**Galilean Transformations**

The IRF transformations that preserve “invariance” of Newton’s Laws are known as **Galilean Transformations (G.Tr.)** (N. Laws are invariant under G. Tr.)

\[
\begin{align*}
    t' &= t \
    x' &= x - Vt \
    y' &= y \
    z' &= z
\end{align*}
\]

- the “absolute” time
- because the space is uniform and isotropic, the IRFs can move with respect to one another with constant velocity

Let’s check that the 2nd Newton’s Law is invariant under Galilean Transformations:

1. Differentiate with respect to the (absolute) time:
   
   \[ \frac{d}{dt} = \frac{d}{dt} - V \]
   
   - **Galilean velocity addition rule**

2. Differentiate again:
   
   \[ \frac{d^2}{dt^2} = \frac{d}{dt} \frac{d}{dt} \]
   
   - acceleration is the same in all IRFs

   The force in N. mechanics can depend (only!) on the difference of two radius-vectors and velocities. Thus

\[ \vec{F}' = \vec{F} \]

That makes vectors so useful in classical mechanics: if one can formulate a law that looks like "vector = vector", it automatically means that this law is invariant under G.Tr.

**Importance of Vectors in Classical Mechanics**

Galilean transformations do not affect the relations between vectors.

In particular, G.Tr. do not affect the length of a vector:

\[ \text{length} = \sqrt{(x_1 - x'_1)^2 + (y_1 - y'_1)^2 + (z_1 - z'_1)^2} \]

This is not the case in non-inertial RFs – an example of an accelerated RF.

**Maxwell’s Equations: challenge to Galilean PR**

In 1873, Maxwell formulated Equations of Electromagnetism. On one hand, Maxwell’s Equations describe very well all observed e.-m. phenomena, on the other hand, they are not invariant under G.Tr!

Some odd things:

At first glance, there is a built-in asymmetry: a charge in motion produces a magnetic field, whereas a charge at rest does not. Also, it follows from M.Eqs that the speed of light is the same in all IRFs, at odds with Galilean velocity addition.

This asymmetry brought into being an idea of a unique stationary RF (ether), with respect to which all velocities are to be measured, and where M. Eqs can be written in their usual form. However, the famous Michelson-Morley experiment (1887) did not detect any motion of the Earth with respect to the ether.

What are the options? At least one of the following statements must be wrong: (a) the principle of relativity applies to both mechanical and e.-m. phenomena; (b) M.Eqs are correct; (c) G. Tr. are correct.

**Einstein’s Principle of Relativity**

Einstein (1905) assumed that (a) and (b) are correct, and put forward the following

Einstein’s Principle of Relativity (the first postulate of the Special Theory of Relativity): “The laws of physics are the same (covariant) in all IRFs”.

Covariance is less restrictive than invariance: Let A=B. If, under RF transformation, both A and B are transformed into A’ and B’, but still A’=B’, then the law is covariant.

One of the consequence of Einstein’s Principle of Relativity (being applied to Maxwell’s Equations): the speed of light in vacuum is the same in all IRFs and doesn’t depend on the motion of the source of light or an observer (in line with the experimental evidence that the ether does not exist).

However, this applies to all (not necessarily e.-m.) phenomena. Thus,

The second postulate: “The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source”.

Thus, Maxwell’s Equations are in line with Einstein’s Principle of Relativity.

Conclusion: **Galilean Transformations must go. The idea of universal and absolute time is wrong!** One has to come up with correct transformations that “work” for both mechanical and e.-m. phenomena (any speed up to ~c). Consequently, the laws of mechanics have to be modified to be covariant under new (correct) transformations.
The Lorentz Transformation Equations

\[ x' = \gamma_v (x - vt) \]  \hspace{1cm} (1-8a)
\[ t' = \gamma_v \left( -\frac{v}{c^2} x + t \right) \]  \hspace{1cm} (1-8b)
\[ x = \gamma_v (x' + vt') \]  \hspace{1cm} (1-9a)
\[ t = \gamma_v \left( +\frac{v}{c^2} x' + t' \right) \]  \hspace{1cm} (1-9b)

**IMPORTANT:** space and time coordinates mix together!
(Not in Classical Physics)

In the “Classical Limit”

\[ \gamma_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 \]

“Classical Limit”:

\[ v \ll c \]
The Lorentz Transformation Equations

In the “Classical Limit”

General case: $v << c$, $\gamma \sim 1$

$$x' = \frac{x}{\gamma}(x - vt) \quad (i-8a)$$
$$x' = \gamma(x' + vt') \quad (i-9a)$$

$$t' = \gamma\left(-\frac{v}{c^2}x + t\right) \quad (i-8b)$$
$$t = \gamma\left(\frac{v}{c^2}x' + t'\right) \quad (i-9b)$$

$x' \sim x - vt \quad t' \sim t$

OK

Lorentz Transformation of Distances and Time Intervals

IMPORTANT: space and time distances mix together

$$x_2' - x_1' = \gamma_v [(x_2 - x_1) - v(t_2 - t_1)]$$
$$t_2' - t_1' = \gamma_v \left[-\frac{v}{c^2} (x_2 - x_1) + (t_2 - t_1)\right]$$
$$x_2 - x_1 = \gamma_v [(x_2' - x_1') + v(t_2' - t_1')]$$
$$t_2 - t_1 = \gamma_v \left[+\frac{v}{c^2} (x_2' - x_1') + (t_2' - t_1')\right]$$

Example 1.1

Anna is on a flatcar moving east at $0.6c$ relative to Bob (Figure 1.6). She holds a flashbulb in each hand and causes them to flash simultaneously. Anna’s hands are 2 m apart and her arms are oriented in an east-west direction. According to Bob, which flashbulb flashes first and by how much?

Solution

This time using Lorentz Transformations

Let Anna be $S'$ and Bob $S$. Event 1 is the east flash and event 2 the west flash. According to Anna, the two events occur simultaneously, so $t_1' - t_1 = 0$, and

Figure 1.6 Bob viewing two events that occur simultaneously according to Anna.

Lorentz Transformation of Distances and Time Intervals

$$0 = t_2' - t_1'$$

$$0 = \gamma_v \left[-\frac{v}{c^2} (x_2 - x_1) + (t_2 - t_1)\right]$$
Lorentz Transformation of Distances and Time Intervals

\[-2 \text{ m} = x_2' - x_1' = \gamma_0 \left[ (x_2 - x_1) - v(t_2 - t_1) \right] \]
\[0 = t_2' - t_1' = \gamma_0 \left[ -\frac{v}{c^2} (x_2 - x_1) + (t_2 - t_1) \right] \]
\[x_2 - x_1 = \gamma_0 \left[ x_2' - x_1' + \frac{v}{c^2} (t_2 - t_1') \right] \]
\[t_2 - t_1 = \gamma_0 \left[ +\frac{v}{c^2} (x_2' - x_1') + (t_2' - t_1') \right] \]

Summary

Einstein’s Postulates of Relativity:

1. The form of each physical law is the same in all inertial frames.
2. Light moves at the same speed relative to all observers.

- Michelson-Morley Experiment – NO AETHER!
- Consequences of Einstein’s Postulates:
  1. Relative Simultaneity
  2. Time Dilation
  3. Length Contraction