PHYS-3301

Lecture 14

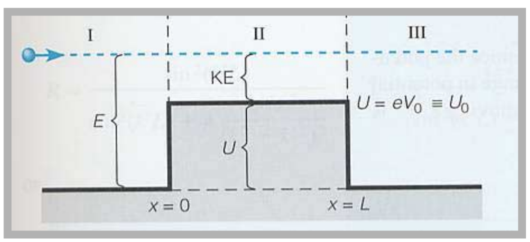
Oct. 10, 2024

Chapter. 6
Unbound States

Outline:

- The Potential Step
- The Potential Barrier & Tunneling
- Alpha Decay & Other Applications
- Particle-Wave Propagation

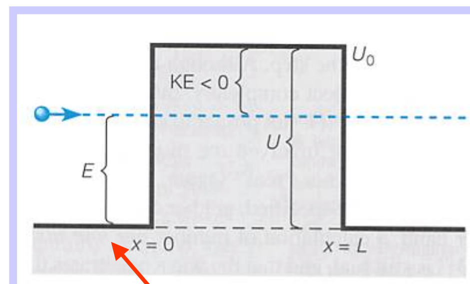
Resonant Transmission



T = 1, R = 0
for:

$$\frac{\sqrt{2m(E-U_0)}}{\hbar}L = n\pi \quad \text{or} \quad E = U_0 + \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Barrier Penetration – "Tunneling"

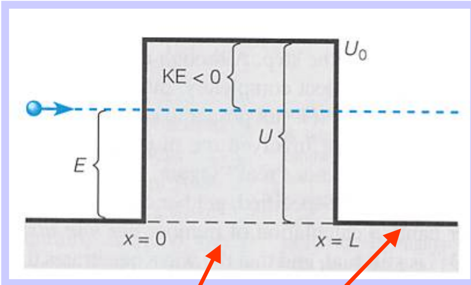


Region I ($x < 0$)

$$\psi_1(x) = Ae^{+ikx} + Be^{-ikx}$$

Incident **Reflected**

Barrier Penetration – "Tunneling"

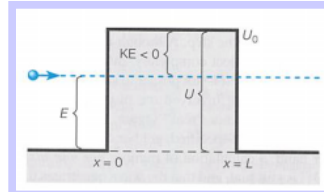


$$\psi_{II}(x) = Ce^{+\alpha x} + De^{-\alpha x}$$

Region III ($x > L$)

$$\psi_{III}(x) = Fe^{+ikx}$$

Barrier Penetration – "Tunneling"



Smoothness

$\psi(x)$ continuous at $x = 0$: $\psi_I(0) = \psi_{II}(0)$

$$A + B = C + D$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = 0$: $\left.\frac{\partial\psi_I}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = \alpha Ce^{+\alpha 0} - \alpha De^{-\alpha 0}$$

$$\Rightarrow \quad ik(A - B) = \alpha(C - D)$$

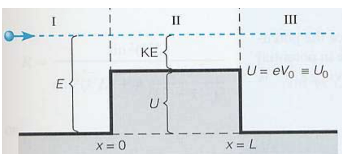
$\psi(x)$ continuous at $x = L$: $\psi_{II}(L) = \psi_{III}(L)$

$$Ce^{+\alpha L} + De^{-\alpha L} = Fe^{+ikL}$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = L$: $\left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=L} = \left.\frac{\partial\psi_{III}}{\partial x}\right|_{x=L}$

$$\alpha Ce^{+\alpha L} - \alpha De^{-\alpha L} = ikFe^{+ikL}$$

$$\Rightarrow \quad \alpha(Ce^{+\alpha L} - De^{-\alpha L}) = ikFe^{+ikL}$$



Smoothness

$\psi(x)$ continuous at $x = 0$: $\psi_I(0) = \psi_{II}(0)$

$$Ae^{+ik0} + Be^{-ik0} = Ce^{+ik'0} + De^{-ik'0}$$

$$\Rightarrow \quad A + B = C + D$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = 0$: $\left.\frac{\partial\psi_I}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = ik'Ce^{+ik'0} - ik'De^{-ik'0}$$

$$\Rightarrow \quad k(A - B) = k'(C - D)$$

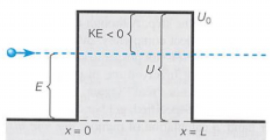
$\psi(x)$ continuous at $x = L$: $\psi_{II}(L) = \psi_{III}(L)$

$$Ce^{+ik'L} + De^{-ik'L} = Fe^{+ikL}$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = L$: $\left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=L} = \left.\frac{\partial\psi_{III}}{\partial x}\right|_{x=L}$

$$ik'Ce^{+ik'L} - ik'De^{-ik'L} = ikFe^{+ikL}$$

$$\Rightarrow \quad k'(Ce^{+ik'L} - De^{-ik'L}) = kFe^{+ikL}$$



Smoothness

$\psi(x)$ continuous at $x = 0$: $\psi_I(0) = \psi_{II}(0)$

$$A + B = C + D$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = 0$: $\left.\frac{\partial\psi_I}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = \alpha Ce^{+\alpha 0} - \alpha De^{-\alpha 0}$$

$$\Rightarrow \quad ik(A - B) = \alpha(C - D)$$

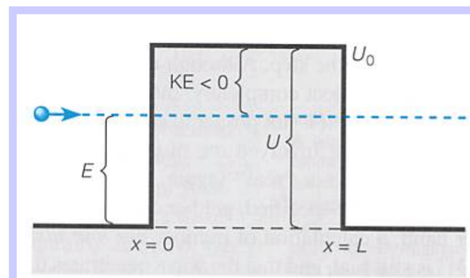
$\psi(x)$ continuous at $x = L$: $\psi_{II}(L) = \psi_{III}(L)$

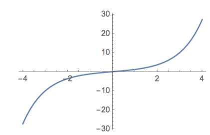
$$Ce^{+\alpha L} + De^{-\alpha L} = Fe^{+ikL}$$

$\frac{\partial\psi(x)}{\partial x}$ continuous at $x = L$: $\left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=L} = \left.\frac{\partial\psi_{III}}{\partial x}\right|_{x=L}$

$$\alpha Ce^{+\alpha L} - \alpha De^{-\alpha L} = ikFe^{+ikL}$$

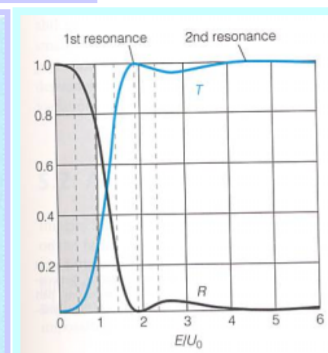
$$\Rightarrow \quad \alpha(Ce^{+\alpha L} - De^{-\alpha L}) = ikFe^{+ikL}$$

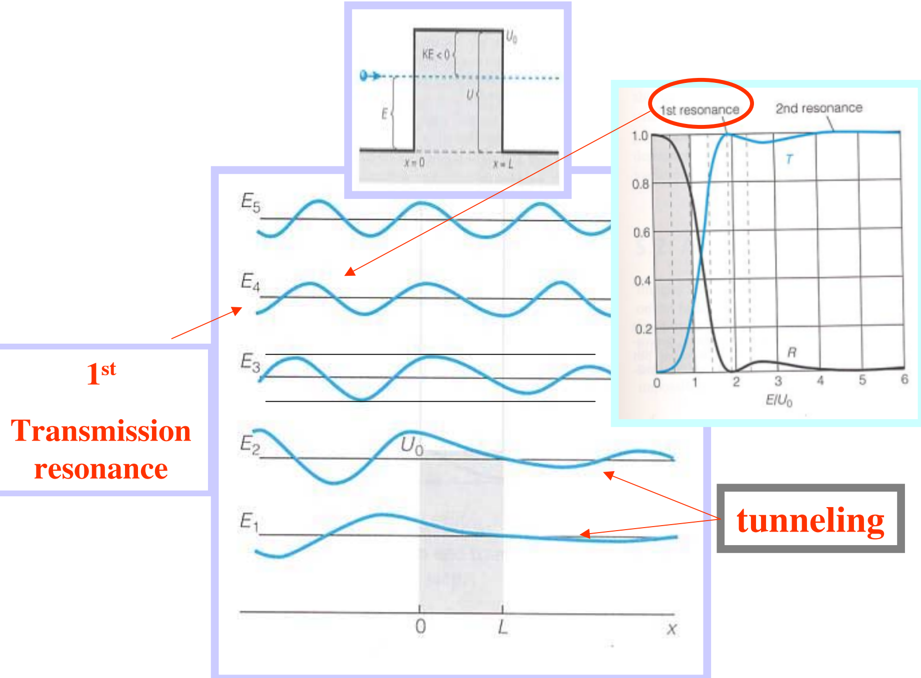




$$R = \frac{\sinh^2\left[\frac{\sqrt{2m(U_0 - E)}}{\hbar}L\right]}{\sinh^2\left[\frac{\sqrt{2m(U_0 - E)}}{\hbar}L\right] + 4\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)}$$

$$T = \frac{4\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)}{\sinh^2\left[\frac{\sqrt{2m(U_0 - E)}}{\hbar}L\right] + 4\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)}$$





Chapter. 6 Unbound States

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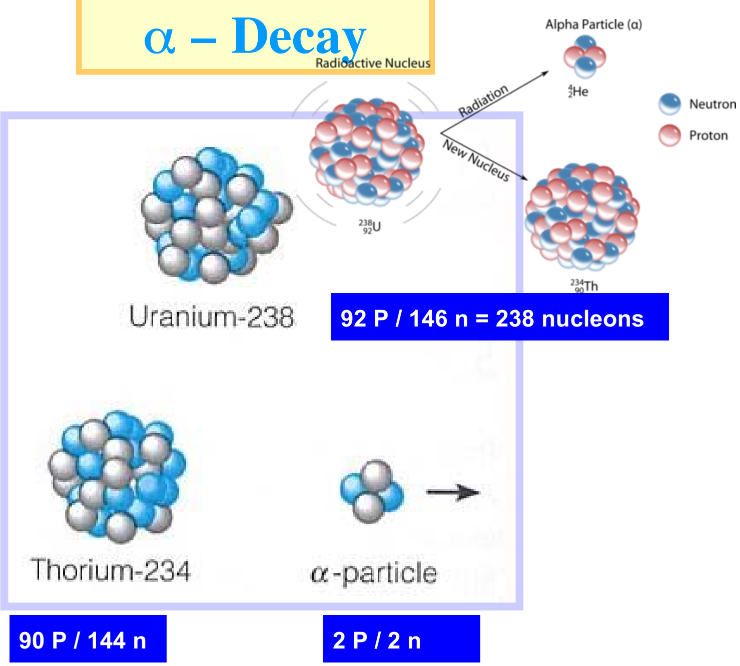
- **Alpha Decay & Other Applications**
- Particle-Wave Propagation

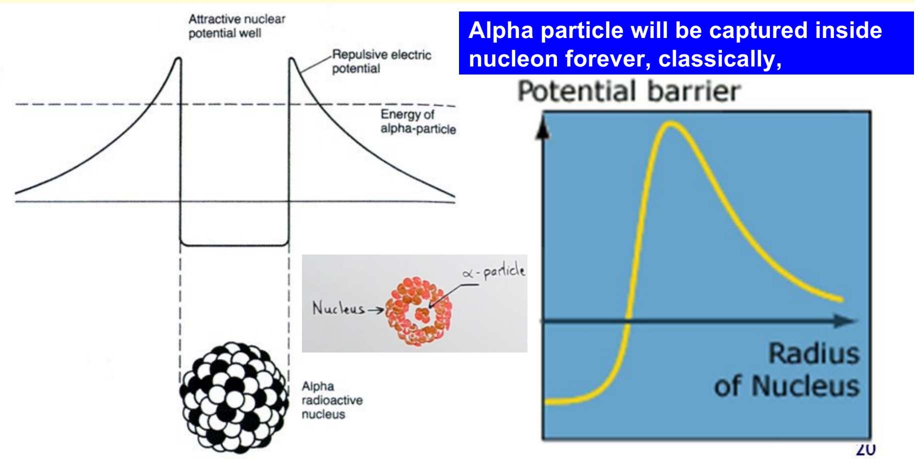
α – Decay

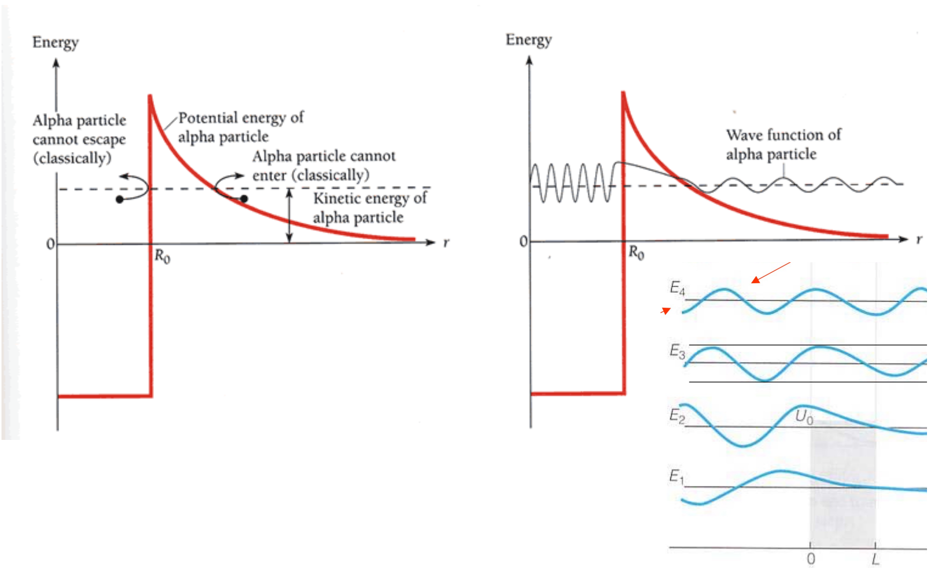


Alpha Decay - Quantum Tunnelling

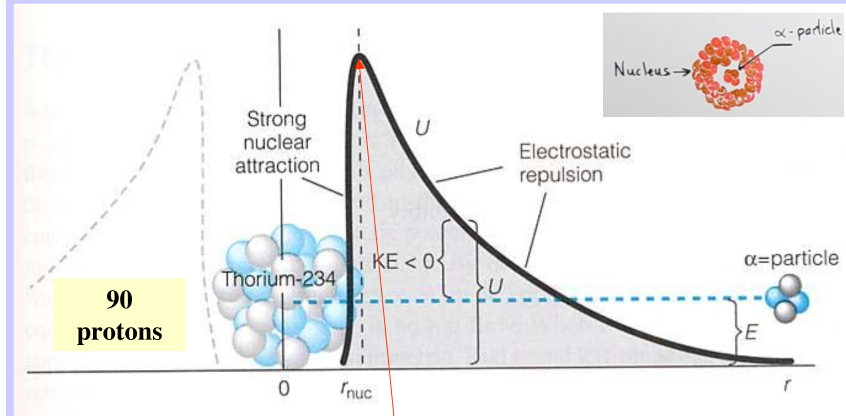
α decay of radioactive nuclei such as uranium is an example of **tunnelling.** First proposed by George Gamow in 1928.

The α particle is held inside the nucleus by strong short-range nuclear forces. Outside of the nucleus, the repulsive EM force dominates.





α – Decay



Electrostatic P.E. ~ minimum E we might classically except the alpha particle to have

$$U_{elec} = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{(2 \times 1.6 \times 10^{-19}\ \mathrm{C})(90 \times 1.6 \times 10^{-19}\ \mathrm{C})}{4\pi(8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2})(7.4 \times 10^{-15}\ \mathrm{m})}$$

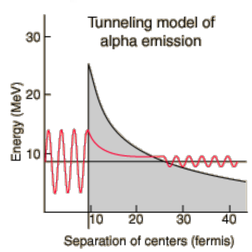
$$= 5.6 \times 10^{-12}\ \mathrm{J} = 35\ \mathrm{MeV}$$

The radius of the uranium **nucleus**

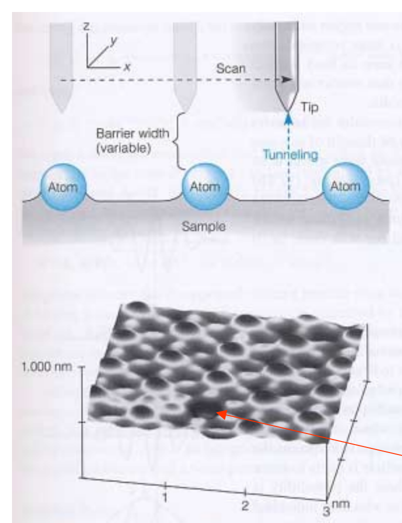
α – Decay

	α-emitting nucleus	α-particle energy (MeV)	Mean time to decay
polonium	Po-212	8.8	4.4×10^{-7} seconds
radon	Rn-220	6.3	79 seconds
Radium	Ra-224	5.7	5.3 days
	Ra-226	4.8	2300 years
	U-238	4.3	6.5×10^{9} years

Only?



Scanning Tunneling Microscope



Tunneling Prob. is very sensitive to barrier width.

Application- STM (Scanning Tunneling Microscope) – 86' Nobel prize

As the tip of a TM is scanned laterally over a samples' surface, it's able to see individual atoms.

scan x,y,z→ topological map
good for DNA, nano structure ..

a missing atom