6. Find the eigenvalues and normalized eigenvectors for each of the matrices M_1 and M_2 listed below:

$$\mathbf{M_1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{M_2} = \begin{bmatrix} 1 & i & 1 \\ -i & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Finally, find the transformation matrices U_1 and U_2 such that $U_1^{-1} M_1 U_1 = \lambda_1$ and $U_2^{-1} M_2 U_2 = \lambda_2$, where λ_1 and λ_2 are diagonal matrices with the eigenvalues of M_1 and M_2 on the diagonal, respectively.

7. The scattering matrix for $V(x) = -\alpha\delta(x)$ that we derived in class is

$$\mathbf{S} = \begin{bmatrix} \frac{i\beta}{1-i\beta} & \frac{1}{1-i\beta} \\ \frac{1}{1-i\beta} & \frac{i\beta}{1-i\beta} \end{bmatrix},$$

where $\beta \equiv m\alpha/\hbar^2 k$. Is S Hermitian? Unitary? Prove it!