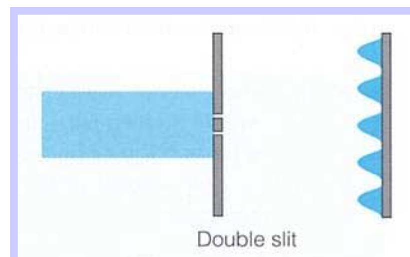




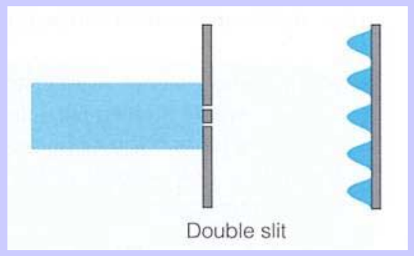








Although diffraction of light is a *wave phenomenon*, there is no smooth distribution of light in the diffraction pattern, but the pattern is rather formed of many *individual hits of particles – the photons*



A *single photon*
DOES NOT
get "disintegrated" in the
Diffraction process
to make a smooth
diffraction pattern

When a phenomenon is detected as *particles*, it cannot be predicted with certainty where a given particle will be found. The most that can be determined is a probability of finding it in a given region, and this "probability density" is proportional to the square of the amplitude of the associated *wave* in that region.

$$\text{probability density of finding } particle \propto (\text{amplitude of } wave)^2$$

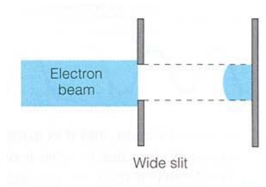
Coming back to that soon…

Chapter. 4
Wave & Particles II

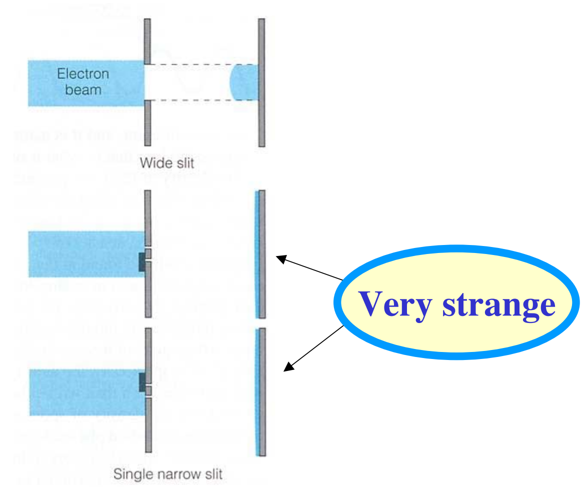
"Matter" behaving as "Waves"

Outline:

- A Double-Slit Experiment (watch "video")
- Properties of Matter Waves
- The Free-Particle Schrödinger Equation
- **Uncertainty Principle**
- **The Bohr Model of the Atom**
- **Mathematical Basis of the Uncertainty Principle – The Fourier Transform**



Electron passing through a wide slit arrive at a screen one by one and produce a bright region essentially the same width as the slit



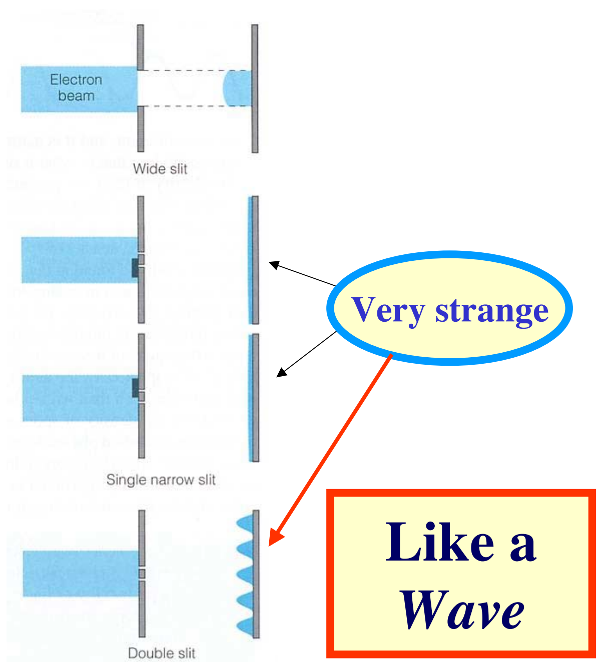
With a single narrow slit,
Electrons arrive one by one all over the screen.
They don't avoid any points …

…

But when 2 slits are open,

there are certain points at which no electron ever arrives.

Each electron wave is destructively interfering with itself.



Electrons producing a double-slit interference pattern - one particle at a time