PHYS-3301

Lecture 13

Oct. 8, 2024

PHYS 3301 - Lecture Notes & Homework [3]

• Lee, Sungwon

To: • Saldivar, Adrian; • Torres-Rodriguez, Caroline; • Rucker, Chase; • Sory, Daniel;

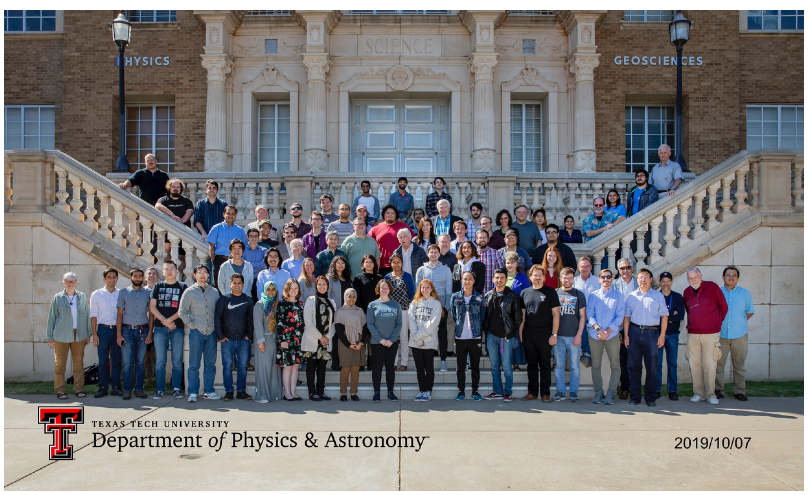
Hi PHYS-3301 students,

Two announcements:
1. Homework [3] :: **Chapter 5 :: 24, 28, 31, 34, 50, 57, 59** (due: Oct 17)
2. Lecture notes are posted on the web: http://www.phys.ttu.edu/~slee/3301/

Cheers,
Sung-Won

Sung-Won Lee, PhD | Professor and Department Chair
Department of Physics and Astronomy | Texas Tech University
Box 41051 | Lubbock | Texas 79409-1051
806.834.8188 | sungwon.lee@ttu.edu
http://www.depts.ttu.edu/phas/

Important reminder, we will take our departmental photo **today**, **October 8th, at promptly 9:20am** out on the front steps of the Science building. We haven't taken a department photo since 2019. We strongly hope you will join us to freeze frame out time as a Physics & Astronomy family!



DEPARTMENT OF PHYSICS & ASTRONOMY

Physics & Astronomy
Colloquium

Professor Benjamin Rose
Baylor University

3:30 - 4:30 p.m. | Tuesday, Oct. 8
Science Building 234

Supernova Cosmology and the Nancy Grace Roman Space Telescope

Type Ia Supernovae (SN Ia) have been used as cosmic distance probes for over 30 years. They showed the first evidence of Dark Energy and the accelerated expansion of the Universe. They have also been a key part of measuring the current expansion rate (the Hubble constant, H0) to less than 1% precision. Though SN Ia are a well-tested cosmic probe, they are also a part of several controversies, namely the disagreement of the measured and predicted values of H0 and the lack of a physical model for Dark Energy. Recent SN Ia data sets have made stringent measurements of Dark Energy and H0 but there are both astrophysical and instrumentation limitations to continuing to improve our measurements. In this talk, I will describe my research into improving our astrophysical models and hardware calibration in preparation for the SN Ia data sets into the 2030s, particularly from NASA's next flagship mission, the Nancy Grace Roman Space Telescope.

Refreshments at 3 p.m. | SC 103

TEXAS TECH
College of Arts & Sciences

The Nobel Prize in Physics 2024

Ill. Niklas Elmehed © Nobel Prize Outreach

John J. Hopfield

Prize share: 1/2

Ill. Niklas Elmehed © Nobel Prize Outreach

Geoffrey E. Hinton

Prize share: 1/2

The 2024 Nobel Prize in Physics :: Machine Learning with Artificial Neural Networks using Physics

• Lee, Sungwon — Today at 6:02 AM

To: • Lee, Sungwon

Dear all,

The 2024 Nobel Prize in Physics was awarded to **John J. Hopfield** and **Geoffrey E. Hinton** "*for foundational discoveries and inventions that enable machine learning with artificial neural networks*".

Machine learning has long been important for research, including the sorting and analysis of vast amounts of data. John Hopfield and Geoffrey Hinton used tools from physics to construct methods that helped lay the foundation for today's powerful machine learning. In announcing the prize, the Nobel committee described Hopfield and Hinton's work as not just helping advance research across different fields of physics — from material science to particle physics to astrophysics — but as something that was already changing daily life, with technology including facial recognition and language translation building off of the research.

Press release: https://www.nobelprize.org/prizes/physics/2024/press-release/
Scientific background: https://www.nobelprize.org/uploads/2024/09/advanced-physicsprize2024.pdf
Popular Information: https://www.nobelprize.org/uploads/2024/10/popular-physicsprize2024.pdf

Happy reading!

**** As in previous years, we will soon hold a special public lecture on the Nobel Prize in Physics this year. ****

Cheers,
Sung-Won

Sung-Won Lee, PhD | Professor and Department Chair
Department of Physics and Astronomy | Texas Tech University
Box 41051 | Lubbock | Texas 79409-1051
806.834.8188 | sungwon.lee@ttu.edu
http://www.depts.ttu.edu/phas/

The Nobel Prize in Physics 2024 was awarded to John J. Hopfield and Geoffrey E. Hinton "for foundational discoveries and inventions that enable machine learning with artificial neural networks"

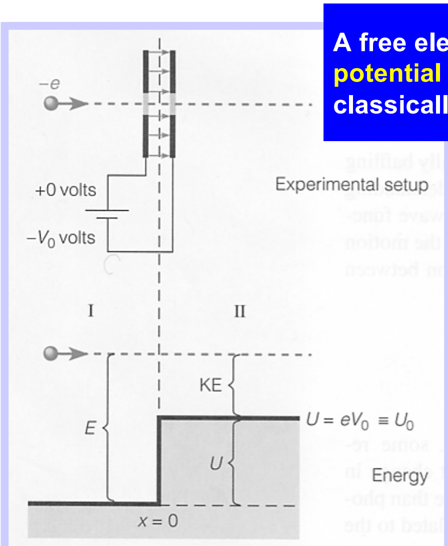
Chapter. 6
Unbound States

Outline:

- The Potential Step
- The Potential Barrier & Tunneling
- Alpha Decay & Other Applications
- Particle-Wave Propagation

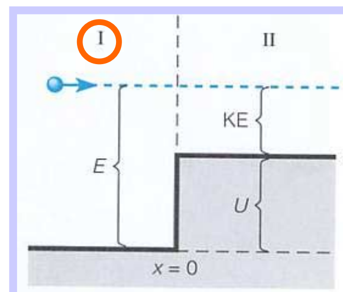
Potential Wall

A free electron encounters a potential step that is classically surmountable.



Case (1)

$E > U$



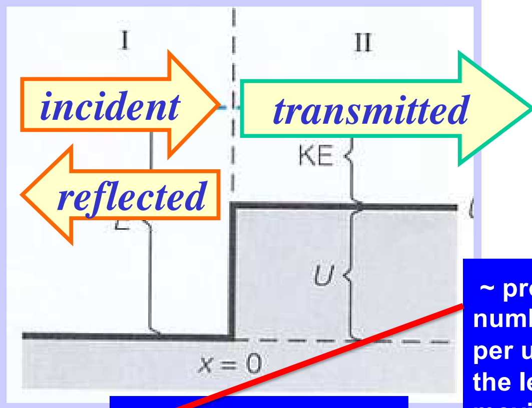
$$\frac{d^2\psi(x)}{dx^2} = \left[\frac{2m(U(x) - E)}{\hbar^2}\right]\psi(x)$$

< 0

Region I ($x < 0$)

In this region, the quantity in brackets is the negative constant $-2mE/\hbar^2$. As noted, to distinguish right-moving from left-moving particles, we do not choose $\sin(kx)$ and $\cos(kx)$ as we did in the potential well, but rather

$$\psi_1(x) = \underbrace{Ae^{+ikx}}_{\text{Incident}} + \underbrace{Be^{-ikx}}_{\text{Reflected}} \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$



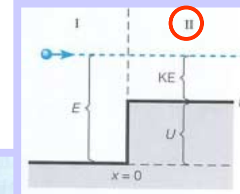
~ proportional to the number of particles per unit distance on the left of the step and moving to the right

Probability Density

$$|\psi|^2_{\text{inc}} = A^*A \qquad |\psi|^2_{\text{refl}} = B^*B$$

incident — reflected

$$\frac{d^2\psi(x)}{dx^2} = \underbrace{\left[\frac{2m(U(x)-E)}{\hbar^2}\right]}_{<0}\psi(x)$$



Region II ($x > 0$)

In this region, the quantity in brackets in equation (5-1) is the negative constant $-[2m(E-U_0)/\hbar^2]$. (It has been rearranged merely as a convenience.) An appropriate solution would seem to be $Ce^{+ik'x} + De^{-ik'x}$ where $k' \equiv \sqrt{2m(E-U_0)/\hbar^2}$. But there is a physical argument against one of these functions. Beyond $x = 0$, there is no change in potential energy—no force to cause a wave moving to the right to reflect. Because there can thus be no left-moving wave in this region, $e^{-ik'x}$ is physically unacceptable. Therefore,

$$\psi_{\text{II}}(x) = Ce^{+ik'x} \qquad \text{where } k' \equiv \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

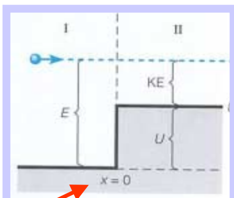
Transmitted

The probability density of finding a right-moving particle in this region is C^*C.

$$|\psi|^2_{\text{trans}} = C^*C \tag{5-3}$$

transmitted

The wave function and its derivative must be continuous at the point where the two regions meet.



Smoothness

The combined wave function must be smooth. We have found its pieces, and we must ensure that they match as they should.

$\psi(x)$ continuous at $x = 0$: $\quad \psi_{\text{I}}(0) = \psi_{\text{II}}(0)$

$$Ae^{+ik0} + Be^{-ik0} = Ce^{+ik'0} \quad \Rightarrow \quad A + B = C \tag{5-4}$$

$\dfrac{\partial\psi(x)}{\partial x}$ continuous at $x = 0$: $\quad \left.\dfrac{\partial\psi_{\text{I}}}{\partial x}\right|_{x=0} = \left.\dfrac{\partial\psi_{\text{II}}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = ik'Ce^{+ik'0} \quad \Rightarrow \quad k(A-B) = k'C \tag{5-5}$$

$$\frac{\text{number}}{\text{time}} = \frac{\text{number}}{\text{distance}}\,\frac{\text{distance}}{\text{time}} \propto |\psi|^2 v$$

$$\frac{\text{number}}{\text{time}} \propto |\psi|^2 k$$

v = p / m= hk / m ~ k

Transmission Prob.

$$T = \frac{\dfrac{\text{number transmitted}}{\text{time}}}{\dfrac{\text{number incident}}{\text{time}}} = \frac{|\psi|^2_{\text{trans}}\, k_{\text{II}}}{|\psi|^2_{\text{inc}}\, k_{\text{I}}} = \frac{C^*C\, k'}{A^*A\, k}$$

Reflection Prob.

$$R = \frac{\dfrac{\text{number reflected}}{\text{time}}}{\dfrac{\text{number incident}}{\text{time}}} = \frac{|\psi|^2_{\text{refl}}\, k_{\text{I}}}{|\psi|^2_{\text{inc}}\, k_{\text{I}}} = \frac{B^*B}{A^*A}$$

$$T = \frac{\left(\frac{2k}{k+k'}A\right)^*\left(\frac{2k}{k+k'}A\right)}{A^*A}\frac{k'}{k} = \frac{4kk'}{(k+k')^2}$$

and

$$R = \frac{\left(\frac{k-k'}{k+k'}A\right)^*\left(\frac{k-k'}{k+k'}A\right)}{A^*A} = \frac{(k-k')^2}{(k+k')^2}$$

C = [2k/(k+k')]A

B = [(k-k')/(k+k')]A

$T + R = ?$

$$T + R = \frac{4kk'}{(k+k')^2} + \frac{(k-k')^2}{(k+k')^2} =$$

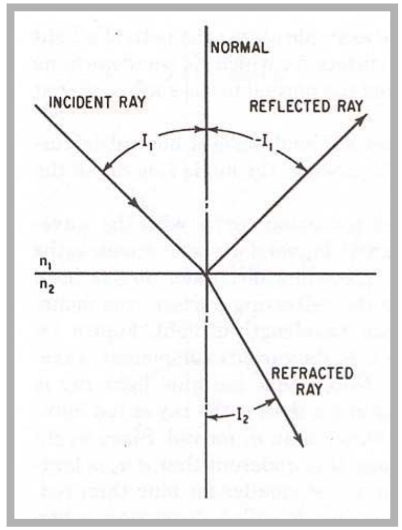
$$= \frac{4kk' + k^2 + k'^2 - 2kk'}{(k+k')^2} =$$

$$= \frac{(k+k')^2}{(k+k')^2} = 1$$

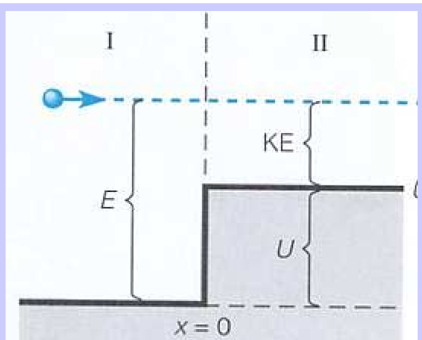
T + R = 1

Finally, we express the Prob. in terms of U0 & E *using the definitions of k, k')

$$T = 4\frac{\sqrt{E(E-U_0)}}{(\sqrt{E}+\sqrt{E-U_0})^2} \qquad R = \frac{(\sqrt{E}-\sqrt{E-U_0})^2}{(\sqrt{E}+\sqrt{E-U_0})^2}$$



Like in Optics



5 eV

3 eV

2 eV

$$T = 4\frac{\sqrt{E(E-U_0)}}{(\sqrt{E}+\sqrt{E-U_0})^2} \qquad R = \frac{(\sqrt{E}-\sqrt{E-U_0})^2}{(\sqrt{E}+\sqrt{E-U_0})^2}$$

Exam

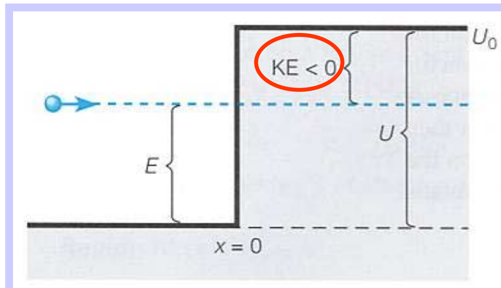
An electron whose kinetic energy is 5 eV encounters a 2-eV potential step. What is the probability that it will be reflected?

Solution

The electron's total energy is its initial kinetic energy of 5 eV (since it starts by convention where $U = 0$), and U_0 is given to be 2 eV.

$$R = \frac{(\sqrt{5}-\sqrt{5-2})^2}{(\sqrt{5}+\sqrt{5-2})^2} = 0.016$$

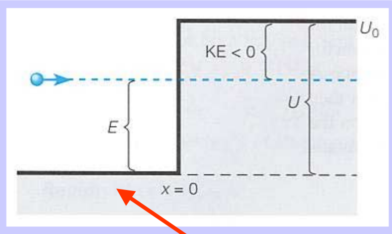
Potential Wall (*KE<U*) – "Potential Step"



Case (2)

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x > 0 \end{cases}$$

$$\frac{d^2\psi(x)}{dx^2} = \left[\frac{2m(U(x)-E)}{\hbar^2}\right]\psi(x)$$



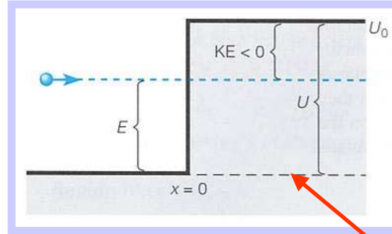
$$\frac{d^2\psi(x)}{dx^2} = \left[\frac{2m(U(x) - E)}{\hbar^2}\right]\psi(x)$$

< 0

The wave function here is the same as before:

$$\psi_I(x) = Ae^{+ikx} + Be^{-ikx} \quad \text{where } k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$

Incident (Ae^{+ikx}) **Reflected** (Be^{-ikx})



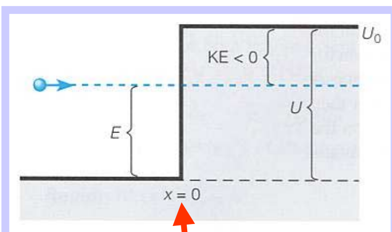
$$\frac{d^2\psi(x)}{dx^2} = \left[\frac{2m(U(x) - E)}{\hbar^2}\right]\psi(x)$$

> 0

C = 0

$$\psi(x) = Ce^{+\alpha x} + De^{-\alpha x} \quad \text{where } \alpha \equiv \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$$

However, the region to the right of the step extends forever, so we must throw out $e^{+\alpha x}$ since it diverges as x → + infinity.



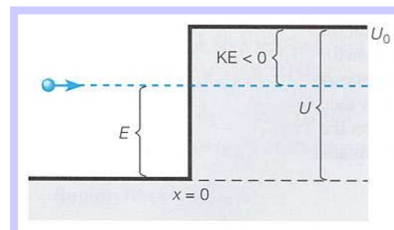
Smoothness

$\psi(x)$ continuous at $x = 0$: $\psi_I(0) = \psi_{II}(0)$

$$Ae^{+ik0} + Be^{-ik0} = De^{-\alpha 0} \Rightarrow A + B = D$$

$\frac{\partial \psi(x)}{\partial x}$ continuous at $x = 0$: $\left.\frac{\partial \psi_I}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_{II}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = -\alpha De^{-\alpha 0} \Rightarrow ik(A - B) = -\alpha D$$

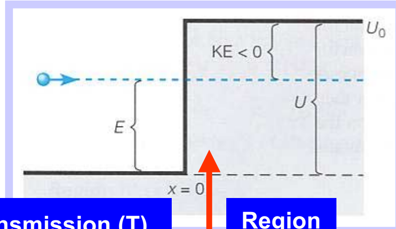


$$|B| = \sqrt{B^*B} = \sqrt{\left(-\frac{\alpha + ik}{\alpha - ik}A\right)^*\left(-\frac{\alpha + ik}{\alpha - ik}A\right)}$$

$$= \sqrt{\left(\frac{\alpha - ik}{\alpha + ik}A^*\right)\left(\frac{\alpha + ik}{\alpha - ik}A\right)} = \sqrt{A^*A} = |A|$$

$$R = \frac{B^*B}{A^*A} = 1$$

100% Reflected



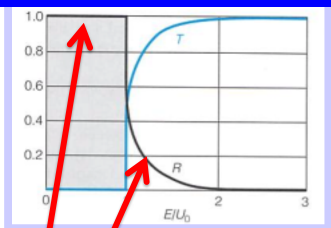
Exponentially decaying wave function outside the wall has the form

α|x| = 1, where x = distance from wall

δ should be come deeper as the E nears the value of the confining U_0

Reflection (R) &transmission (T) probabilities for a potential step

Region II



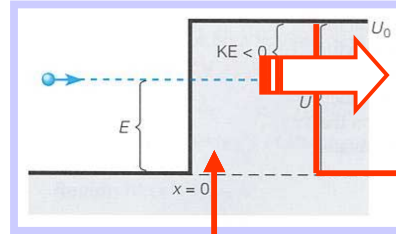
$$e^{-\alpha x} = e^{-x/\delta}.$$

$$\delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

If E < U0→ wave is totally reflected. When E > U0→ R falls rapidly with increasing E

"Penetration depth"

Section 5.6



$$e^{-\alpha x} = e^{-x/\delta}$$

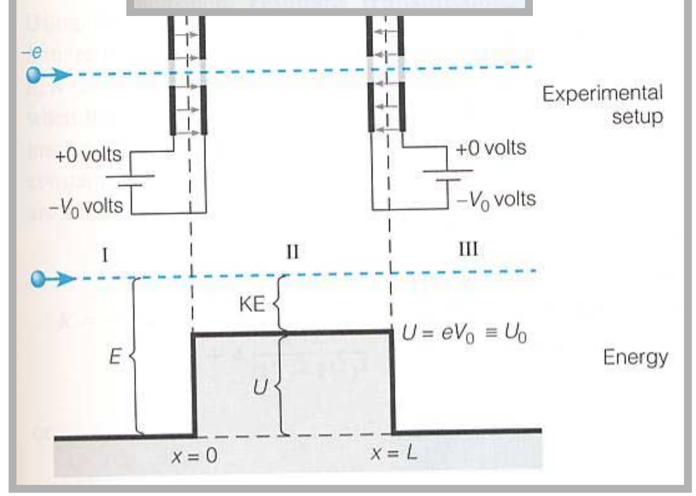
Would a particle penetrate all the way through?

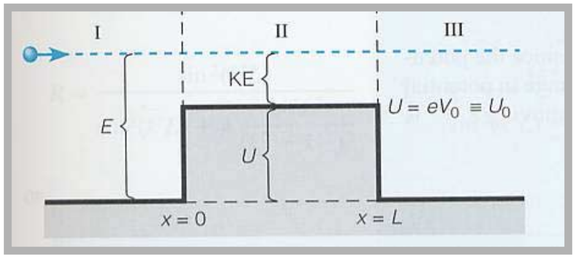
6.2 The Potential Barrier & Tunneling

- Tunneling is one of the most important & startling ideas in QM.
- The simplest solution is a potential barrier, a PE jumps that is only temporary.
- If a particle's E < Barrier's height.
  it should not get through - classically

Potential Barrier

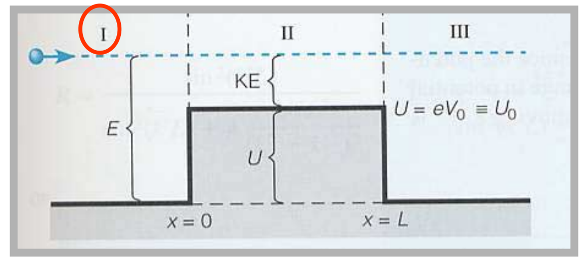
$$U(x) = \begin{cases} 0 & x < 0,\ x > L \\ U_0 & 0 < x < L \end{cases}$$





$$U(x) = \begin{cases} 0 & x < 0,\ x > L \\ U_0 & 0 < x < L \end{cases}$$

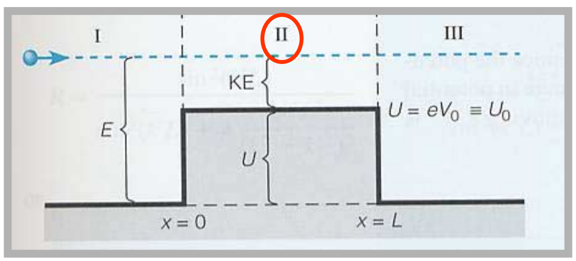
$$\frac{d^2\psi(x)}{dx^2} = \left[\frac{2m(U(x) - E)}{\hbar^2}\right]\psi(x)$$



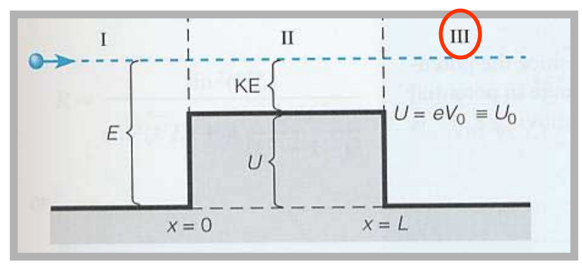
Region I ($x < 0$)

The solution is the same here as for the potential step.

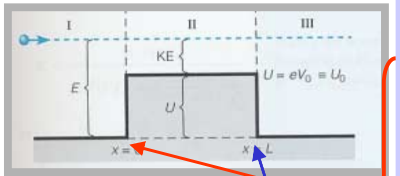
$$\psi_{\text{I}}(x) = \underset{\textbf{Incident}}{Ae^{+ikx}} + \underset{\textbf{Reflected}}{Be^{-ikx}} \quad \text{where} \quad k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$



$$\psi_{\text{II}}(x) = \underset{\textbf{Right-moving}}{Ce^{+ik'x}} + \underset{\textbf{Left-moving}}{De^{-ik'x}} \quad \text{where} \quad k' \equiv \sqrt{\frac{2m(E - U_0)}{\hbar^2}}$$



$$\psi_{\text{III}}(x) = \underset{\textbf{Transmitted}}{Fe^{+ikx}} \quad \text{where} \quad k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$



Smoothness

$\psi(x)$ continuous at $x = 0$: $\quad \psi_{\text{I}}(0) = \psi_{\text{II}}(0)$

$$Ae^{+ik0} + Be^{-ik0} = Ce^{+ik'0} + De^{-ik'0}$$

$$\Rightarrow \quad A + B = C + D$$

$\dfrac{\partial \psi(x)}{\partial x}$ continuous at $x = 0$: $\quad \left.\dfrac{\partial \psi_{\text{I}}}{\partial x}\right|_{x=0} = \left.\dfrac{\partial \psi_{\text{II}}}{\partial x}\right|_{x=0}$

$$ikAe^{+ik0} - ikBe^{-ik0} = ik'Ce^{+ik'0} - ik'De^{-ik'0}$$

$$\Rightarrow \quad k(A - B) = k'(C - D)$$

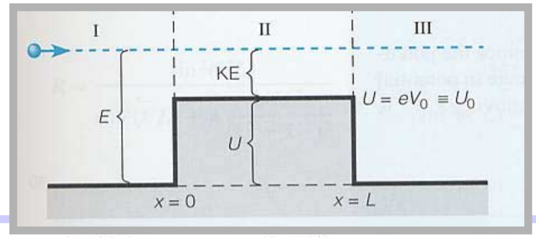
$\psi(x)$ continuous at $x = L$: $\quad \psi_{\text{II}}(L) = \psi_{\text{III}}(L)$

$$Ce^{+ik'L} + De^{-ik'L} = Fe^{+ikL}$$

$\dfrac{\partial \psi(x)}{\partial x}$ continuous at $x = L$: $\quad \left.\dfrac{\partial \psi_{\text{II}}}{\partial x}\right|_{x=L} = \left.\dfrac{\partial \psi_{\text{III}}}{\partial x}\right|_{x=L}$

$$ik'Ce^{+ik'L} - ik'De^{-ik'L} = ikFe^{+ikL}$$

$$\Rightarrow \quad k'(Ce^{+ik'L} - De^{-ik'L}) = kFe^{+ikL}$$



$$R = \frac{B^*B}{A^*A} \qquad T = \frac{|\psi|^2_{\text{trans}}\, k_{\text{III}}}{|\psi|^2_{\text{inc}}\, k_{\text{I}}} = \frac{F^*F}{A^*A}$$

$$R = \frac{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right]}{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}$$

$$T = \frac{4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}$$

(!)

R(L)

T(L)

$$\frac{\text{number}}{\text{time}} = \frac{\text{number}}{\text{distance}}\,\frac{\text{distance}}{\text{time}} \propto |\psi|^2 v$$

$$\frac{\text{number}}{\text{time}} \propto |\psi|^2 k$$

v = p / m= hk / m ~ k

Transmission Prob.

$$T = \frac{\dfrac{\text{number transmitted}}{\text{time}}}{\dfrac{\text{number incident}}{\text{time}}} = \frac{|\psi|^2_{\text{trans}}\, k_{\text{II}}}{|\psi|^2_{\text{inc}}\, k_{\text{I}}} = \frac{C^*C\,k'}{A^*A\;\,k}$$

Reflection Prob.

$$R = \frac{\dfrac{\text{number reflected}}{\text{time}}}{\dfrac{\text{number incident}}{\text{time}}} = \frac{|\psi|^2_{\text{refl}}\, k_{\text{I}}}{|\psi|^2_{\text{inc}}\, k_{\text{I}}} = \frac{B^*B}{A^*A}$$

Resonant Transmission

$$R = \frac{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right]}{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}$$

$$T = \frac{4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}{\sin^2\left[\dfrac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}$$

Is there any "L" that would give

T = 1, R = 0

?

Like in OPTICS

Resonant Transmission

$$R = \frac{\sin^2\left[\frac{\sqrt{2m(E-U_0)}}{\hbar}L\right]}{\sin^2\left[\frac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\frac{E}{U_0}\left(\frac{E}{U_0}-1\right)}$$

$$T = \frac{4\frac{E}{U_0}\left(\frac{E}{U_0}-1\right)}{\sin^2\left[\frac{\sqrt{2m(E-U_0)}}{\hbar}L\right] + 4\frac{E}{U_0}\left(\frac{E}{U_0}-1\right)}$$

= 0

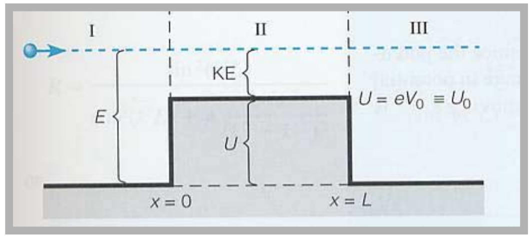
Is there any "L" that would give

T = 1, R = 0

?

Like in OPTICS

Resonant Transmission



T = 1, R = 0

for:

$$\frac{\sqrt{2m(E-U_0)}}{\hbar}L = n\pi \quad \text{or} \quad E = U_0 + \frac{n^2\pi^2\hbar^2}{2mL^2}$$