Announcements

HW1: Ch.2-20, 26, 36, 41, 46, 50, 51, 55, 58, 63, 65
HW1 due: 2/2 (by class hour)
Lab start-up meeting with TA – Next Monday
Lab manual is posted on the course web

*** Course Web Page ***
http://highenergy.phys.ttu.edu/~slee/2402/
Lecture Notes, HW Assignments,
Schedule for the Physics Colloquium, etc..

Lecture 5 – Chapter. 2
Special Relativity

Outline:
• Relativistic Momentum
• Relativistic Kinetic Energy
• Total Energy
• Momentum and Energy in Relativistic Mechanics
• Thursday: General Theory of Relativity
• Next Week – Quantum Physics

Relativistic Dynamics

Relativistic Momentum

\[ \vec{p} = m\vec{\nu} \]
the momentum of a particle, \( m \) is invariant (does not depend on the velocity)

Newton’s 2nd Law:
\[ \vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{\nu}}{dt} = m\vec{\alpha} \]
expressed in terms of 3-vectors, invariant under G.Tr. (but not L.Tr.!) (1)

Relativistic form of the 2nd Law (introduced by Einstein):
\[ \vec{F} = \frac{d\vec{p}}{dt} = \frac{d (m\vec{\nu})}{dt} \]
where
\[ \vec{p} = \frac{m\vec{\nu}}{\sqrt{1 - \frac{\nu^2}{c^2}}} = \gamma m\vec{\nu} \]
definition of the momentum in relativistic mechanics

Example: Calculate the momentum of an electron moving with a speed of 0.98c.
\[ p = \frac{m_e (0.98c)}{\sqrt{1-0.98^2}} = 4.9m_e c \]
By ignoring relativistic effects, one would get
\[ p = 0.98m_e c \]

Relativistic Kinetic Energy

In relativistic mechanics, the concept of energy is more useful than the force:

\[ K = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = mc^2 (\gamma - 1) \]
kinetic energy of a particle of the mass \( m \) moving with speed \( v \)
Relativistic Kinetic Energy

In relativistic mechanics, the concept of energy is more useful than the force:

\[ K = \frac{1}{2} m \left( \gamma v^2 - c^2 \right) \]

where \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \) and \( \beta = \frac{v}{c} \).

Kinetic energy of a particle of the mass \( m \) moving with speed \( v \):

\[ K = \frac{1}{2} m \left( \frac{v^2}{c^2} - 1 \right) \]

Total and Rest Energies

We expect this result to be reduced to the classical KE at low speed:

\[ K = \frac{mc^2}{\sqrt{1 - \beta^2}} - mc^2 (\beta << 1) = mc^2 \left( 1 + \frac{\beta^2}{2} - 1 \right) = \frac{m v^2}{2} \]

Let's rewrite the expression for \( K \) in the form:

\[ K = \frac{mc^2}{\sqrt{1 - \beta^2}} - mc^2 \]

Limit of small speed:

\[ E = mc^2 + m \frac{v^2}{2} \]

The energy and momentum are conserved (the consequence of uniform and isotropic space).

For an isolated system of particles:

\[ E = \sum_{i} E_i = \text{const} \]

\[ \vec{p} = \sum_{i} \vec{p}_i = \text{const} \]

Relativistic Kinetic Energy

Show that \( E^2 = p^2 c^2 + m^2 c^4 \) follows from \( p = \gamma \mu \) and \( E = \gamma mc^2 \) for momentum and energy in terms of \( m \) and \( u \).
\[ E^2 - p_x^2 c^2 - p_y^2 c^2 - p_z^2 c^2 = \text{INARIANT} = m^2 c^4 \]

\[ E^2 - p_x^2 c^2 - p_y^2 c^2 - p_z^2 c^2 = E'^2 - p_x'^2 c^2 - p_y'^2 c^2 - p_z'^2 c^2 \]

\[ E^2 - \vec{p} \cdot \vec{c}^2 = (mc^2)^2 \]

\[ E^2 = \vec{p} \cdot \vec{c}^2 + (mc^2)^2 \]

\[ E = \sqrt{\vec{p} \cdot \vec{c}^2 + (mc^2)^2} \]

**Revolutionary Concept**

**What about**

\[ E = mc^2 \]

\[ E = \sqrt{p \cdot c^2 + (mc^2)^2} \]

\[ E = \text{INTERNAL ENERGY} \]

\[ E = mc^2 \] (when \( p = 0 \))

\[ E = \text{TOTAL ENERGY} \]

\[ E = \sqrt{p \cdot c^2 + (mc^2)^2} \]
Expressions for (total) Energy and Momentum of a particle of mass $m$, moving at velocity $u$

$$p = \gamma_u m u \quad \text{where} \quad \gamma_u = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$E = \gamma_u m c^2$$

Classical Limit

$$\approx 1$$

$$p = \gamma_u m u \quad \text{where} \quad \gamma_u = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\vec{p} \approx m \vec{u}$$

$$E = \gamma_u m c^2$$

$$E \approx m c^2 + \frac{m u^2}{2}$$

Kinetic Energy = KE

$$KE = (\text{energy moving}) - (\text{energy at rest})$$

$$= \gamma_u m c^2 - m c^2 = (\gamma_u - 1) m c^2$$

$$E_{\text{internal}} = m c^2$$

NEW

FAMILIAR
kinetic energy
Atomic Bomb (Chapter 10):
Energy is *created*
From the Mass of Nuclei
(Internal energy is transformed into kinetic energy)

Is there *Absolute Causality*?

Might *cause* precede *effect* in one reference frame but *effect* precede *cause* in different reference frame(s)?

*E.g.* can someone see you first die, and then see you get born?

Let’s assume that the order of events is changed in some reference frame S’

$$\Delta t > 0$$

$$\Delta t' < 0$$

Is that possible?

*Δt* and *Δt’* are the time intervals between the same two events observed in S and S’, respectively.
Using Lorentz transformations….

\[
\Delta t' = \gamma \left( -\frac{v}{c^2} \Delta x + \Delta t \right)
\]

\[
\Delta t' = \gamma \Delta t \left( -\frac{v}{c^2} \frac{\Delta x}{\Delta t} + 1 \right)
\]

if \( \Delta t > 0 \) then \( \Delta t' < 0 \)

\( \Delta t' < 0 \)

if \( \Delta t > 0 \) then

Impossible

Is there *Absolute Causality*?

Might *cause* precede *effect* in one reference frame but *effect* precede *cause* in different reference frame(s)?

e.g. can someone see you first die, and then see you get born?
Some Examples

1. What is the momentum of an electron with $K = mc^2$?

$$p = \sqrt{\left(\frac{E}{c}\right)^2 - m^2c^2} = \sqrt{\left(\frac{mc^2 + K}{c}\right)^2 - m^2c^2} = \sqrt{4m^2c^2 - m^2c^2} = \sqrt{3}mc$$

2. How fast is a proton traveling if its kinetic energy is 2/3 of its total energy?

$$K = \frac{2}{3}E = \frac{2}{3}(mc^2 + K) \quad E = 3mc^2$$

$$E = \frac{mc^2}{\sqrt{1 - (V/c)^2}} \quad \frac{1}{\sqrt{1 - (V/c)^2}} = 3 \quad 1 - \left(\frac{V}{c}\right)^2 = \frac{1}{9} \quad V = \frac{\sqrt{8}}{3}c$$

Problem

An electron whose speed relative to an observer in a lab RF is $0.8c$ is also being studied by an observer moving in the same direction as the electron at a speed of $0.5c$ relative to the lab RF. What is the kinetic energy (in MeV) of the electron to each observer?
Problem

An electron whose speed relative to an observer in a lab RF is 0.8c is also being studied by an observer moving in the same direction as the electron at a speed of 0.5c relative to the lab RF. What is the kinetic energy (in MeV) of the electron to each observer?

\[ v' = \frac{v - v'}{1 - \frac{v' c}{c}} = \frac{0.8c - 0.5c}{1 - (0.8)(0.5)} = 0.5c \]

In the lab IRF K:

\[ K = mc^2 \left( 1 - \frac{v^2}{c^2} \right) \approx 0.5MeV \left( 1 - \frac{(0.5c)^2}{c^2} \right) \approx 0.34 MeV \]

In the moving IRF K':

\[ K' = mc^2 \left( 1 - \frac{v'^2}{c^2} \right) \approx 0.5MeV \left( 1 - \frac{(0.5c)^2}{c^2} \right) \approx 0.08 MeV \]

Problem

An electron initially moving with momentum \( p=mc \) is passed through a retarding potential difference of \( V \) volts which slows it down; it ends up with its final momentum being \( mc/2 \).

(a) Calculate \( V \) in volts.
(b) What would \( V \) have to be in order to bring the electron to rest?

\[ p = mc, \quad p' = mc/2 \]

Thus, the retarding potential difference \( V = 1.5 \times 10^3 V \)

(a) \( p=mc: \quad E_1 = \sqrt{p^2 c^2 + (mc^2)^2} = \sqrt{(mc^2)^2 + (mc^2)^2} = \sqrt{2}mc^2 \)

(b) \( p=mc/2: \quad E_2 = \sqrt{\left( \frac{1}{2} \frac{mc^2}{c^2} \right)^2 + (mc^2)^2} = \sqrt{\frac{5}{2}}mc^2 \)

\[ \Delta E = E_1 - E_2 = (\sqrt{2} - \frac{\sqrt{5}}{2})mc^2 \approx 0.3mc^2 \approx 0.3 \times 10^9 eV = 1.5 \times 10^9 eV \]

Problem

An unstable particle of mass \( m \) moving with velocity \( v \) relative to an inertial lab RF disintegrates into two gamma-ray photons. The first photon has energy 8 MeV in the lab RF and travels in the same direction as the initial particle; the second photon has energy 4 MeV and travels in the direction opposite to that of the first. Write the relativistic equations for conservation of momentum and energy and use the data given to find the velocity \( v \) and rest energy, in MeV, of the unstable particle.

\[ \text{photon 2} \quad \text{photon 1} \]

before

after

\[ E_1 = \sqrt{2}mc^2 \quad E_2 = mc^2 \quad \Delta E = (\sqrt{2} - 1)mc^2 = 2.1 \times 10^9 eV \quad V = 2.1 \times 10^9 V \]
An unstable particle of mass $m$ moving with velocity $v$ relative to an inertial lab RF disintegrates into two gamma-ray photons. The first photon has energy 8 MeV in the lab RF and travels in the same direction as the initial particle; the second photon has energy 4 MeV and travels in the direction opposite to that of the first. Write the relativistic equations for conservation of momentum and energy and use the data given to find the velocity $v$ and rest energy, in MeV, of the unstable particle.

\[
\begin{align*}
\frac{mv}{\sqrt{1-(v/c)^2}} &= E_{\text{ph1}} - E_{\text{ph2}} \quad \text{momentum conservation} \\
\frac{mc^2}{\sqrt{1-(v/c)^2}} &= E_{\text{ph1}} + E_{\text{ph2}} \quad \text{energy conservation}
\end{align*}
\]

(a) \[ \frac{mc^2}{\sqrt{1-(v/c)^2}} = E_{\text{ph1}} - E_{\text{ph2}} = 8\text{MeV} - 4\text{MeV} = 4\text{MeV} \]

(b) \[ \frac{mc^2}{\sqrt{1-(v/c)^2}} = E_{\text{ph1}} + E_{\text{ph2}} = 8\text{MeV} + 4\text{MeV} = 12\text{MeV} \]

\[ mc^2 = (E_{\text{ph1}} + E_{\text{ph2}})\sqrt{1-(v/c)^2} = (12\text{MeV})\sqrt{1-1/9} \approx 11.3\text{MeV} \]

A moving electron collides with a stationary electron and an electron-positron pair comes into being as a result. When all four particles have the same velocity after the collision, the kinetic energy required for this process is a minimum. Use a relativistic calculation to show that \( K_{\text{min}} = 6mc^2 \), where $m$ is the electron mass.

\[
\begin{align*}
|\text{before}| & \quad |\text{after}| \\
E_i + mc^2 &= 4E_2 \\ p_i &= 4p_2
\end{align*}
\]

\[
\begin{align*}
E_i^2 &= (mc^2)^2 + (p_i c)^2 \\
E_i^2 &= (mc^2)^2 + (p_2 c)^2 \\
E_i + mc^2 &= 4E_2 \\
p_i &= 4p_2
\end{align*}
\]

\[
\begin{align*}
E_i &= 14(mc^2)^2/2mc^2 = 7mc^2 \quad K_i = E_i - mc^2 = 6mc^2
\end{align*}
\]

\[
\begin{align*}
E^2 = c^2 p^2 &= (mc^2)^2 \\
E^2 - c^2 p^2 &= (mc^2)^2
\end{align*}
\]

### From before...

\[
E = \gamma m_o c^2 = KE_{rel} + m_o c^2
\]

\[
E = \gamma m_o c^2, \text{ or } E = mc^2
\]

- The total energy of a particle is dependent on its kinetic energy and its mass.
- Even when the particle is not moving, \textit{i.e. no KE}, it has (internal) energy.
- Mass is another form of energy
  - Moreover, energy can show up as mass.

### Space & Time / Energy & Momentum

- Relativity mixes up space & time - also energy & momentum
  - When converting from one inertial frame to another
  - In the time dilation and length contraction formulas, \textit{time} is in the \textit{length} formula and \textit{length} is in the \textit{time} volume through the velocity (i.e. velocity = length/time)
  - In the total energy formula, momentum (or kinetic energy) and mass energy are related
- There are combinations of space/time and energy/momentum that observers in any inertial frame will measure the as the same
  - For energy and momentum this invariant says that all observers can agree on mass an object has when it’s at rest!
General Relativity

- General relativity is the geometric theory of gravitation published by Albert Einstein in 1916.
- It is the current description of gravitation in modern physics.
- It unifies special relativity and Newton's law of universal gravitation, and describes gravity as a geometric property of space and time.
- In particular, the curvature of space-time is directly related to the four-momentum (mass-energy and momentum).
- The relation is specified by the Einstein's field equations, a system of partial differential equations. (Graduate level course)

Many predictions of general relativity differ significantly from those of classical physics.

- Examples of such differences include gravitational time dilation, the gravitational red-shift of light, and the gravitational time delay.
- General relativity's predictions have been confirmed in all observations and experiments to date.
- However, unanswered questions remain, solution is the quantum gravity!!

Special Theory of Relativity

The two postulates:
1. The form of each physical law is the same in all inertial frames.
2. Light moves at the same speed relative to all observers.

BUT:

- Localized frames of reference

General Theory of Relativity:

1. The form of each physical law is the same in all inertial frames.
2. Light moves at the same speed relative to all observers.
**Special Theory of Relativity:**

Deals exclusively with **globally** INERTIAL FRAMES -

\[ v = \text{constant} \]

1. The form of each physical law is the same in all inertial frames.
2. Light moves at the same speed in all inertial frames.

**General Theory of Relativity:**

Deals also with **locally** INERTIAL FRAMES -

1. Deals with Accelerating -
2. Accelerating reference frames are indistinguishable from a gravitational force.

**Equivalence Principle**

Accelerating reference frames are **indistinguishable** from a gravitational force.

See what this means!!

**Acceleration**

**Gravitational Force**

**Try some experiments**

Constant velocity

Floor accelerates upward to meet ball

Cannot do any experiment to distinguish accelerating frame from gravitational field.
Light follows the same path

Path of light beam in our frame

Velocity = \( v \)

Light

Velocity = \( v + at_0 \)

Path of light beam in accelerating frame \( t=2t_o \)

Is light bent by gravity?

• If we can’t distinguish an accelerating reference frame from gravity…
• and light bends in an accelerating reference frame…
• then light must bend in a gravitational field

But light doesn’t have any mass. How can gravity affect light?

Maybe we are confused about what a straight line is

Which of these is a straight line?

A. A
B. B
C. C
D. All of them

Which of these is a straight line?

A. A
B. B
C. C
D. All of them
Straight is shortest distance!!

- They are the shortest distances determined by wrapping string around a globe. On a globe, they are called ‘great circles’.
- This can be a general definition of straight, and is in fact an intuitive one on curved surfaces.
- It is the one Einstein used for the path of all objects in curved space-time.
- The confusion comes in when you don’t know you are on a curved surface.

Mass and Curvature

- **General relativity** says that any mass will give space-time a curvature.
- Motion of objects in space-time is determined by that curvature.

Idea behind geometric theory

- Matter bends **space** and **time**.
- Bending on a two-dimensional surface is characterized by the radius of curvature: $r$.
- Einstein deduced that $1/r^2$ is proportional to the local **energy** and **momentum** density.
- The proportionality constant is $\frac{8\pi G}{c^2}$.
- where $G$ is Newton's constant.

A test of General Relativity

- Can test to see if the path of light appears curved to us.
- Local massive object is the sun.
- Can observe apparent position of stars with and without the sun.
Space is Curved?

- Einstein said to picture gravity as a warp in space
- Kepler’s Laws can all be explained by movement around these “puckers”
- Everything moving is affected, regardless of mass

Other Consequences of GR

- Time dilation from gravity effects
- Gravitational Radiation!
  - Created when big gravity sources are moved around quickly
  - Similar to the electromagnetic waves that were caused by moving electron charges quickly
- Black Holes
- Expanding Universe (although Einstein missed the chance to predict it! – He didn’t believed)

1. Gravitational Time Dilation

- Gravity warps both space and time!
- At 10,000 km above the Earth’s surface, a clock should run 4.5 parts in $10^{10}$ faster than one on the Earth
- Comparing timing pulses from atomic oscillator clocks confirms the gravitational time dilation in 1976 to within 0.01%.
- Corrections are now standard in the synchronizing satellites
- This correction needed in addition to the special relativity correction for GPS

2. Gravitational Radiation

- When a mass is moved, the curvature of space-time changes
- If a mass is oscillated, ripples of space-time curvature carry the signal
- Gravitational radiation carries energy and momentum and wiggles mass in its path
Evidence for Gravity Waves

• In 1974, Joseph Taylor and his student Russell Hulse discovered a binary neutron star system losing energy as expected from gravitational radiation.

Direct Detection of Gravity Waves

LIGO is a collection of large laser interferometers searching for gravity waves generated by exploding stars or colliding black holes.

The big bang

• In 1929 observation of nearby and far away galaxies indicate that everything is receding from us.
  – Key physics needed to understand this is the simple Doppler shift of light waves. Waves from sources moving away from us are stretched out or lower frequency.
• Extrapolating backwards indicates that all the galaxies originated from the same source 14 billion years ago.
• In 1964 radiation from the early stages of that explosion was detected.
  – Again the Doppler shift was the key since the waves were shifted to low frequency - microwave.

Nobel Prize in 2006

• For the universe to start small and expand space and time must be thing that can expand(or contract)
  – General relativity was key physics needed to understand that process
• However, a simple model of that would predict such a universe would not have clumps of matter(stars, galaxies)
• Unless those clumping were present very early on
• 2006 Nobel prize was given to the people who designed the COBE experiment which was sensitive enough to see those clumping in the CMB.
Conclusion

Maxwell’s Equations of electromagnetism (1873)

Newtonian Mechanics, Thermodynamics, Statistical Mechanics

Relativistic mechanics, El.-Mag. (1905)

Classical physics

Quantum mechanics (1920’s-)

v/c

h/s

Relativistic quantum mechanics (1927-)

Question:

Should we use relativistic or classical approach to describe the motion of an electron in H atom?