How do we *really* look for gravitational waves?

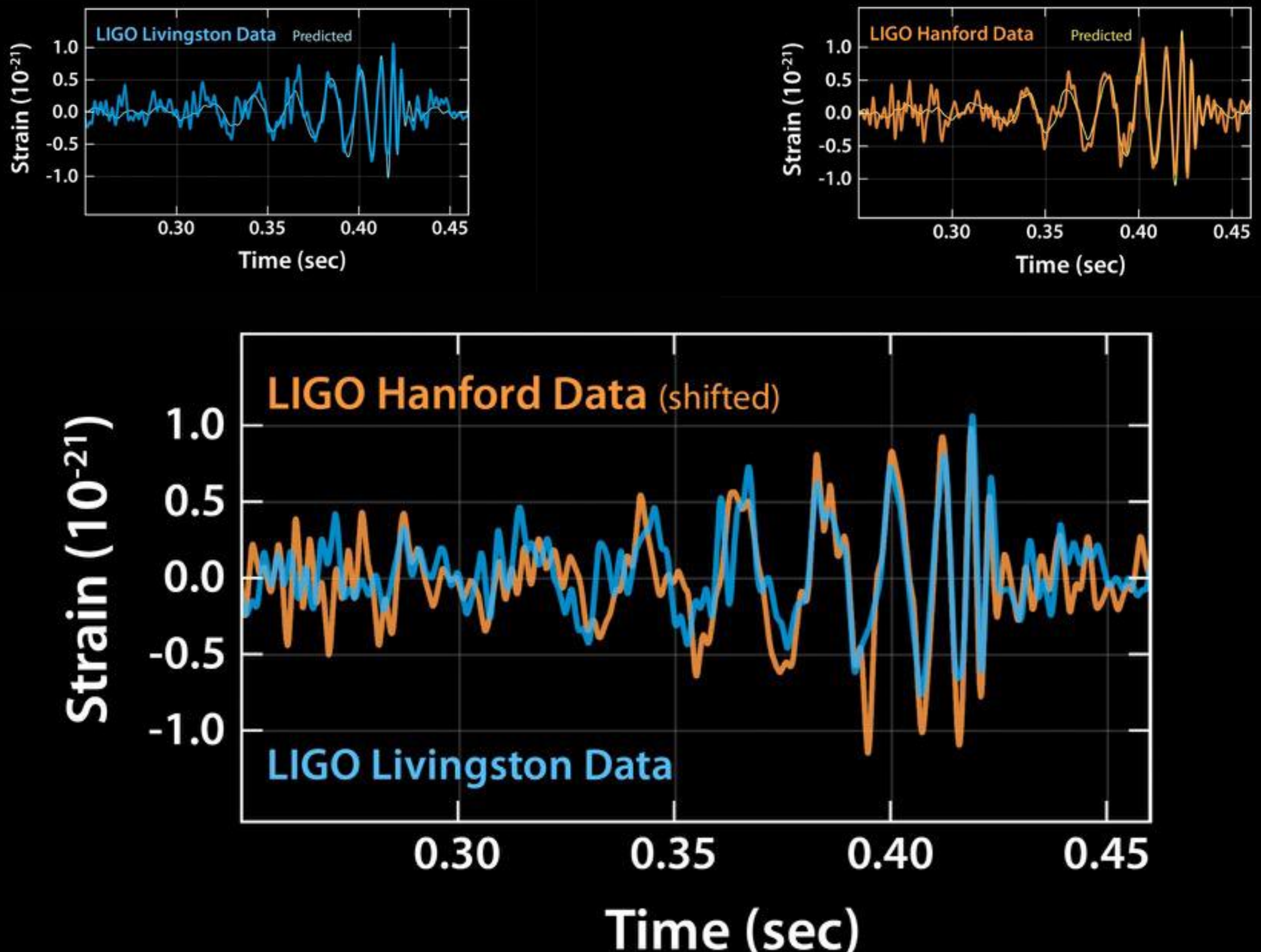
A tour of some applied mathematical tools used within the LIGO and Virgo collaborations

Ra Inta (Texas Tech University)

for the LIGO Scientific Collaboration and the Virgo Collaboration

LIGO Document G1700692-v3

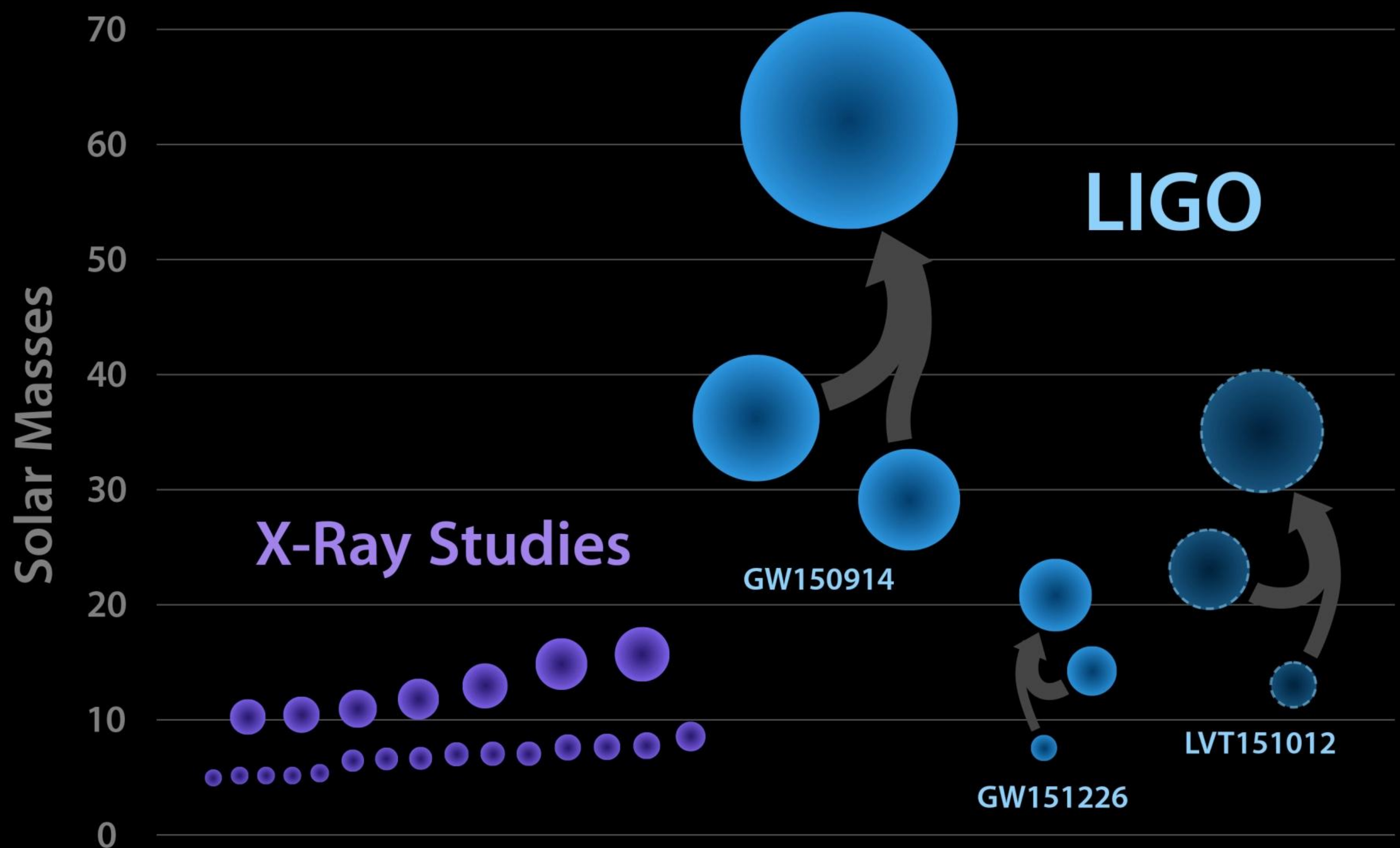
Gravitational waves



Caltech/MIT/LIGO Lab

Abbott, B.P. et al., *PRL* **116**:061102 (2016)

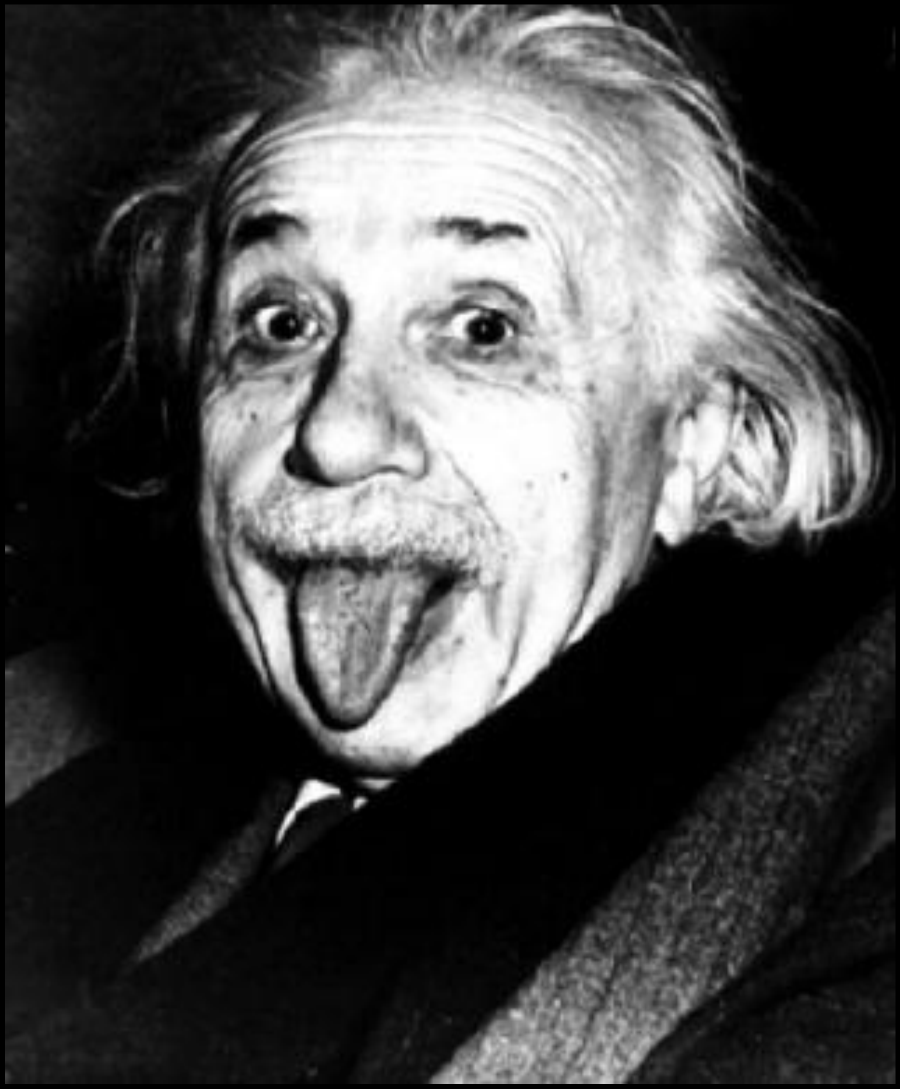
Black Holes of Known Mass



LVC: “The basic physics of the binary black hole merger GW150914,” *Annalen der Physik* **529**(1) (2017)

Image: LIGO

History of Gravitational Waves (GWs)



Einstein, A.: *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)* **1**, 688 (1916)

History of Gravitational Waves (GWs)

Einstein, A. and Rosen, N.:"On Gravitational Waves," *J. Franklin Institute* **223**, pp.43-54 (1937)

search for: "who's afraid of the referee?"

Linearized general relativity

Take small perturbations, **h**, of the space-time metric, **g**:

$$g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon} \qquad \left\| h_{\mu\upsilon} \right\| << 1$$

Put into the Einstein Field Equations:

$$G_{\mu\upsilon} = \frac{8\pi G}{c^4} T_{\mu\upsilon}$$

Get a wave-equation (in transverse-traceless gauge):

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

Linearized general relativity

Vacuum solution $\Box \bar{h}_{\mu\nu} = 0$

Admits plane waves: $\bar{h}_{\mu\upsilon} = A_{\mu\upsilon} \exp(ik_{\sigma}x^{\sigma})$

So: $k_{\sigma}k^{\sigma} = 0$ (i.e. *k* is null)

Harmonic gauge: $A_{\mu\upsilon}k^{\mu} = 0$ (Transverse polarization)

Linearized general relativity

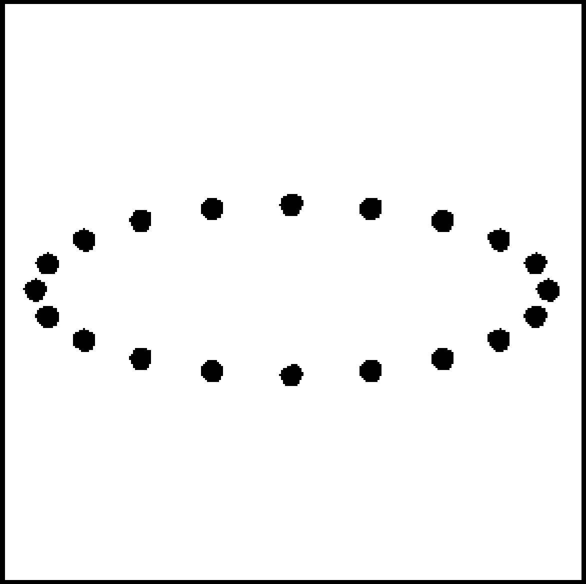
Two polarization states:

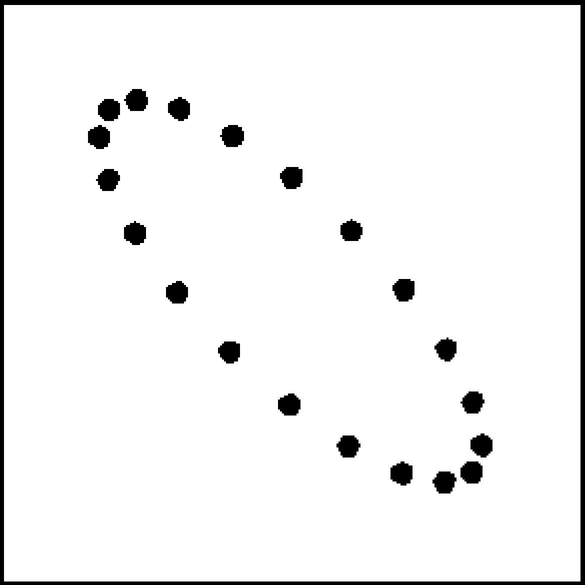
Mass quadrupole:

$$\bar{h}_{ij} = \frac{2G}{c^4 D}\ddot{I}_{ij}$$

Linearized general relativity

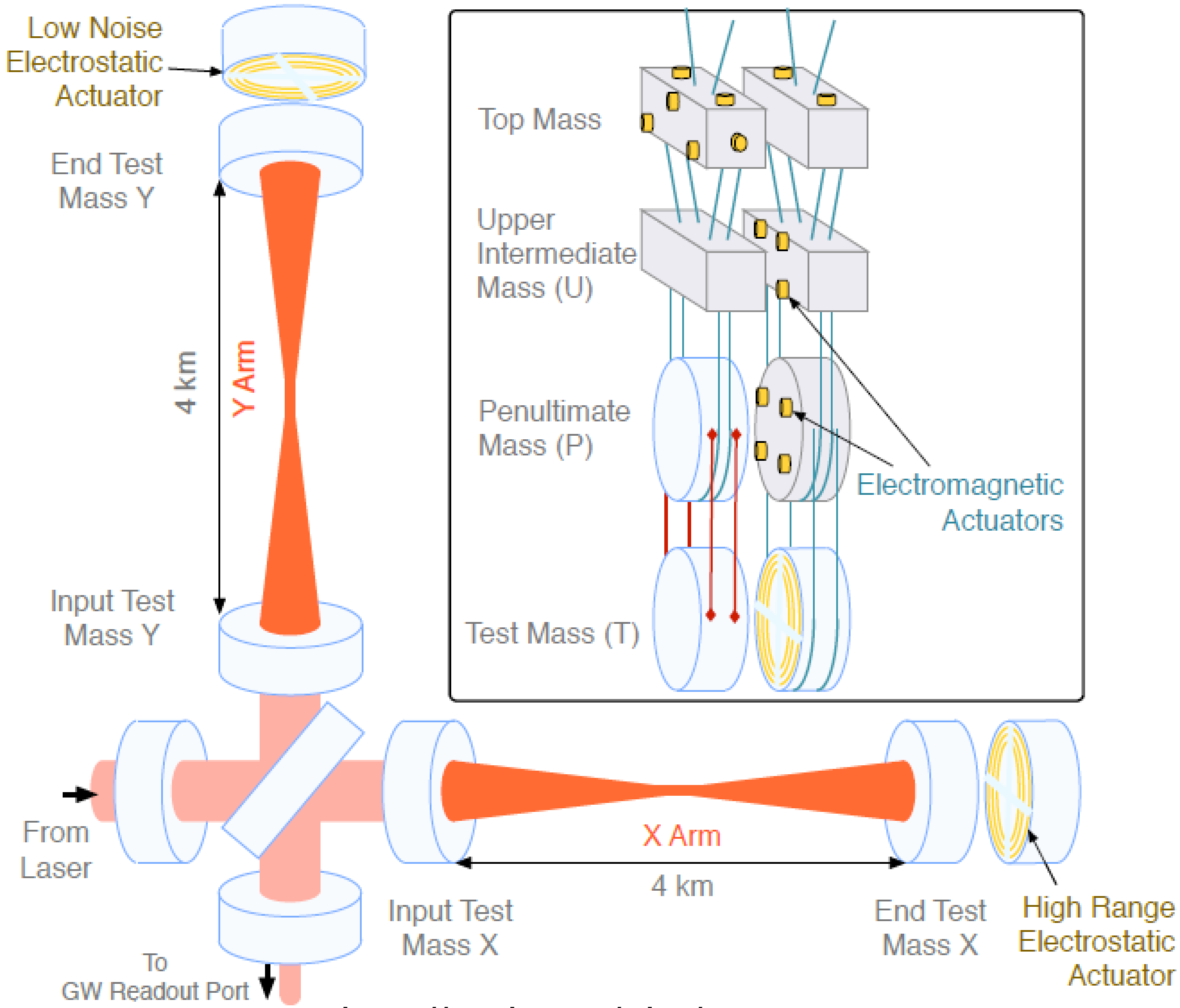
Two polarization states:





Mass quadrupole: $$\bar{h}_{ij} = \frac{2G}{c^4 D} \ddot{I}_{ij}$$

LASER interferometers



http://arxiv.org/abs/1602.03845

The LIGO Network

4 km baseline, seismic isolation

Hanford, WA
4 km
3,030 km
Livingston, LA
4 km
Canada
Saskatchewan
Manitoba
Montana
North Dakota
Minnesota
South Dakota
Wisconsin
Michigan
Toronto
Detroit
Chicago
Wyoming
Nebraska
Iowa
San Francisco
Nevada
Utah
San Jose
California
Colorado
Illinois
Ohio
Indianapolis
Missouri
Kentucky
Los Angeles
San Diego
Arizona
Phoenix
New Mexico
Arkansas
Tennessee
North Carolina
South Carolina
Georgia
Dallas
Texas
San Antonio
Florida
Gulf of California
Image © 2008 TerraMetrics
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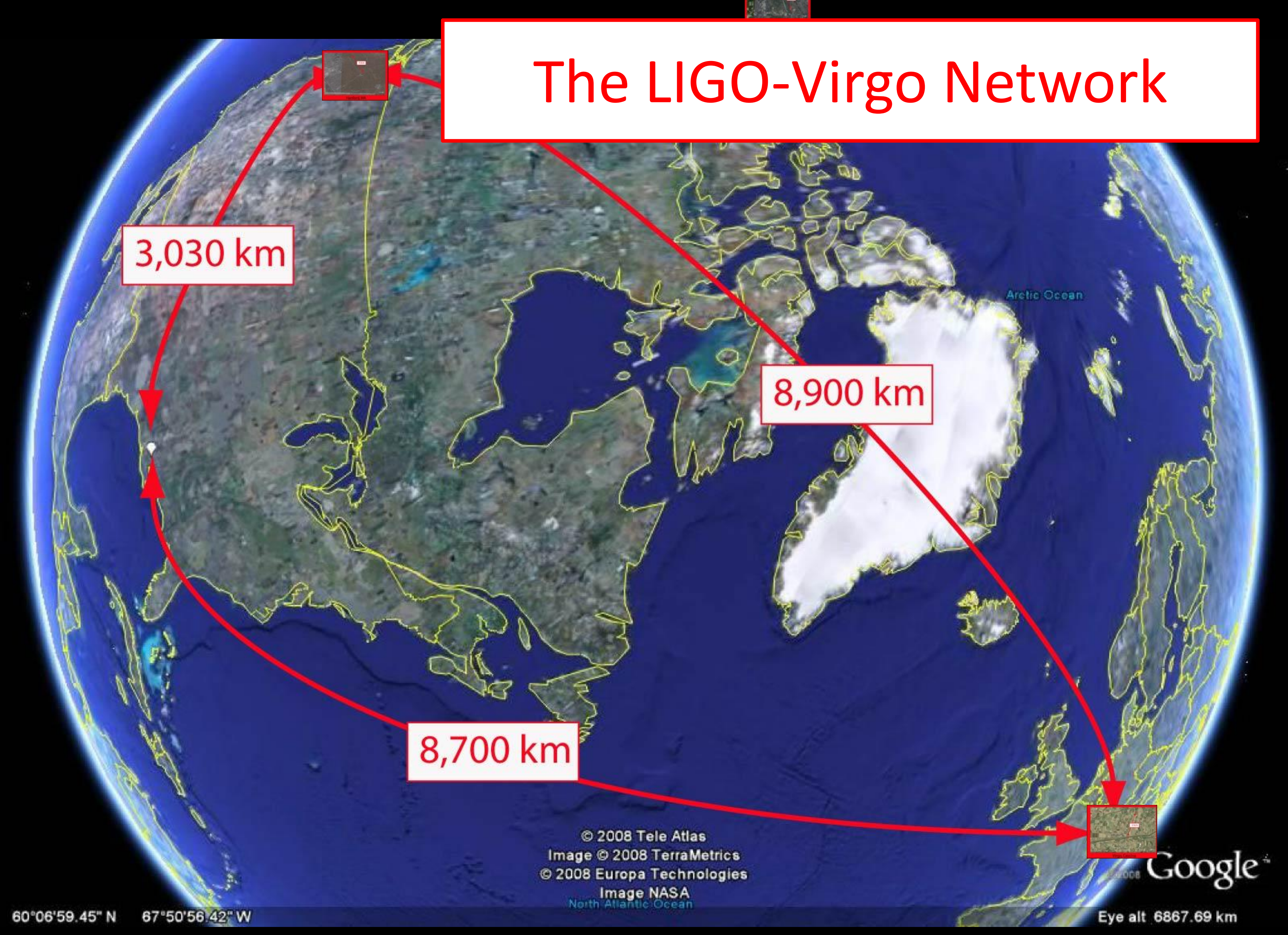
Image NASA
© 2008 Europa Technologies
39°30'35.12" N 98°59'49.96" W
Eye alt 3986.21 km

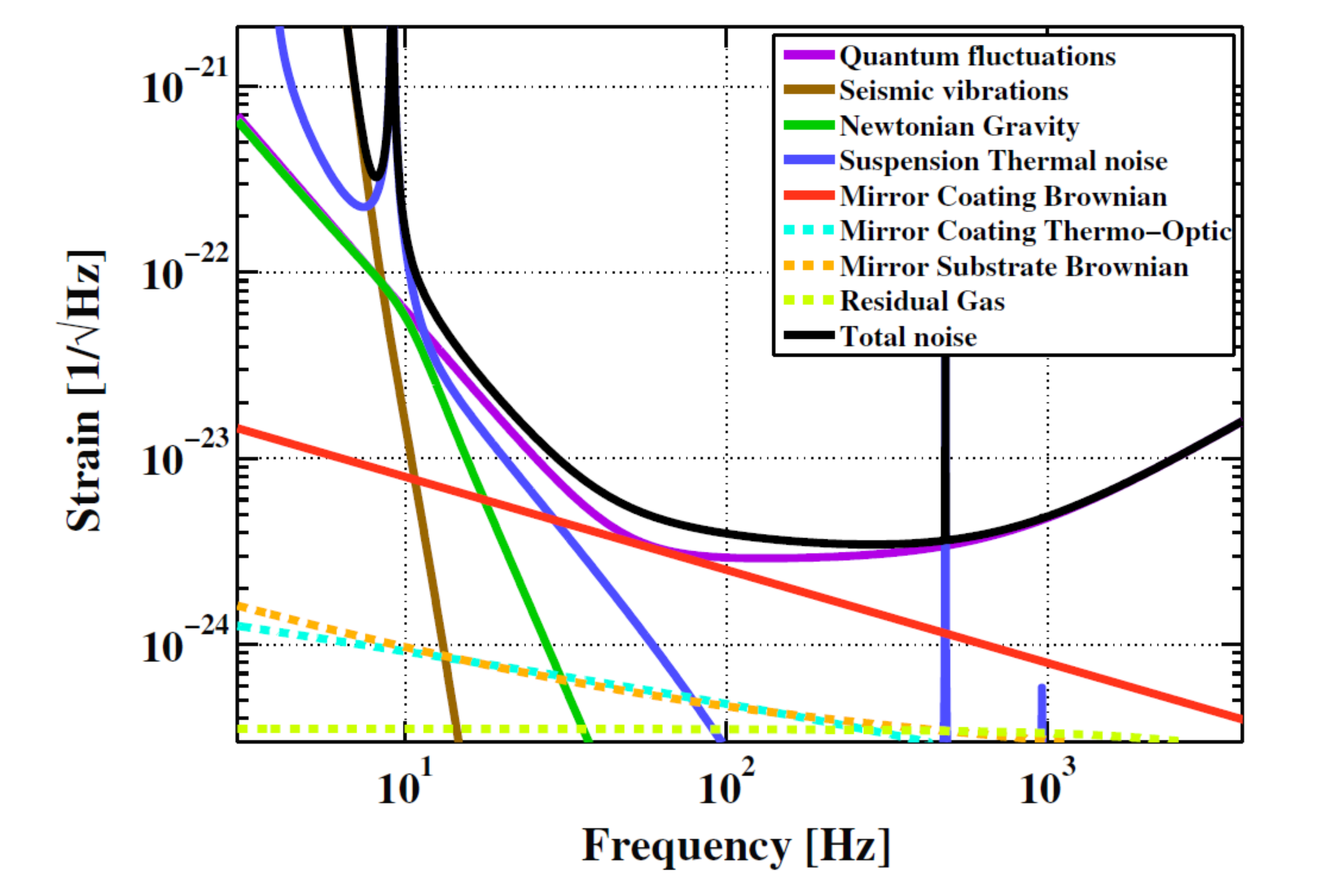
The LIGO-Virgo Network



LIGO/Virgo facts

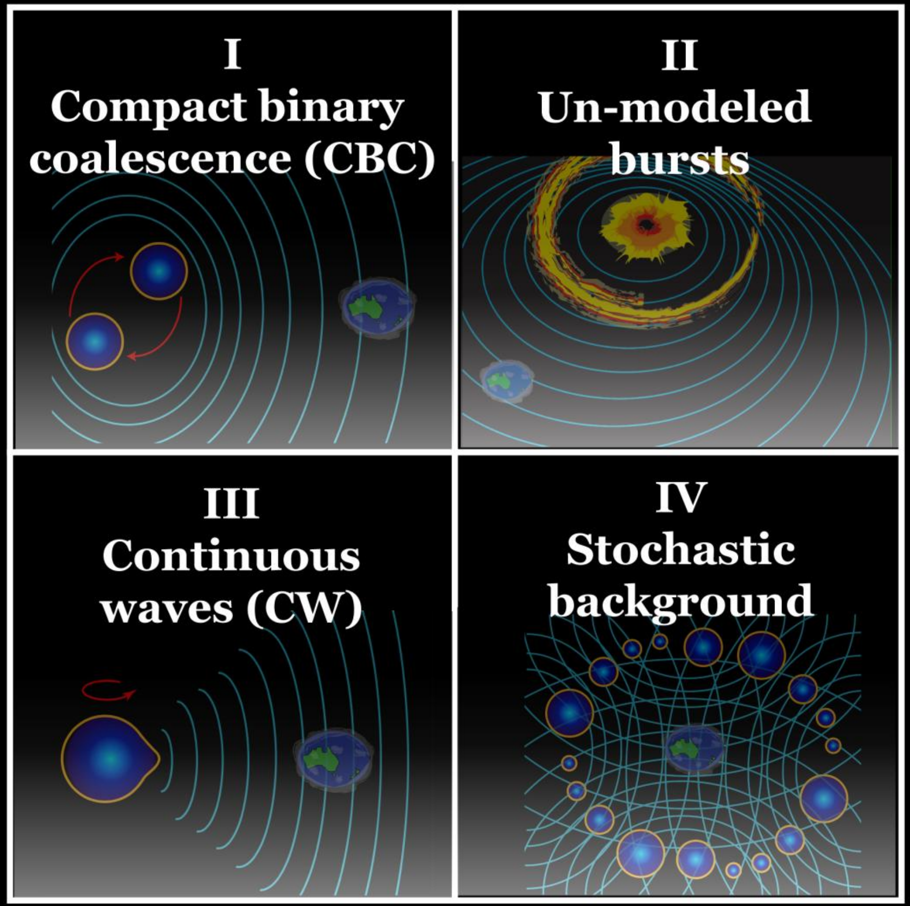
- Largest ultra-high vacuum system
- LIGO/Virgo band: *O(10)* Hz – *O(1)* kHz (audio frequencies)
- Dominant noise source at high frequency: quantum vacuum fluctuations (‘shot noise’)!

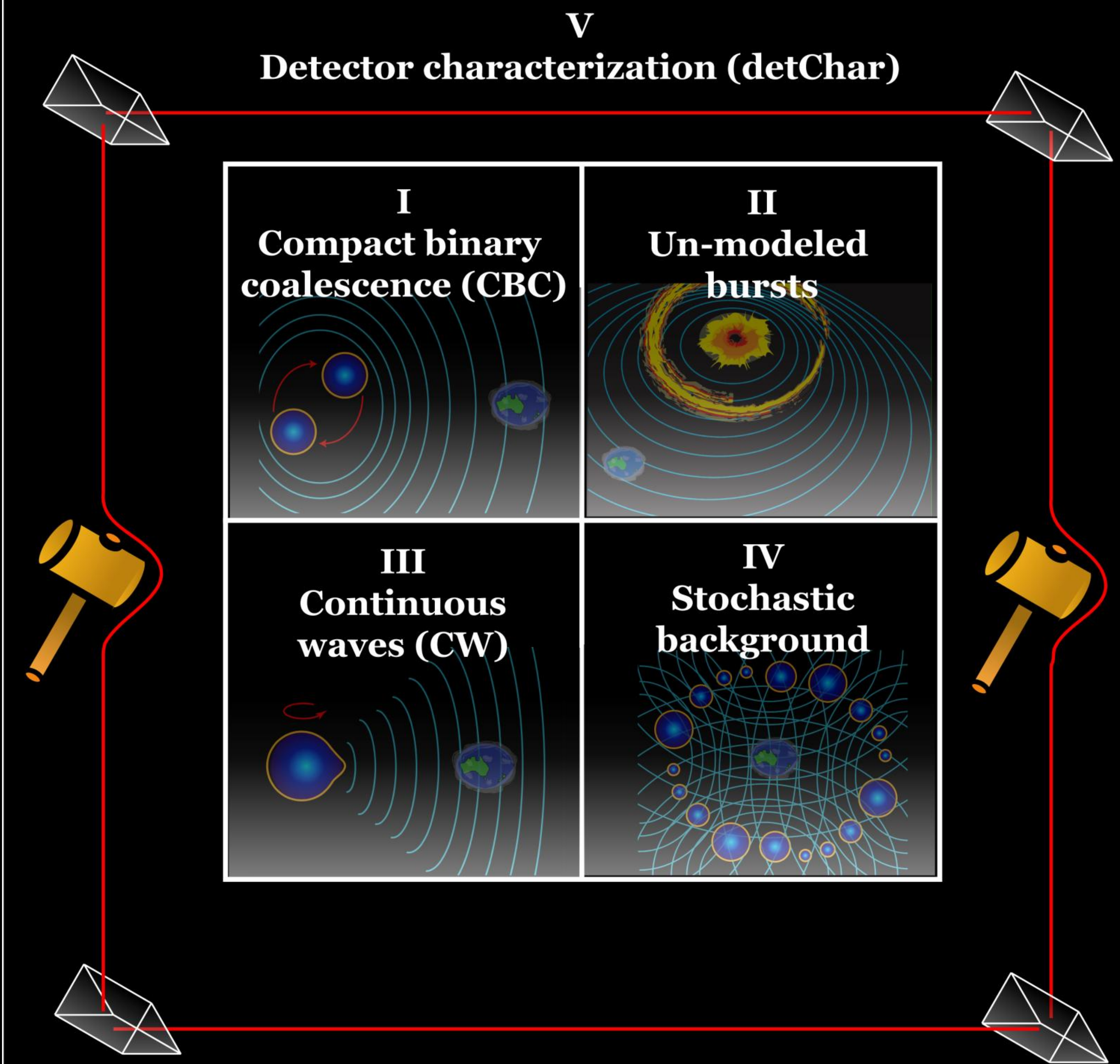
aLIGO noise budget



Adhikari, R.X.: "Gravitational radiation detection with laser interferometry," *Rev. Mod. Phys.* **86** (2014)

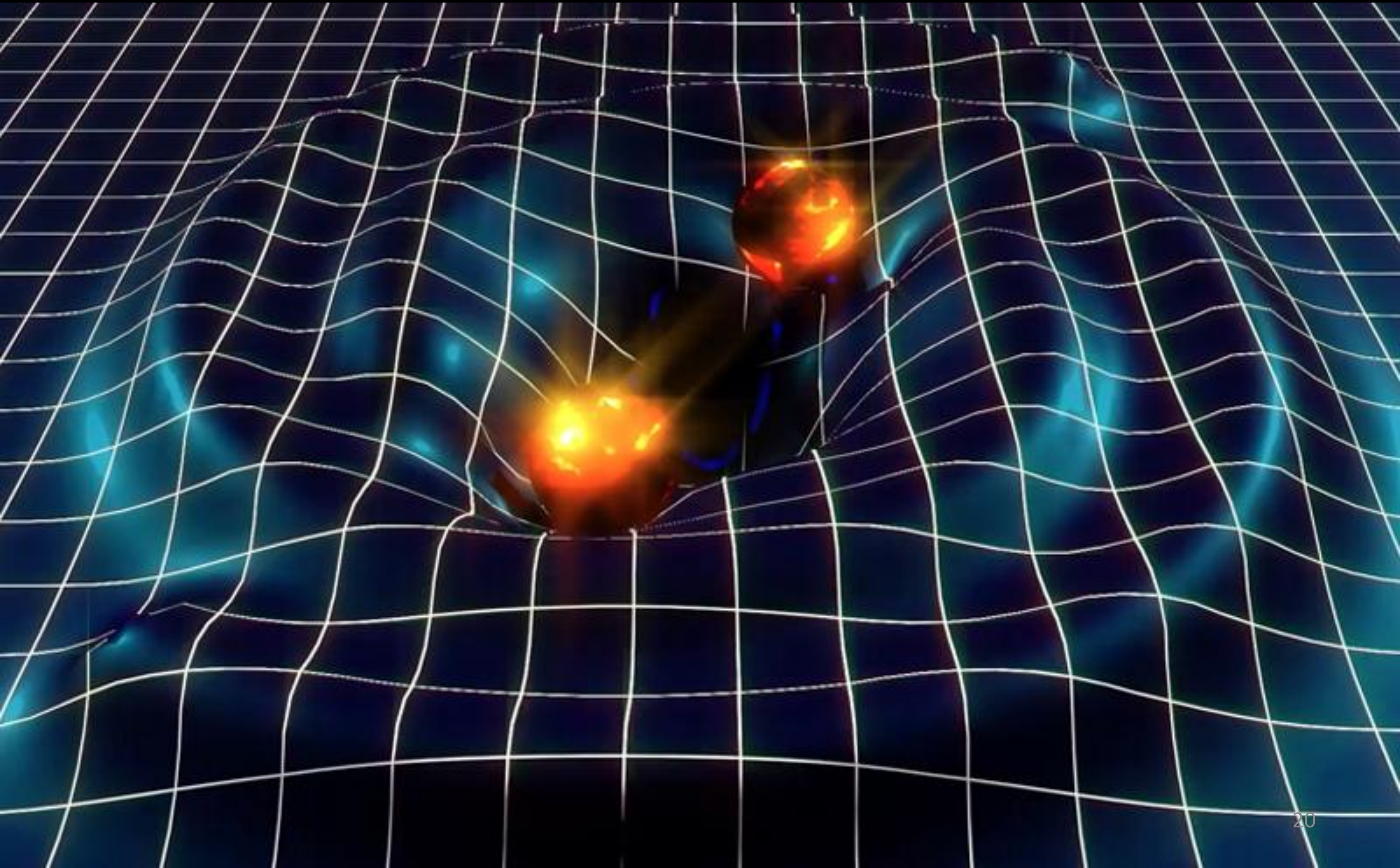
Working groups




V
Detector characterization (detChar)
I
Compact binary
coalescence (CBC)
II
Un-modeled
bursts
III
Continuous
waves (CW)
IV
Stochastic
background

Feature detection

I: Compact Binary Coalescence (CBC)



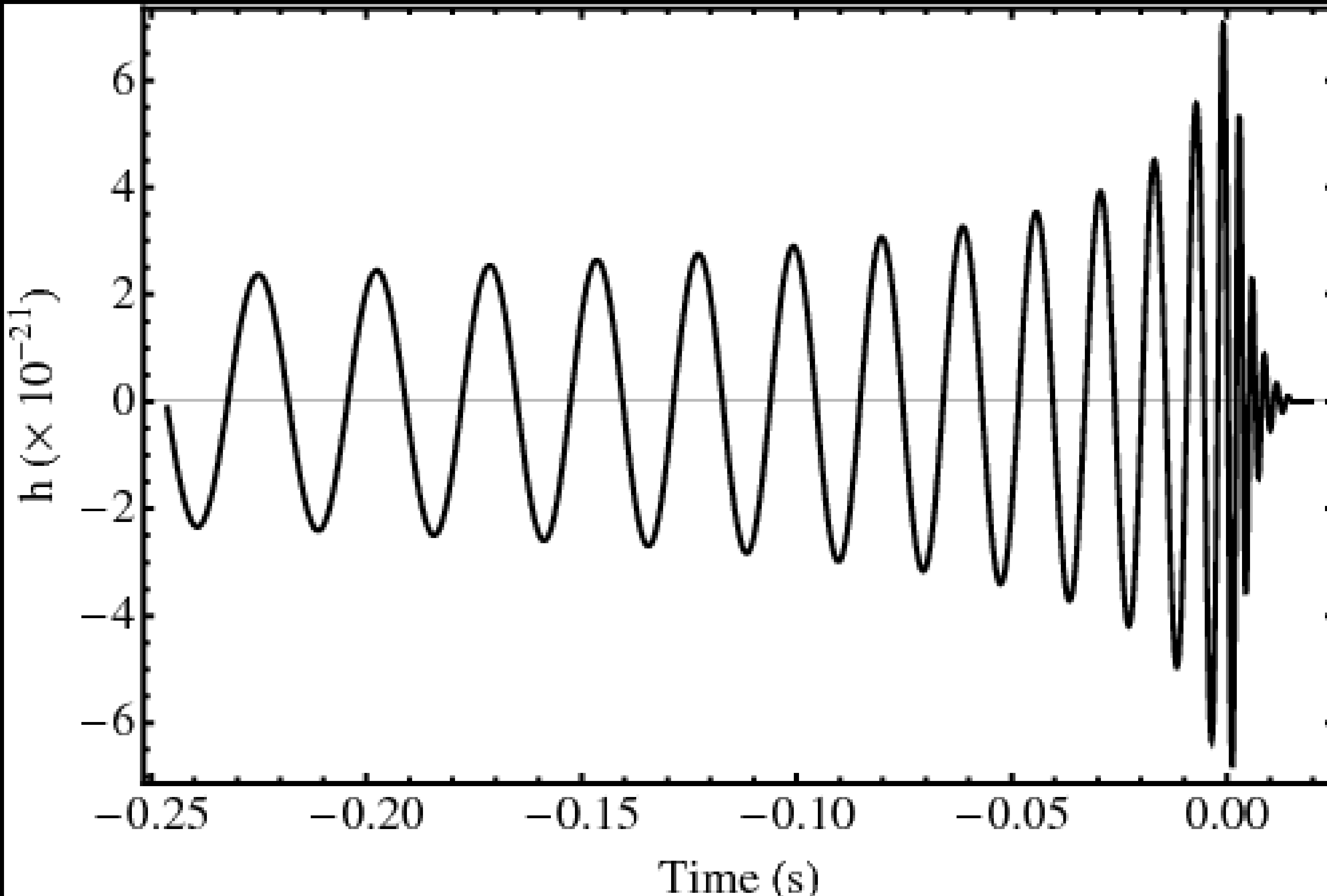


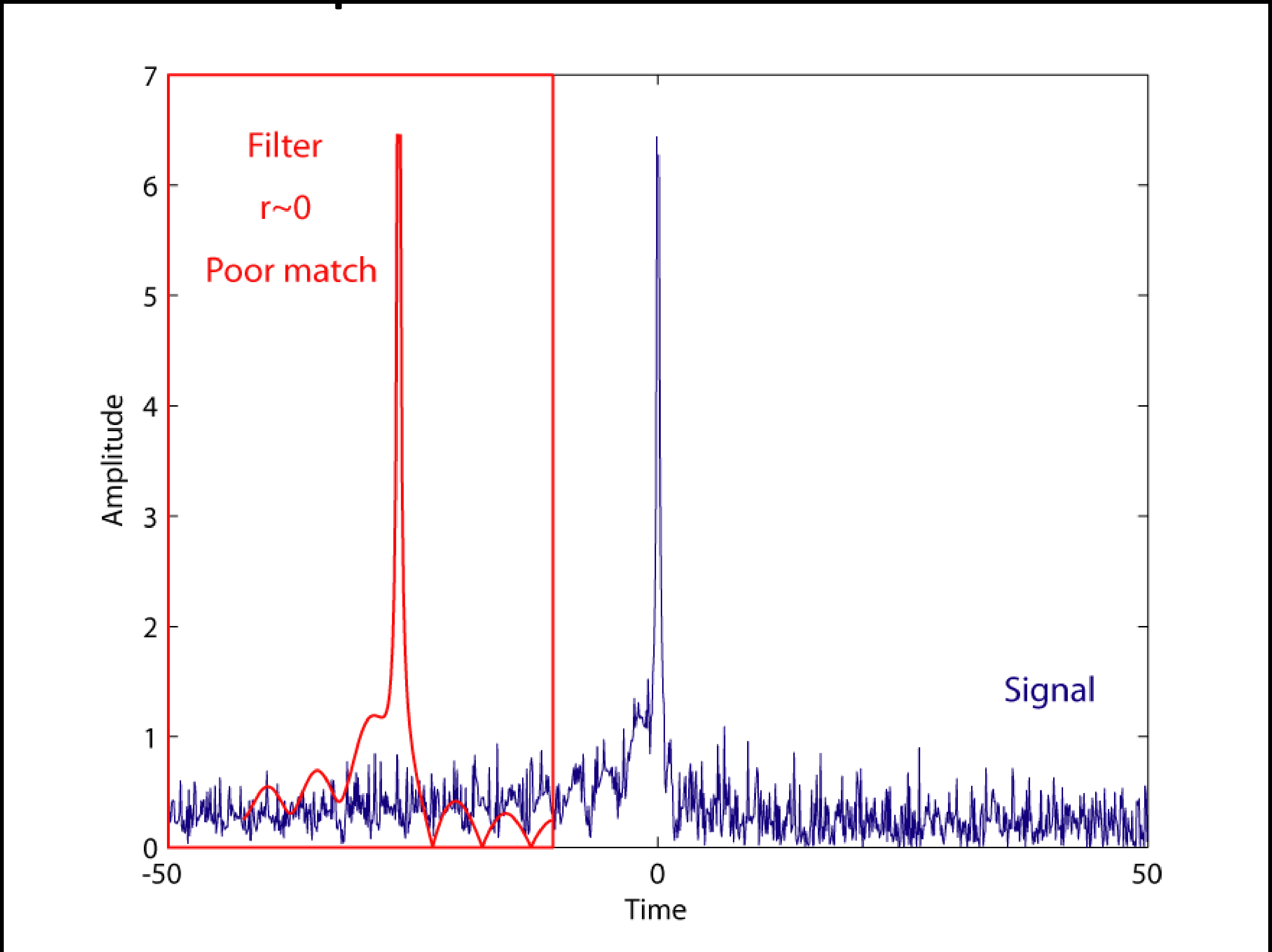
Image: Hannam, Mark *et al.*, *Phys.Rev. D* **79** (2009) 084025

Chirp mass

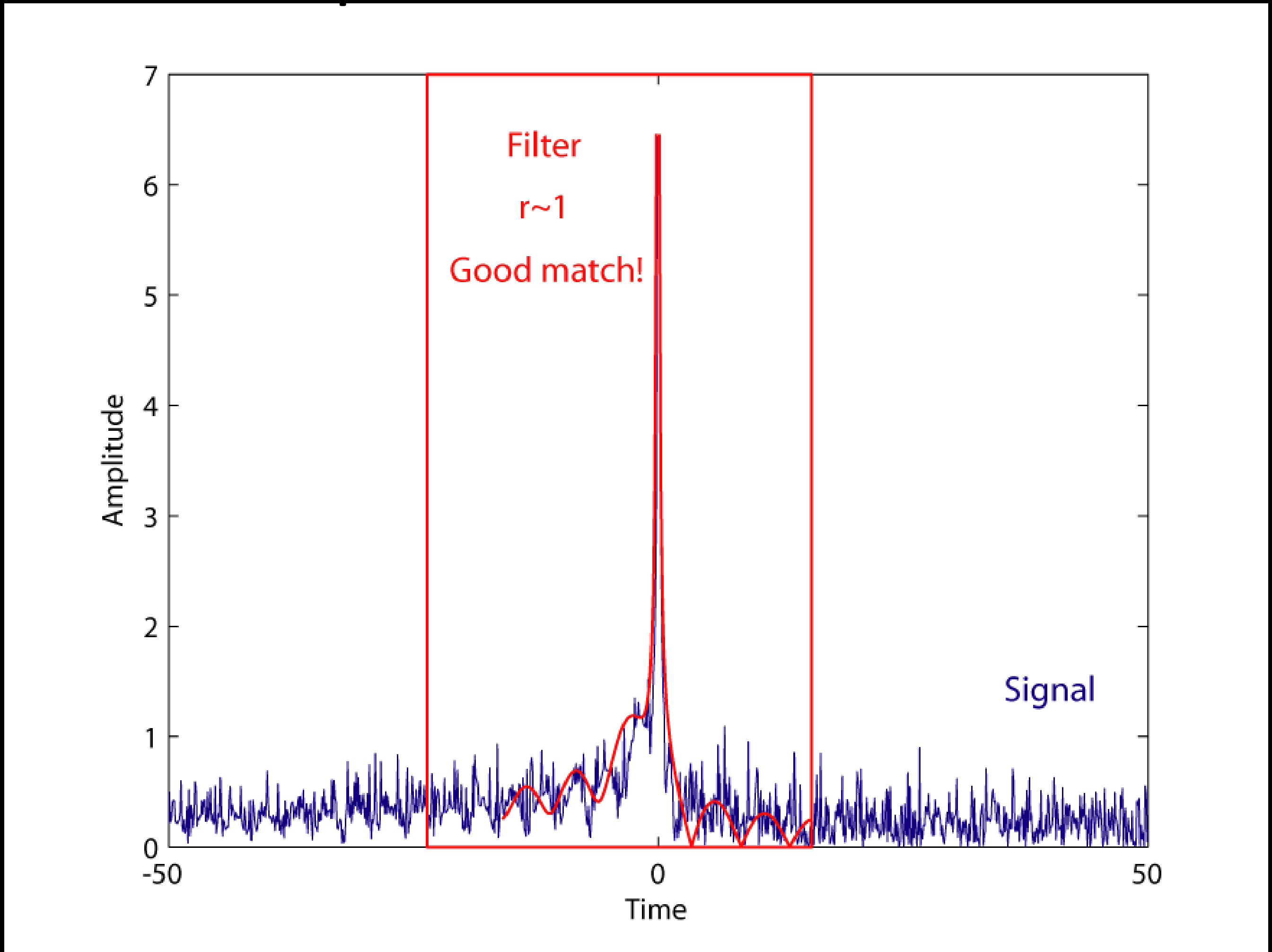
$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$= \frac{c^3}{G}\left(\left(5/96\right)^3 \pi^{-8} f^{-11} \dot{f}^{3}\right)^{1/5}$$

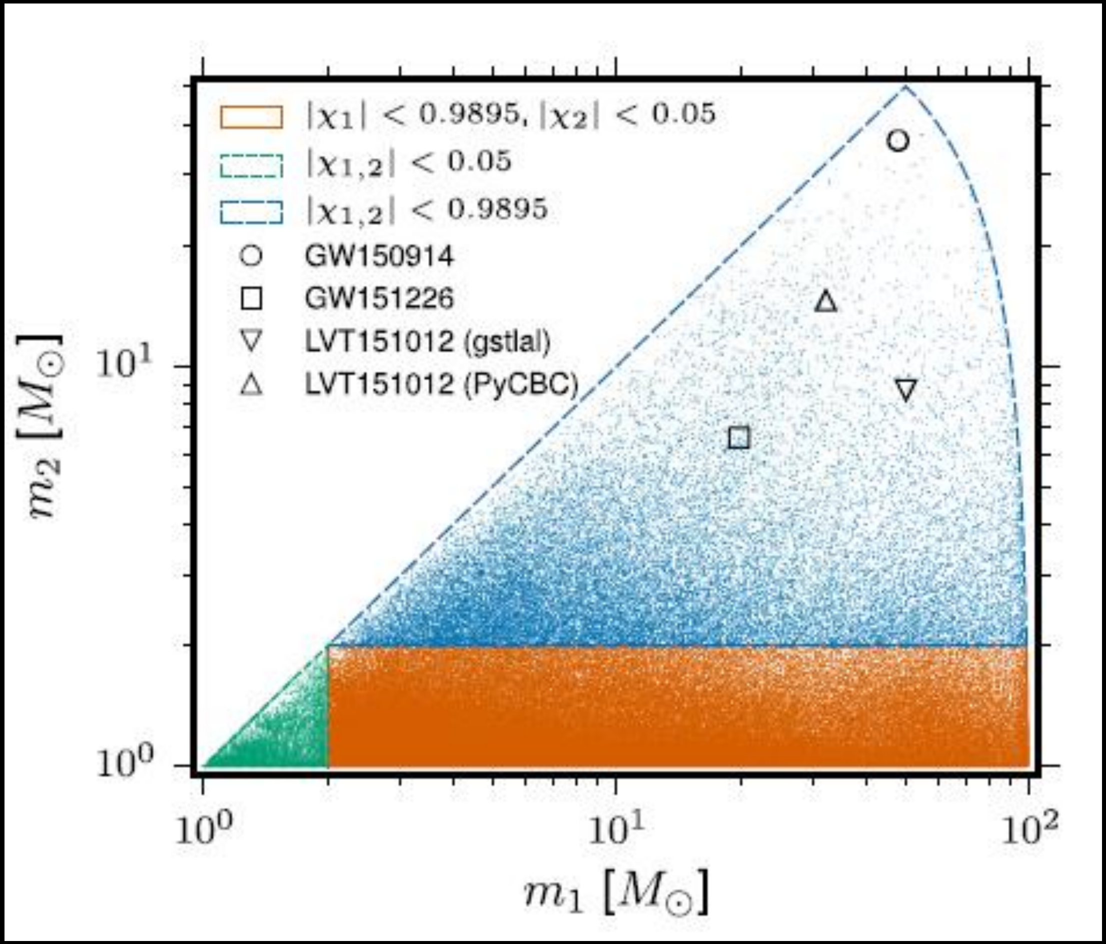
Matched filter



Matched filter

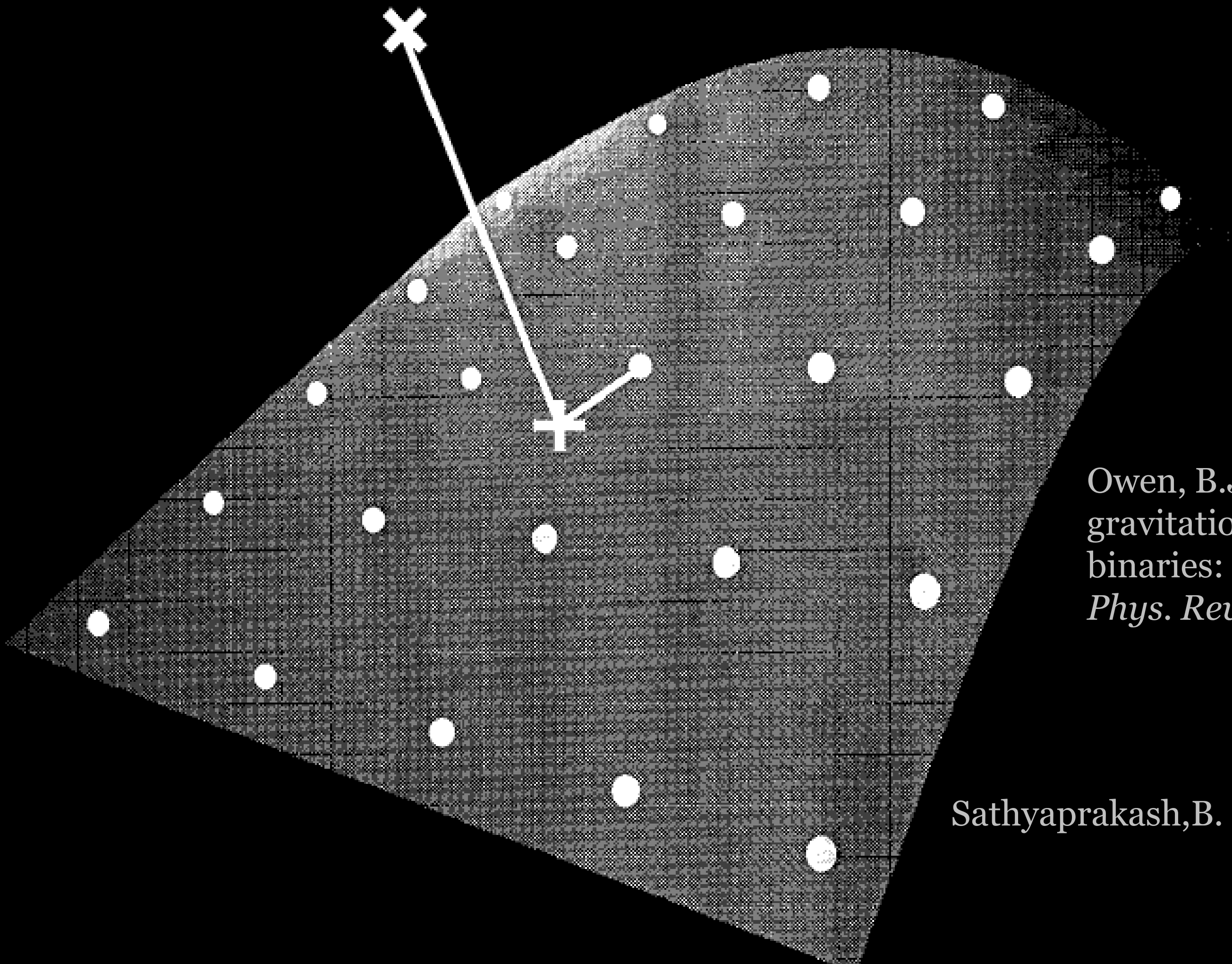


Template banks



LVC: "Binary Black Hole Mergers in the First Advanced LIGO Observing Run," *Phys. Rev. X* **6**(041015) (2016)

Information geometry

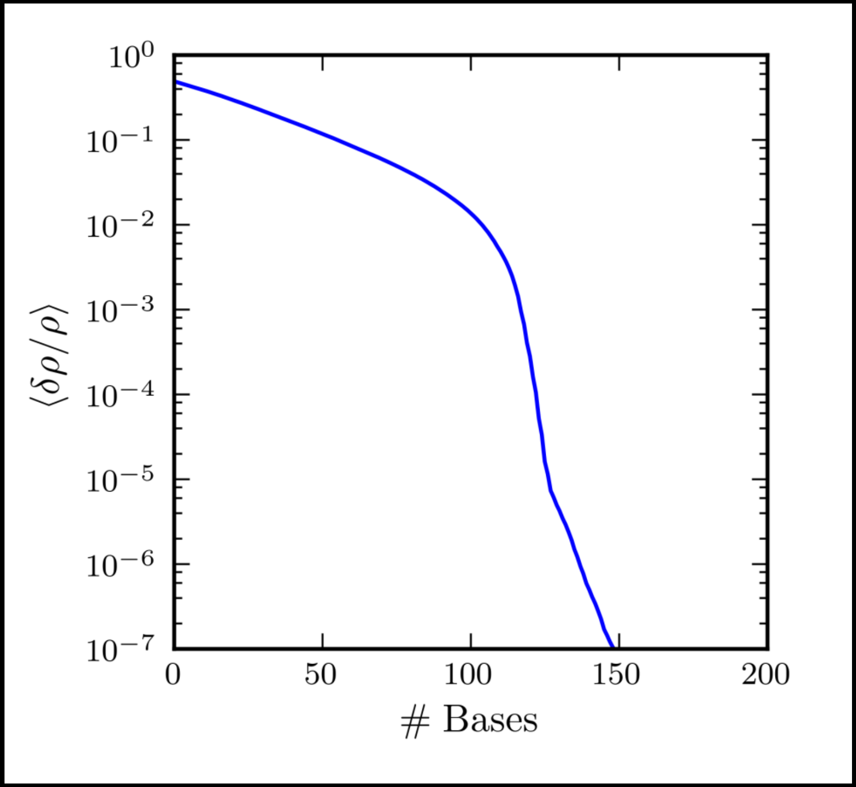


Owen, B.J.: “Search templates for gravitational waves from inspiraling binaries: Choice of template spacing,” *Phys. Rev. D* **53**(12) (1996)

Sathyaprakash,B. S.: *Phys. Rev. D* **50**(R7111) (1994)

Cutler, C. & Flanagan ,E. E. : *Phys. Rev. D* **49**(2658) (1994)

Dimensionality reduction via SVD

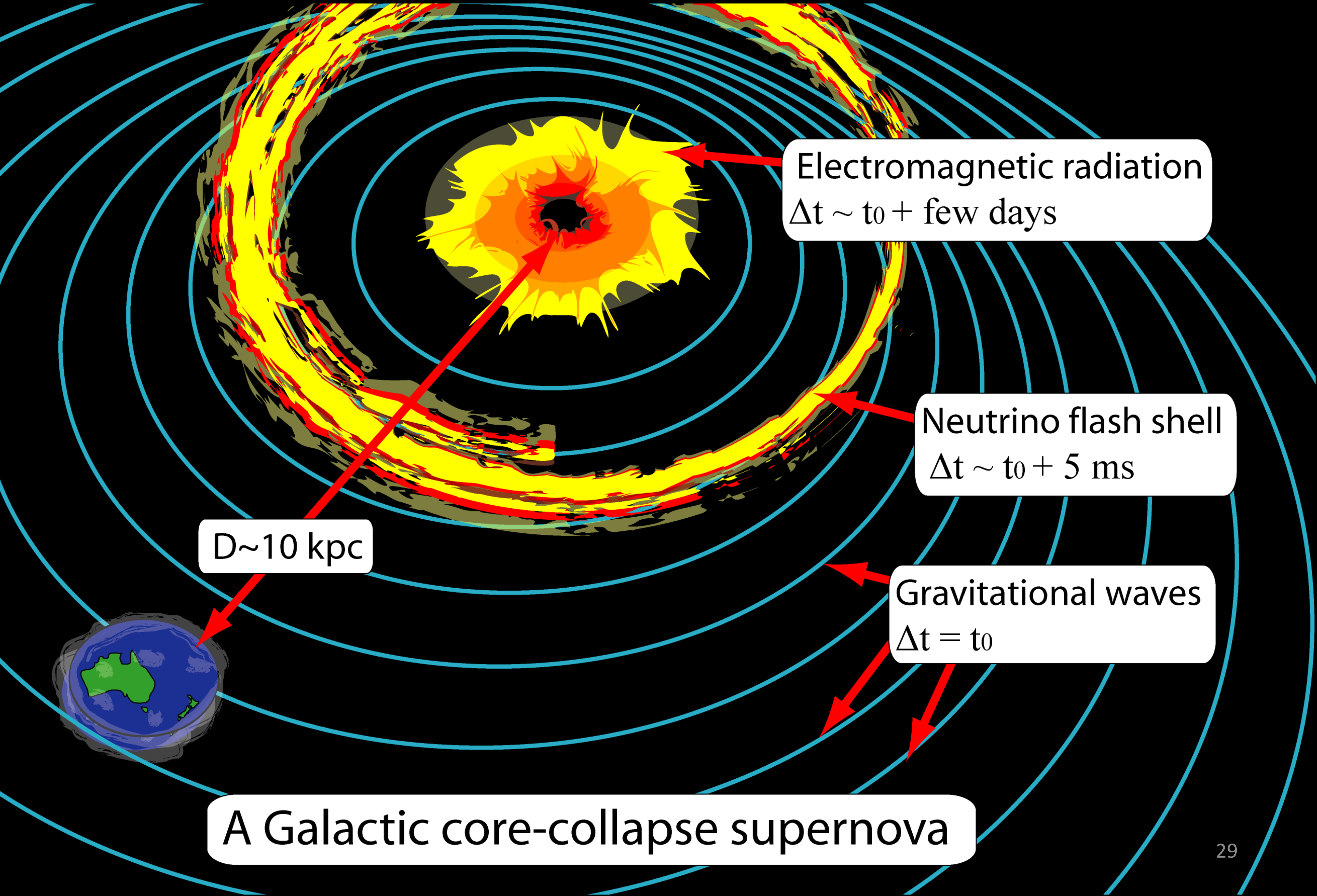


Cannon, K. *et al.*: "Singular value decomposition applied to compact binary coalescence gravitational-wave signals," *Phys. Rev. D* **82**(044025) (2010)

Further improvements

- Abbott, B.P., *et al.*: "GW150914: First results from the search for binary black hole coalescence with Advanced LIGO" *Phys. Rev. D* **93**:122003 (2016)
- Allen, B.: "A chi-squared time-frequency discriminator for gravitational wave detection," *Phys.Rev.D* **71**:06200 (2005)
- Allen, B., *et al.*: "FINDCHIRP: An algorithm for detection of gravitational waves from inspiraling compact binaries," *Phys. Rev. D* **85**:122006 (2012)
- Capano, C., *et al.*: "Implementing a search for gravitational waves from non-precessing, spinning binary black holes," *Phys. Rev. D* **93**:124007 (2016)
- Usman, S.A., *et al.*: "The PyCBC search for gravitational waves from compact binary coalescence," *Classical and Quantum Gravity* **33**(21) (2016)
- Messick, C., *et al.*: "Analysis Framework for the Prompt Discovery of Compact Binary Mergers in Gravitational-wave Data," *Phys. Rev. D* **95**:042001 (2017)

II: Un-modeled Bursts



Wavelets

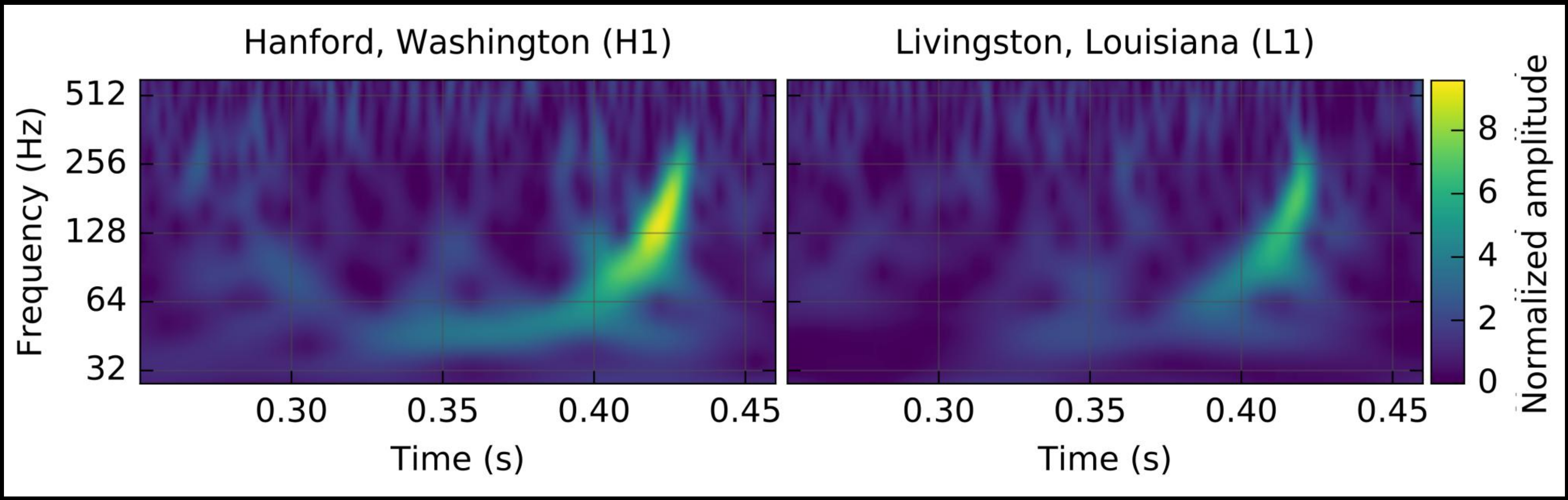
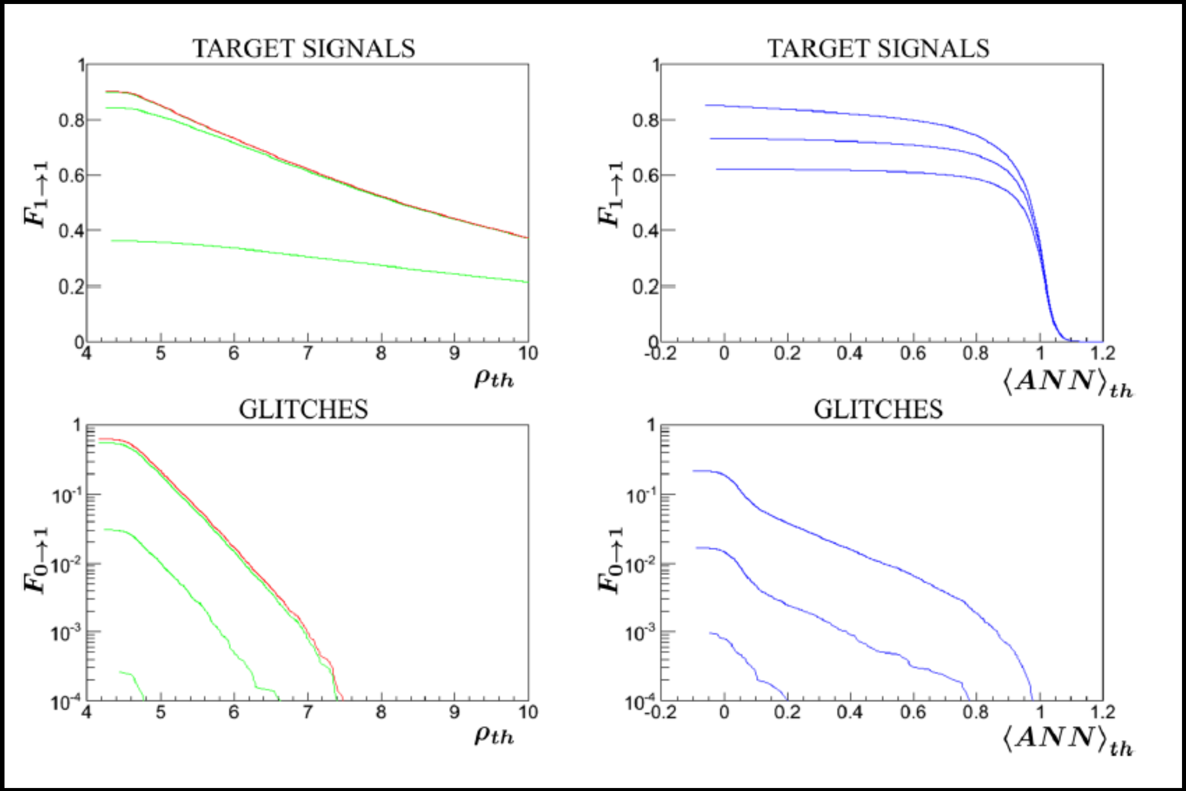


Image: after Abbott, B.P. *et al.*, *PRL* **116**:061102 (2016)

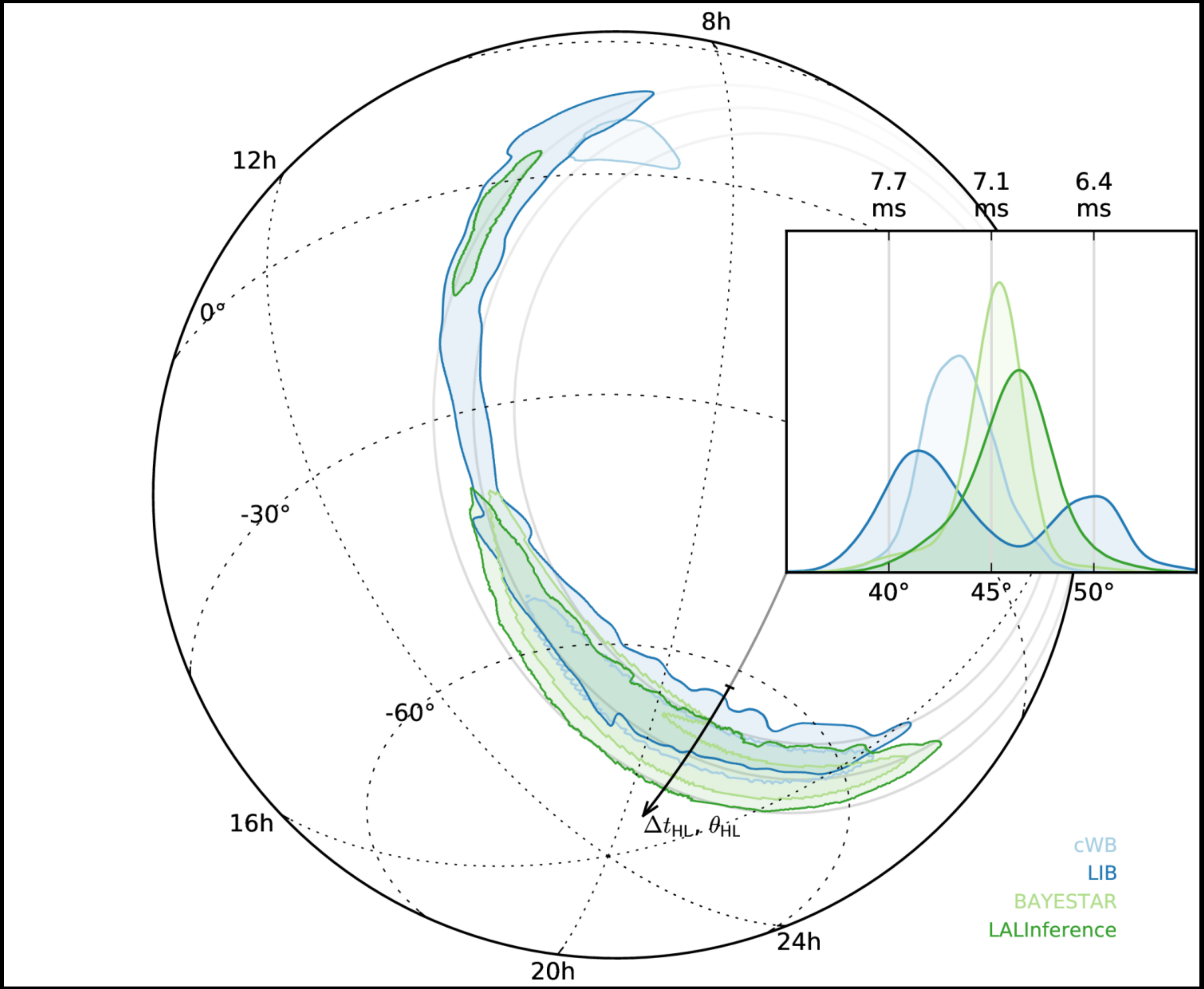
Klimenko, S. & Mitselmakher, G.: "A wavelet method for detection of gravitational wave bursts," *Class. Quant. Grav.* **21**(20) (2004)

ANN+Wavelets



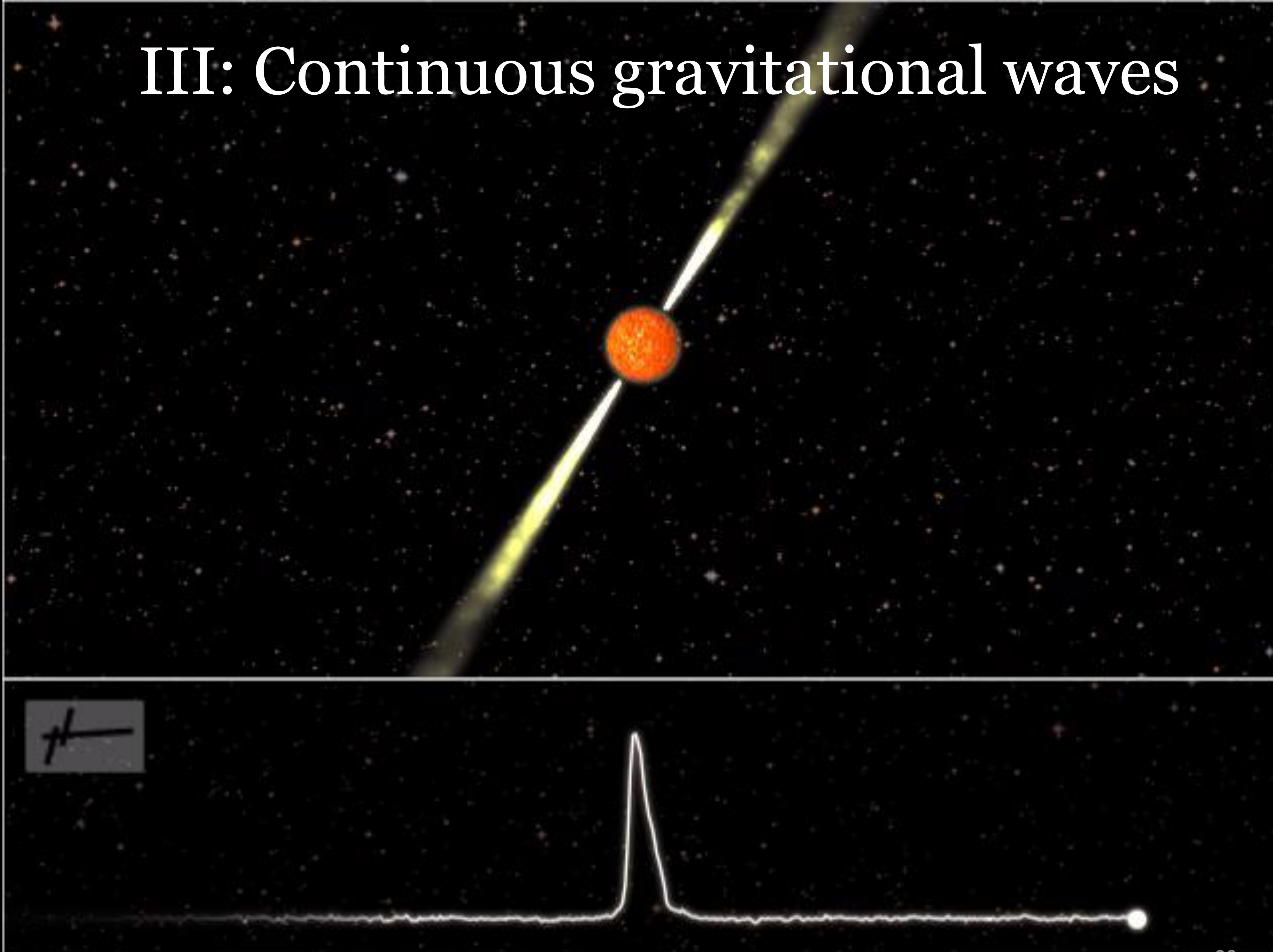
Vinciguerra, S., *et al.*: "Enhancing the significance of gravitational wave bursts through signal classification," *Class. Quantum Grav.* **34**(9) (2017)

Mukund, N. *et al.*: "Transient Classification in LIGO data using Difference Boosting Neural Network," *arXiv:* **1609.07259v2** (2016)



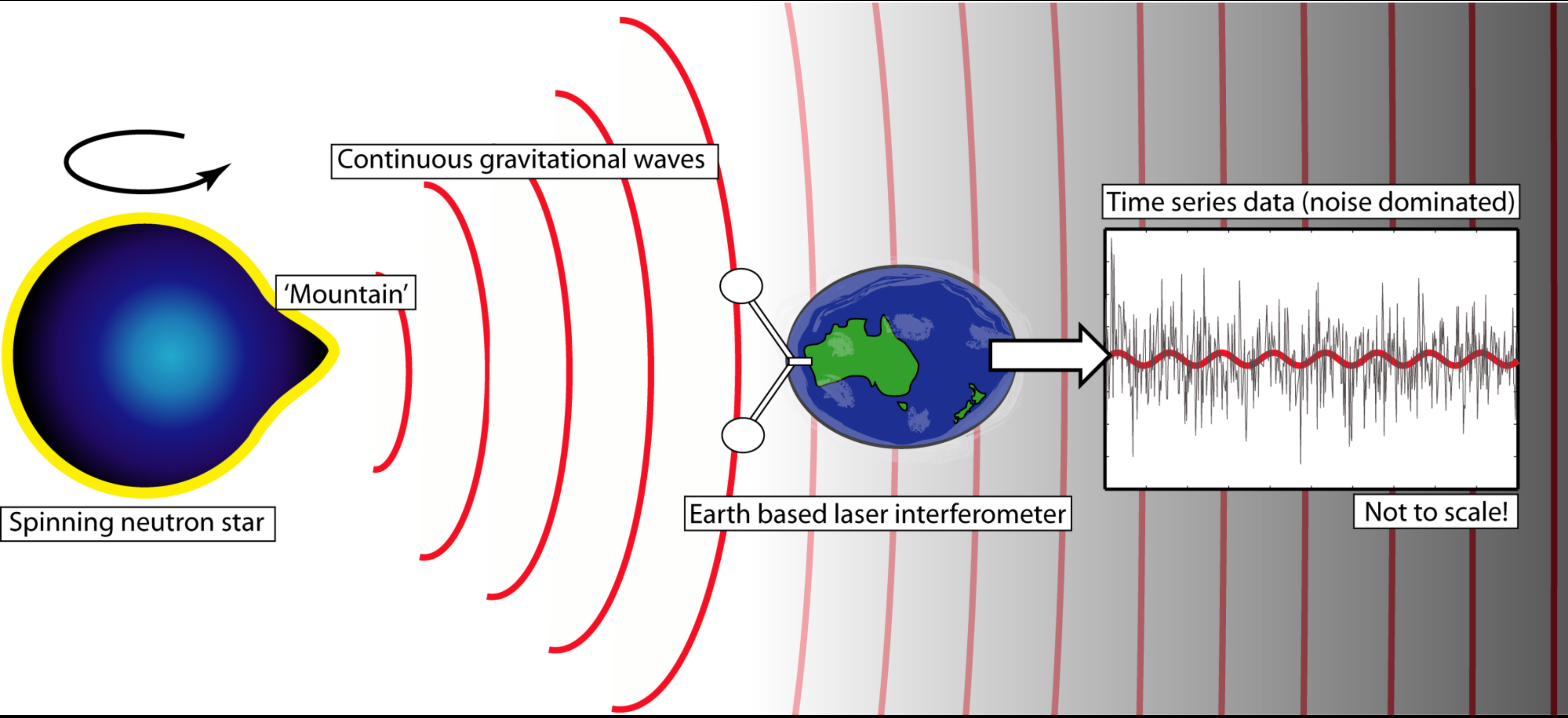
LVC+EM partners: "Localization and broadband follow-up of the gravitational-wave transient GW150914," *Ap. J. Letters* **826**(L13) (2016)

III: Continuous gravitational waves



Animation: Joeri van Leeuwen

III: Continuous gravitational waves



$$f_{GW} = 2\, f_{Rot.}$$

Expected amplitudes

$$h_0 = \frac{4\pi^2 G}{c^4} \cdot \frac{I_{zz} f_{GW}^2}{D} \cdot \varepsilon$$

$$f_{GW} = 100\ \text{Hz},\ D = 8.5\,\text{kpc},\ \varepsilon = 4\times10^{-6}$$

$$h_0 \le 5.0\times10^{-27}$$ Tiny!

Computational bound

Averaging means most CW searches are computationally limited

e.g.: typical, modest, search takes ~400,000 CPU hrs on Albert Einstein Institute's Atlas supercomputer

Bounds often explicitly used to determine a figure of merit for CW searches

Maximum Likelihood Estimation

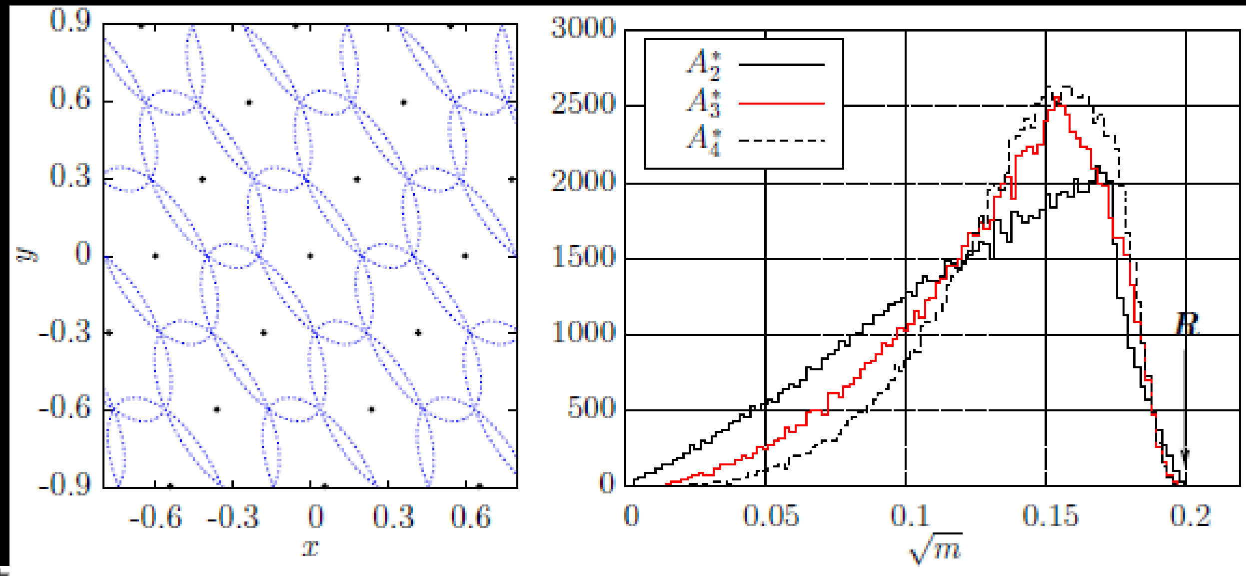
$$\log(\Lambda) = (x \mid h) - \frac{1}{2}(h \mid h)$$

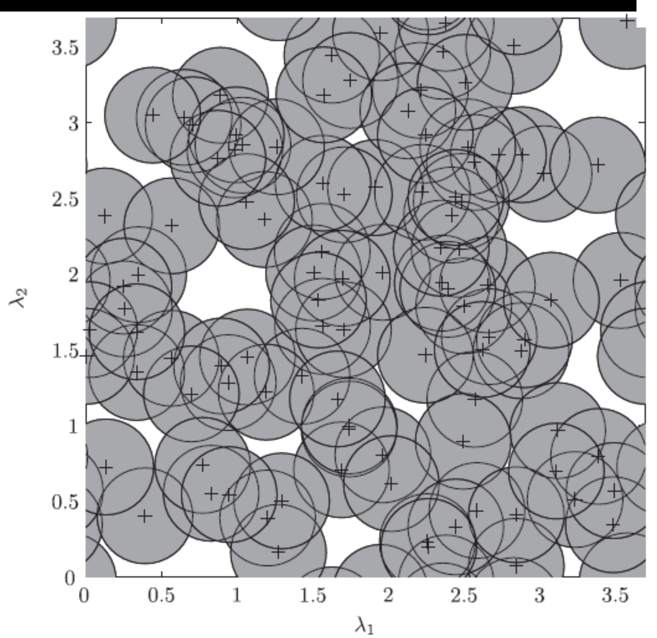
$$(x \mid y) \cong \frac{2}{S_h(f_0)} \int_0^T x(t)y(t)dt$$

$$F = \log(\Lambda(\alpha, \delta, f, \dot{f}, \ldots))\Big|_{h_0, \iota, \phi_0, \psi = \max(\Lambda)}$$

Jaranowski, P., Królak, A. & Schutz, B.F.: "Data analysis of gravitational-wave signals from spinning neutron stars: The signal and its detection," *Phys. Rev. D* **58**(063001) (1998)

Template banks: sphere covering



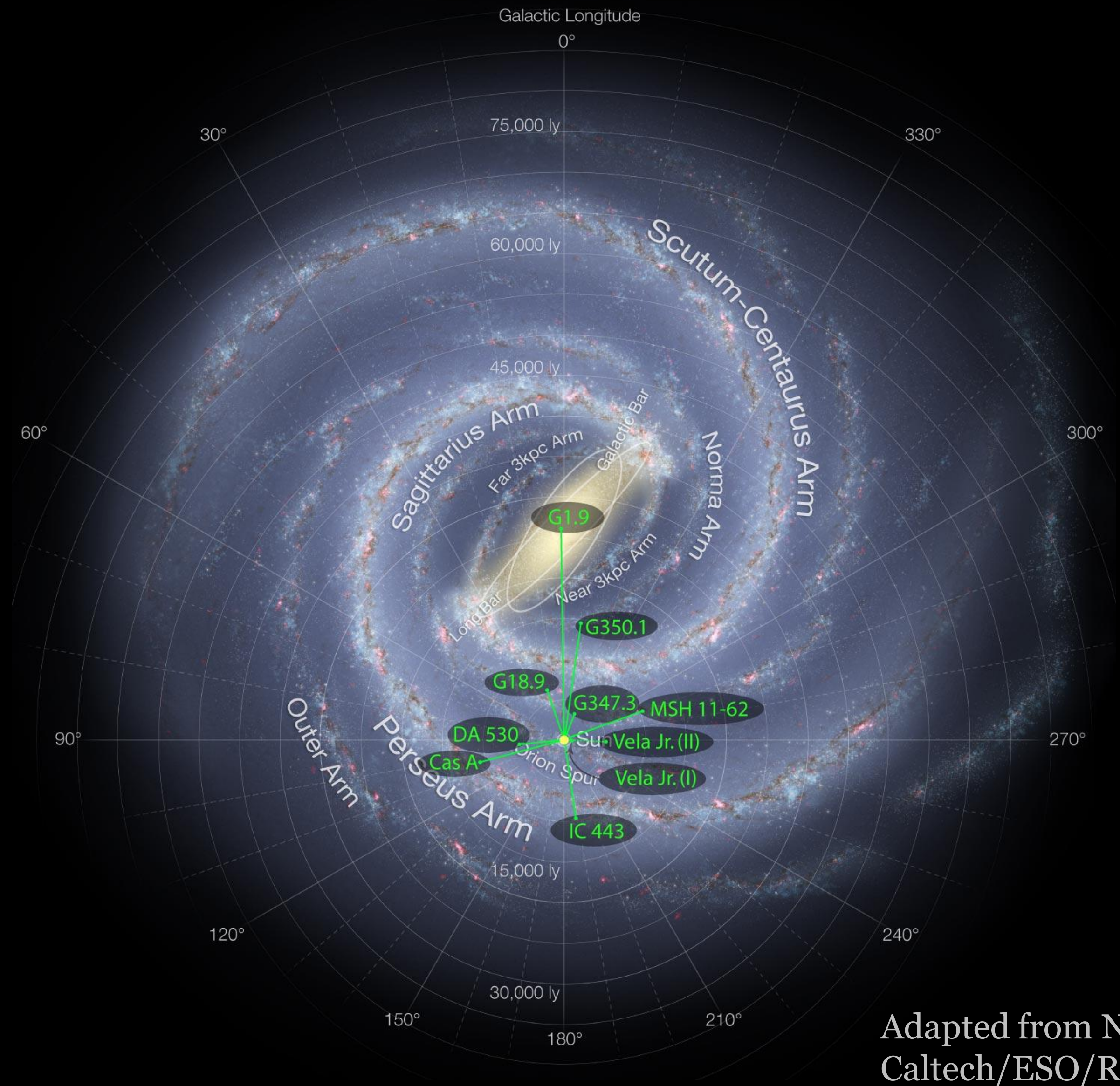


Prix, R.: "Template-based searches for gravitational waves: efficient lattice covering of flat parameter spaces," *Class. Quantum Grav.* **24**(S481–S490) (2007)

Messenger, C., Prix, R. & Papa, M.A.: "Random template banks and relaxed lattice coverings," *Phys. Rev. D* **79**(104017) (2009)

Wette, K.: "Lattice template placement for coherent all-sky searches for gravitational-wave pulsars," *Phys. Rev. D* **90**(122010) (2014)

Young supernova remnants



Aasi, J. et al. *ApJ*. **813**(1) 39 (2015)

Adapted from NASA/JPL-Caltech/ESO/R. Hurt., with permission

Other algorithmic improvements

1. Time series resampling
2. Viterbi tracking of spin wandering
3. Cross correlation
4. Parameter space improvements

2: LVC: "Search for gravitational waves from Scorpius X-1 in the first Advanced LIGO observing run with a hidden Markov model," *arXiv:* **1704.03719** (2017)

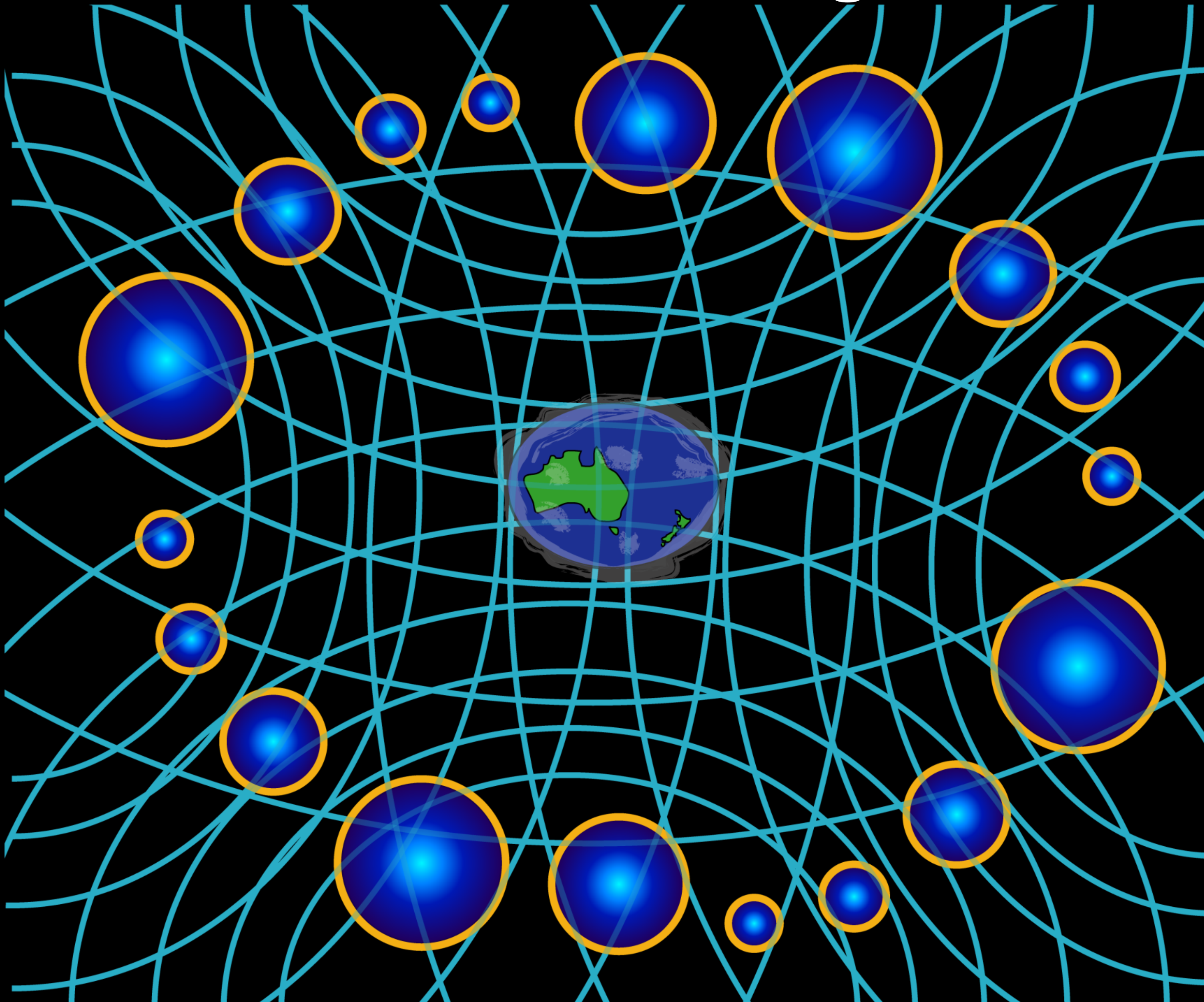
4: Wette, K., *PRD* **92**:082003 (2015); Jones, D.I., *MNRAS* **453**:53 (2015); Leaci, P. & Prix, R., *PRD* **91**:102003 (2015)

+Many more!

How can you get involved?

https://einstein.phys.uwm.edu/

IV: Stochastic Background



Cross-correlation (again)

GW energy density:

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}$$

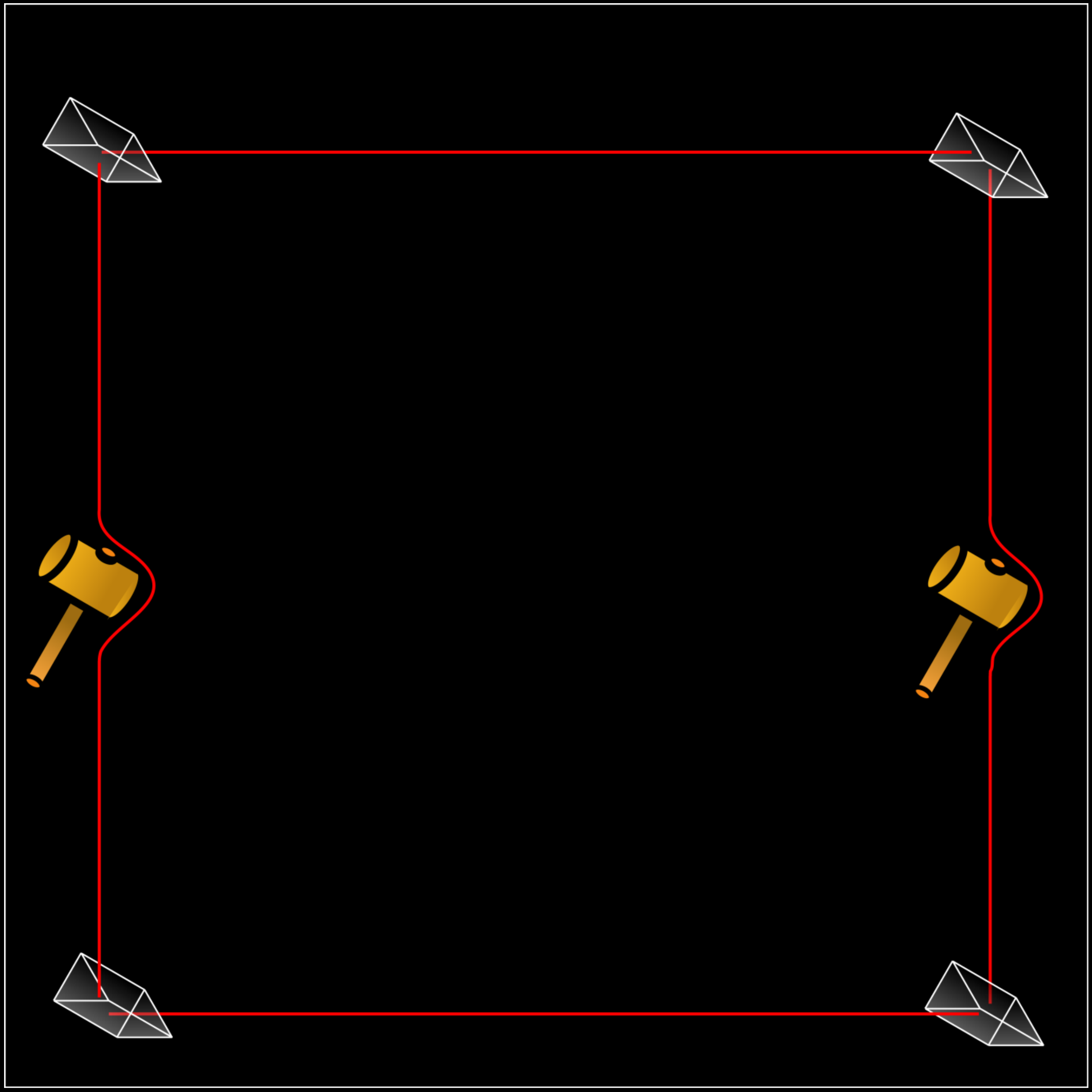
Cross-correlation estimator:

$$Y = \int_{-\infty}^{+\infty} df\ \tilde{s}_1^*(f)\tilde{s}_2(f)\tilde{Q}(f)$$

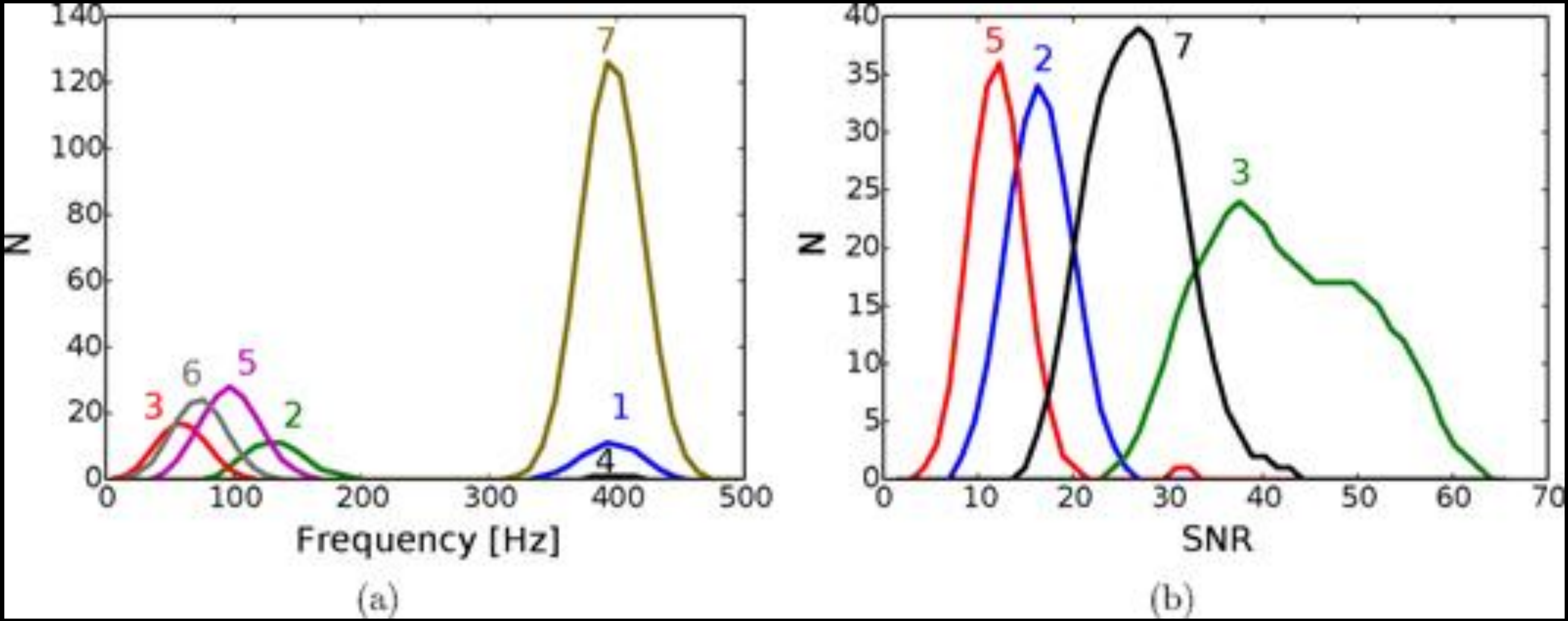
Using an optimal filter:

$$\tilde{Q}(f) = \frac{1}{N}\frac{\gamma(f)\Omega_t(f)}{f^3 P_1(f) P_2(f)}$$

V: Detector Characterization (detChar)



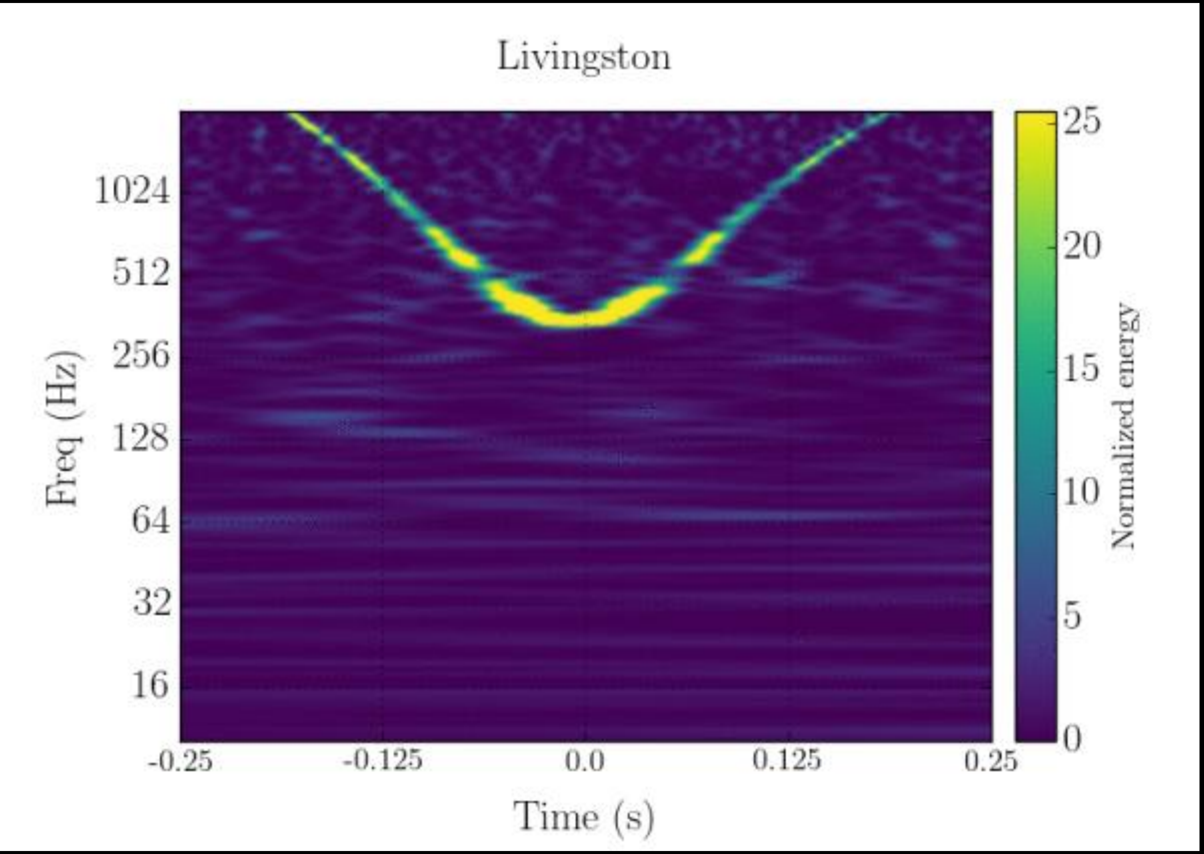
Machine learning



Powell, J.: "Classification methods for noise transients in advanced gravitational-wave detectors," *Class. Quantum Grav.* **32**(21) (2015)

Citizen science: gravitySpy

www.zooniverse.org/projects/zooniverse/gravity-spy



Zevin, M. *et al.* :"Gravity Spy: Integrating Advanced LIGO Detector Characterization, Machine Learning, and Citizen Science," *arXiv:* **1611.04596** (2016)

Conclusion and the future

- Very healthy range of algorithms and signal processing techniques
- Increasingly more computationally efficient
- Rapid adoption of machine learning techniques
- Also good use of citizen science (also good for outreach)