Sparse methods for improving gravitational wave detection

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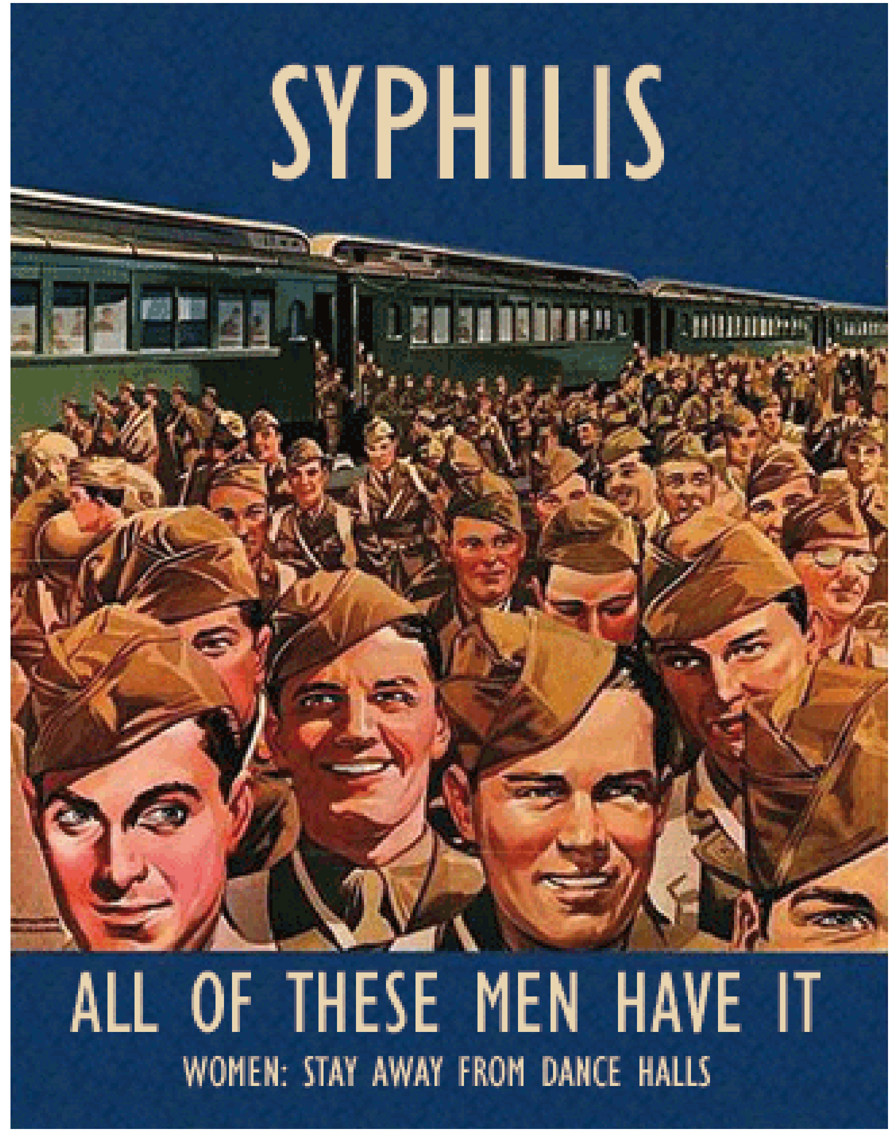
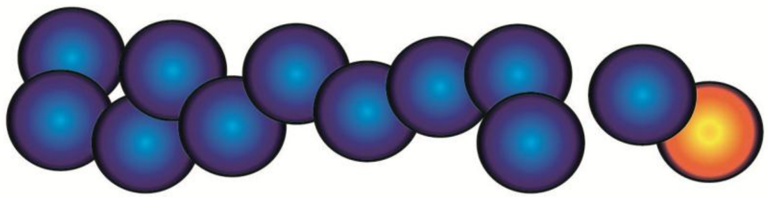
Sparse methods

Reminder: a vector is S-sparse (compressible) if it has at most S non-zero (non-negligible) coefficients (e.g. Yves' and Gabriel's talks)

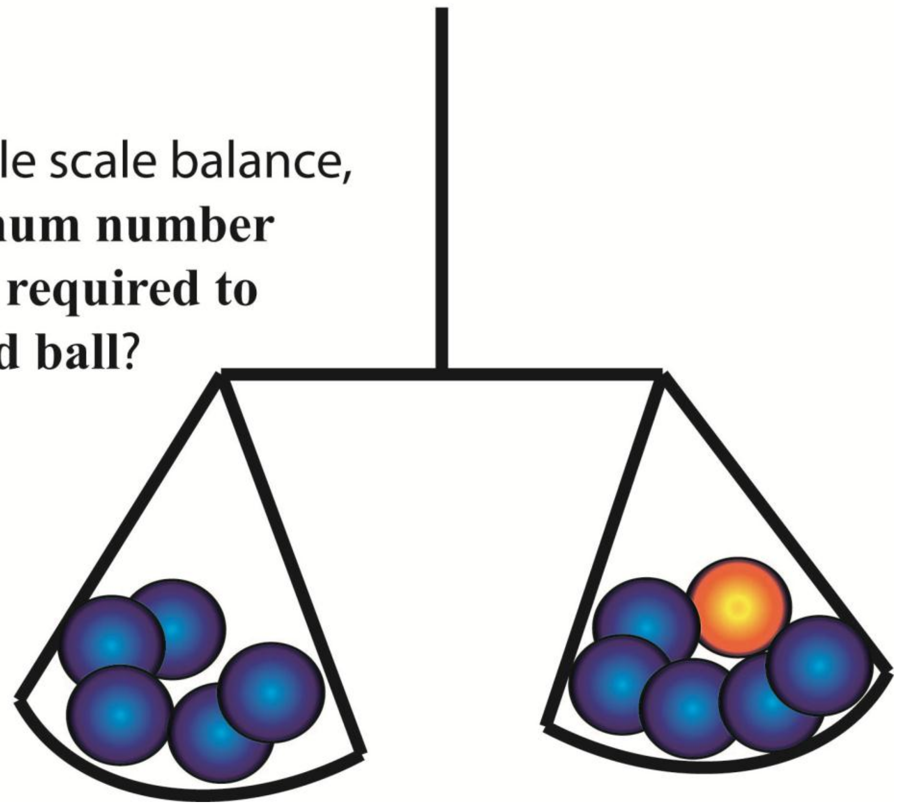
- Syphilis in WWII
- Image/file compression
- Twelve Ball Problem



You're given 12 identical balls, except exactly **one is heavier/lighter than the others.**



Using only a simple scale balance, what is the **minimum number of measurements required to determine the odd ball?**



Brief re-cap on compressive sampling

- Unified framework for encoding/reconstruction of sparse signals
- Applications from radar, Herschel space observatory, to a single pixel camera

Image: M. F. Duarte et al., Rice University

E. Candès, J. Romberg and T. Tao: *IEEE Trans. on Information Theory* **52**(2):489–509 (2006)

Compressive sampling (CS) framework

$$y = \Theta\, x$$

Measurement vector
(1 X M)

Original signal
(1 X N)

$$\Theta = \Phi\, \psi$$

Sparse basis
(N X N)

Sensing matrix
(M X N)

Determined system: M = N
CS ('under-determined'): M << N

To solve for x, impose sparsity:

$$\text{find } \|x\|_0 \quad \text{s.t.} \quad \|y - \Theta x\|_2 = 0$$

...but this is a combinatorial problem!
(Need to guess up to ${}^{N}C_{M}$ coefficients)

Instead, almost as good (esp. for large N):

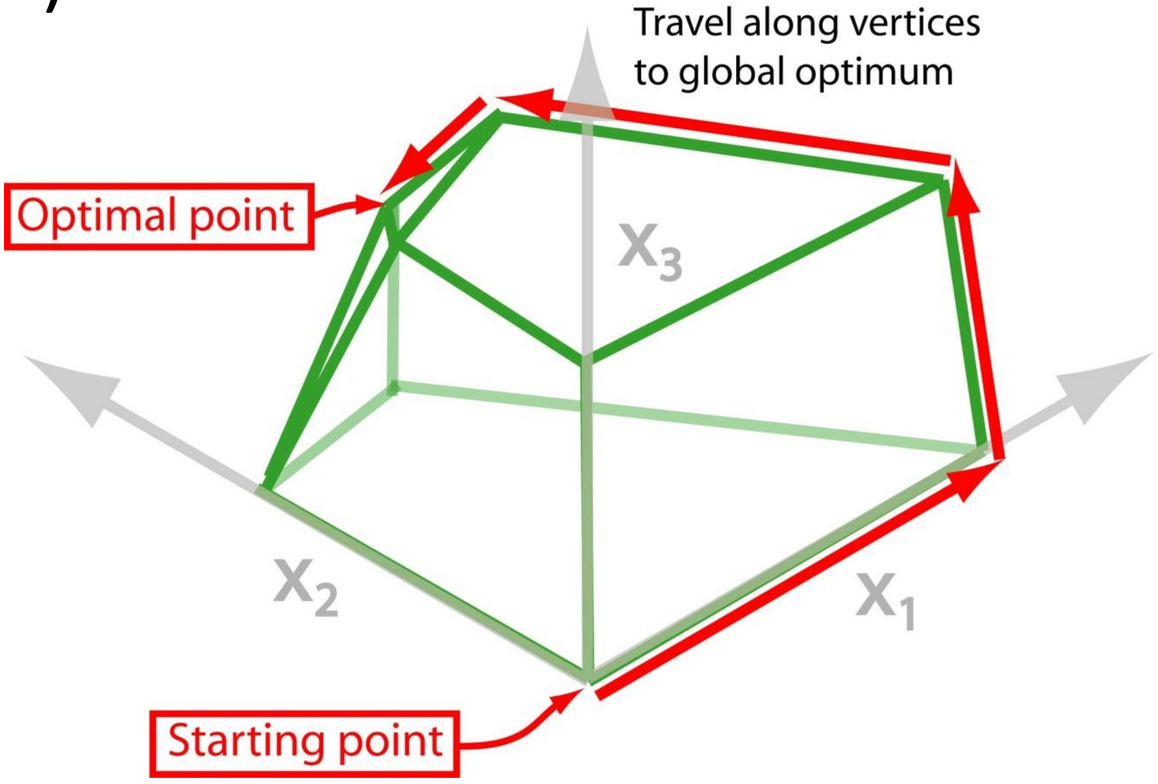
$$\text{find } \|x\|_1 \quad \text{s.t.} \quad \|y - \Theta x\|_2 = 0$$

Compressible/noisy signals

Find: $\|x\|_1$ s.t. $\|y - \Theta x\|_2 \leq \epsilon$

Linear Program

- Optimisation reconstruction methods
- E.g. Simplex method (linear constraints give convex polytope)



Orthogonal Matching Pursuit (OMP)

A recursive greedy algorithm

1. Initialise vectors and determine stopping criterion
2. Get signal estimate by taking most significant column from residual vector
3. Update residual basis
4. Update residual vector

- Similar to CLEAN algorithm used in radio astronomy
- ‘Easy’ to implement noise-based criteria

CS results

Can determine minimum number of samples required for perfect reconstruction:

$$M > C\, S\, \mu^2 \;\log(N)$$

Where *C* is small (~0.5) , and the mutual coherence:

$$\mu \;\stackrel{\text{def}}{=}\; \sqrt{N} \max_{i,j<N} | < \varphi_i, \psi_j > |$$

Measures how ‘spread out’ the signal is in the sampling domain

Mutual coherence

Guarantees sampled low dimensional subspace sufficiently covers sparse basis (Restricted Isometry)

e.g. Delta functions in Fourier domain are minimally coherent with time domain:

$$\mu = \sqrt{N} \max_k \int \delta(f - f_k) e^{i2\pi f t} dt = \sqrt{N} \frac{1}{\sqrt{N}} = 1$$

E. Candès and J. Romberg, *Inverse Problems* **23**:969–985 (2007)

CS applications

Basic ingredients:

1. Sparse/compressible representation
2. Low mutual coherence between sensing and sparsity bases

The Sparse Fast Fourier Transform

- Implements non-recursive OMP
- Computational complexity for known sparsity S: $O(S \log(N))$

Compare to FFT [$O(N \log(N))$]:
Speed-up is $O(N/S)$, so e.g., for $N=10^6$, $S=100$:

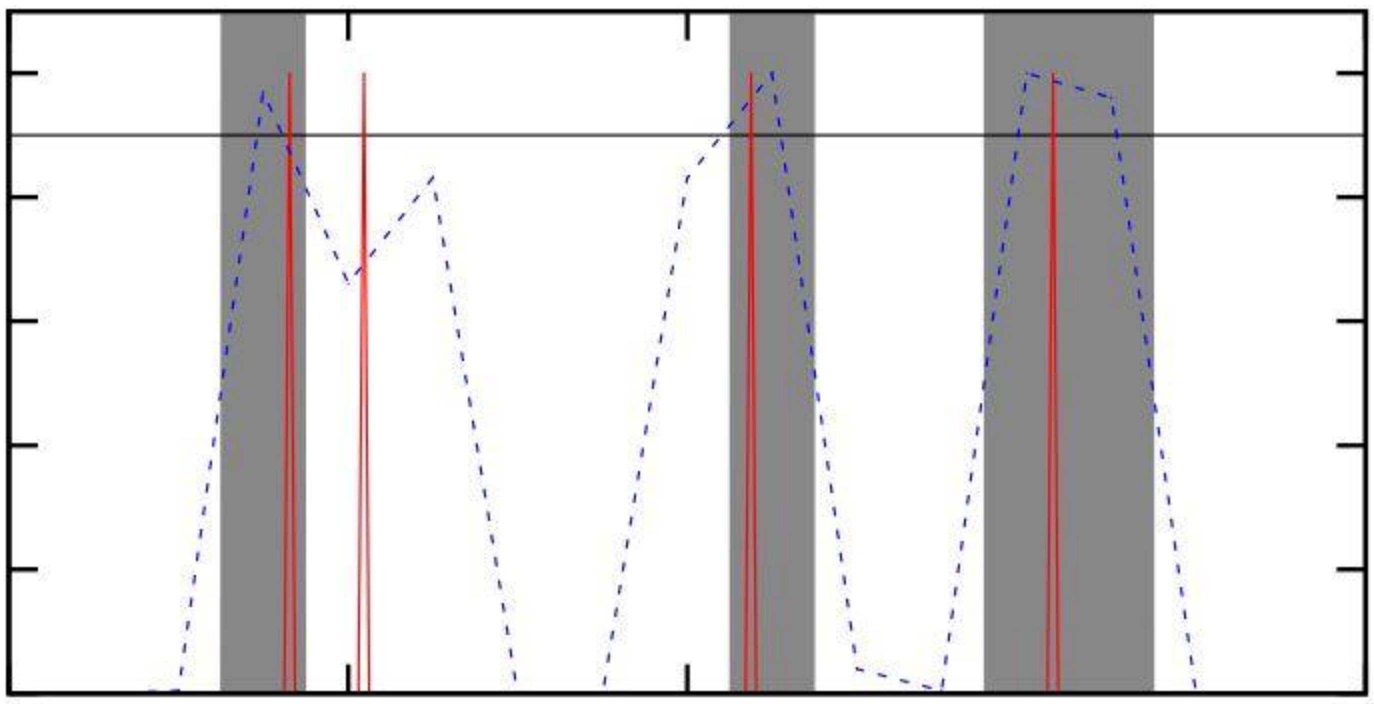
Here sFFT is (theoretically) 10,000 times as fast as FFT!

Hassanieh, H., Indyk, P., Katabi, D., and Price, E.: "Nearly Optimal Sparse Fourier Transform," *arXiv* **1201.2501v1** (12 Jan 2012)

How the sFFT works

FFT: output proportional to N

sFFT: output proportional to S

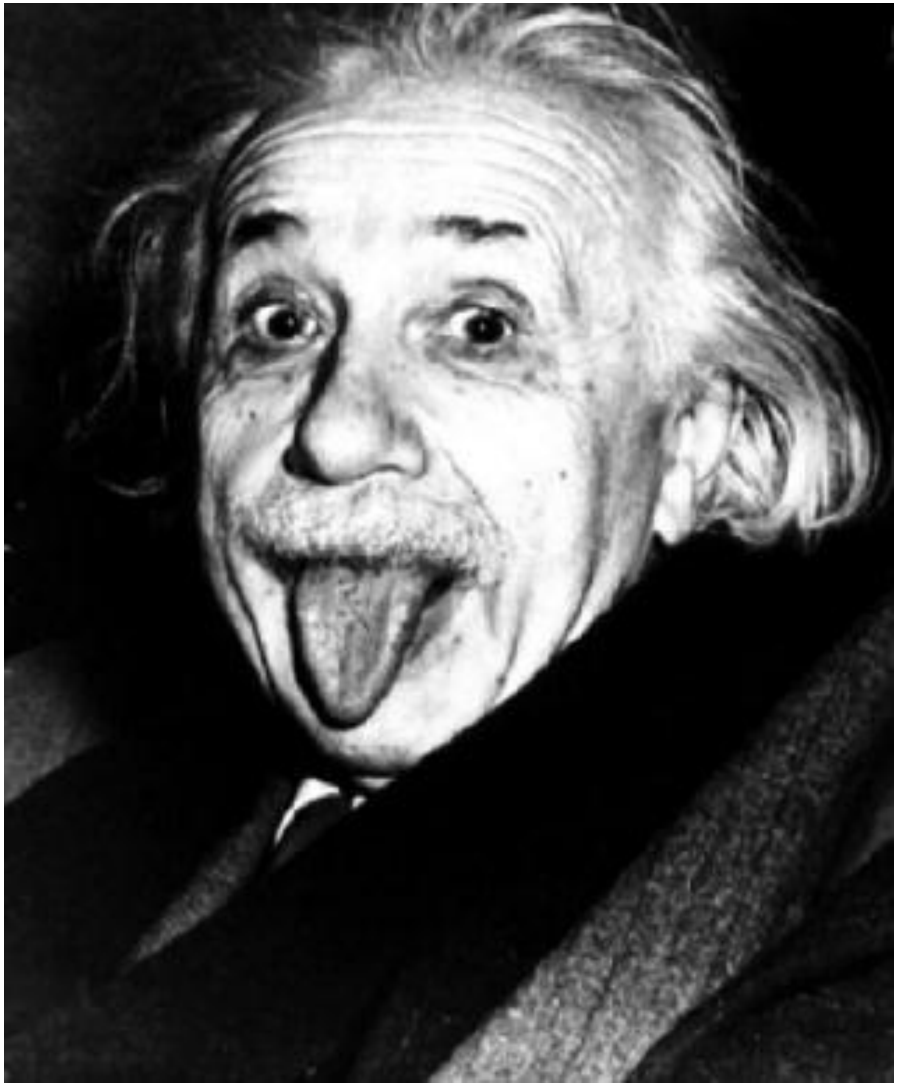


Code available from http://groups.csail.mit.edu/netmit/sFFT/code.html

- Very wide---and tuned---bin size (B ~ $\sqrt{(\mathrm{NS})}$ initially, decreasing as coefficients identified)
- Leakage-free bin filters
- Use block recovery (i.e. not individual coefficients)
- Iteratively remove identified coefficients from *bins*, not the *signal*

G = (Gaussian) * (Box-car)

Gravitational Waves



Einstein, A. and Rosen, N.:"On Gravitational Waves," *Journal of the Franklin Institute* **223**, pp.43-54 (1937)

(Or search for: “who’s afraid of the referee?”)

Linearised general relativity

Take small perturbations, **h**, of the space-time metric:

$$\boldsymbol{g}^{\mu\nu} = \boldsymbol{\eta}^{\mu\nu} + \boldsymbol{h}^{\mu\nu}$$

Put into the Einstein Field Equations:

$$\boldsymbol{G}^{\mu\nu} = \frac{8\pi G}{c^4}\boldsymbol{T}^{\mu\nu}$$

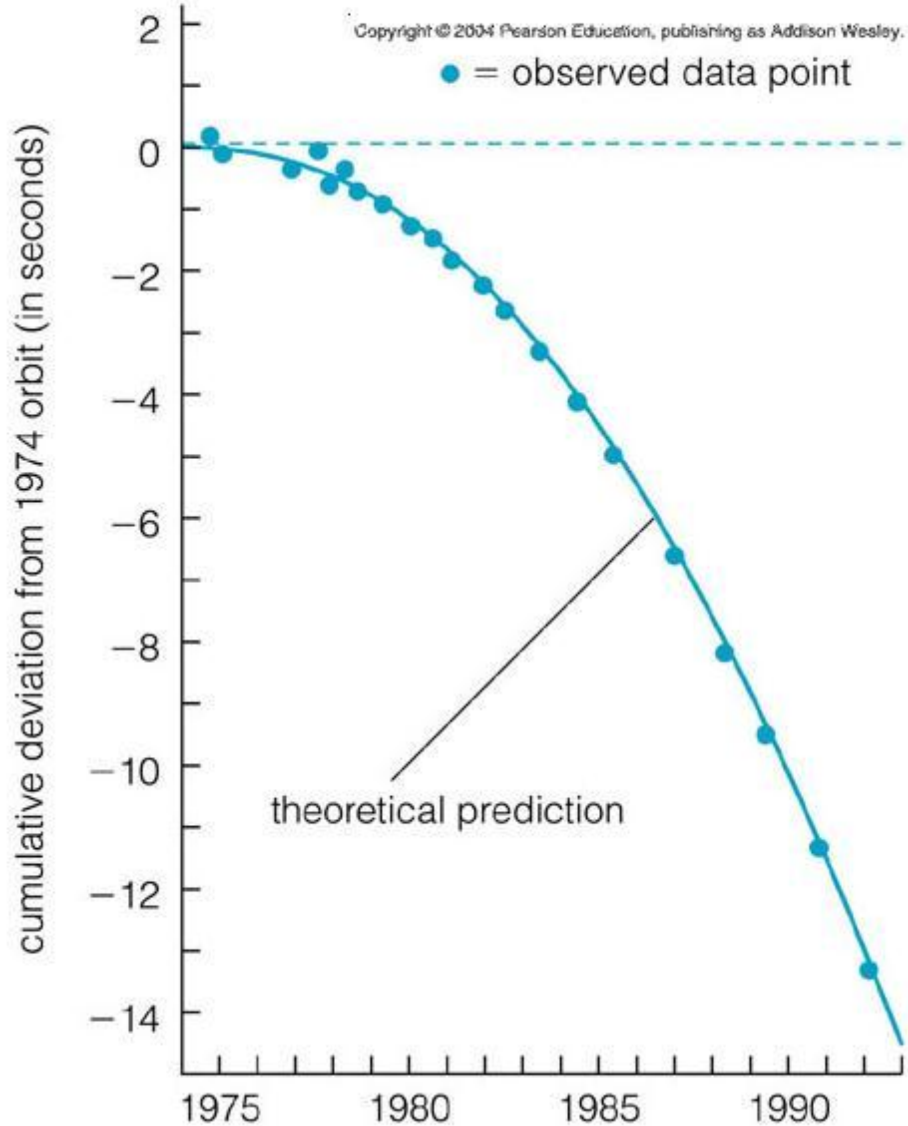
To finally get the wave-equation (transverse-traceless gauge):

$$\Box\bar{\boldsymbol{h}}^{\mu\nu} = 16\pi G\,\boldsymbol{T}^{\mu\nu}$$

Observations: Hulse and Taylor

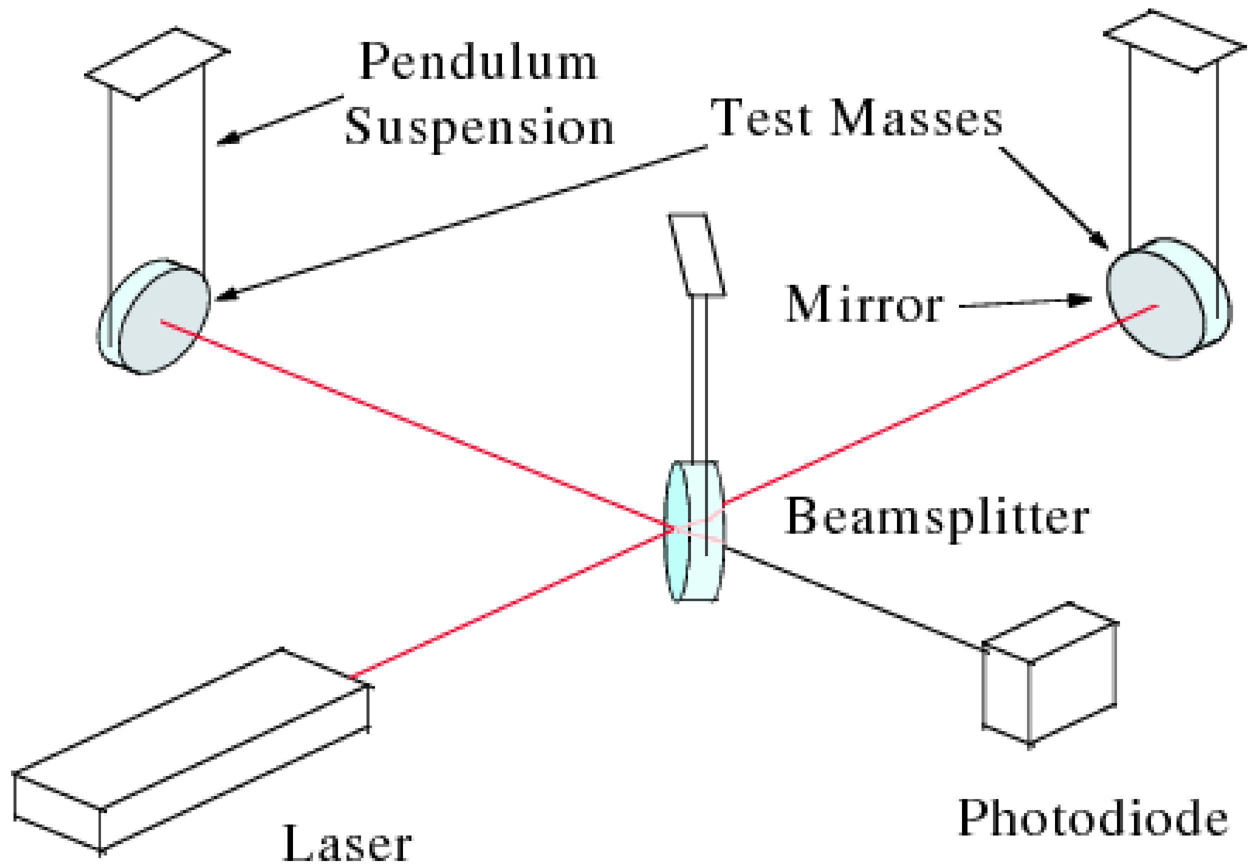
1974: Discovered binary pulsar

1993: Nobel prize



LASER interferometers

Michelson type: sensitive, broad-band



The LIGO Network

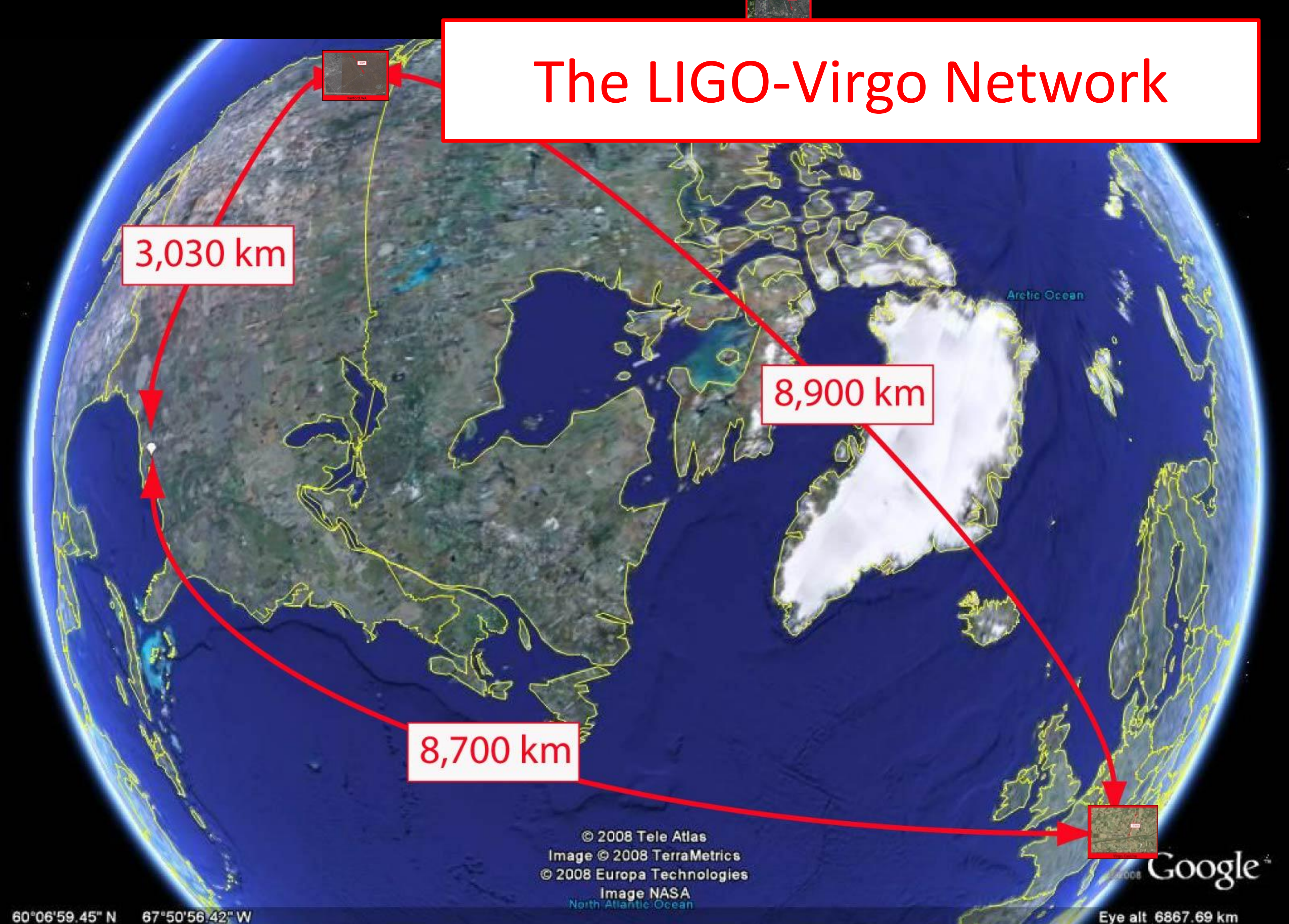
Hanford, WA

Livingston, LA

4 km baseline, seismic isolation

Hanford, WA
4 km
3,030 km
Livingston, LA
4 km
Canada
Saskatchewan
Manitoba
Montana
North Dakota
Minnesota
South Dakota
Wisconsin
Michigan
Toronto
Wyoming
Detroit
Chicago
Iowa
Nebraska
San Francisco
Nevada
Utah
Ohio
San Jose
California
Colorado
Illinois
Indianapolis
Missouri
Kentucky
Los Angeles
North Carolina
Tennessee
Oklahoma
San Diego
Arizona
Phoenix
New Mexico
Arkansas
South Carolina
Georgia
Dallas
Texas
San Antonio
Gulf of California
Florida
Image © 2008 TerraMetrics
© 2008 Tele Atlas
Image NASA
© 2008 Europa Technologies
Google
39°30'35.12" N
98°59'49.96" W
Eye alt 3986.21 km

The LIGO-Virgo Network



Gravitational wave applications

Can analyse GW data within a CS framework:

$$\Phi = D \times A \times W \times \cdots \qquad \text{(time domain)}$$

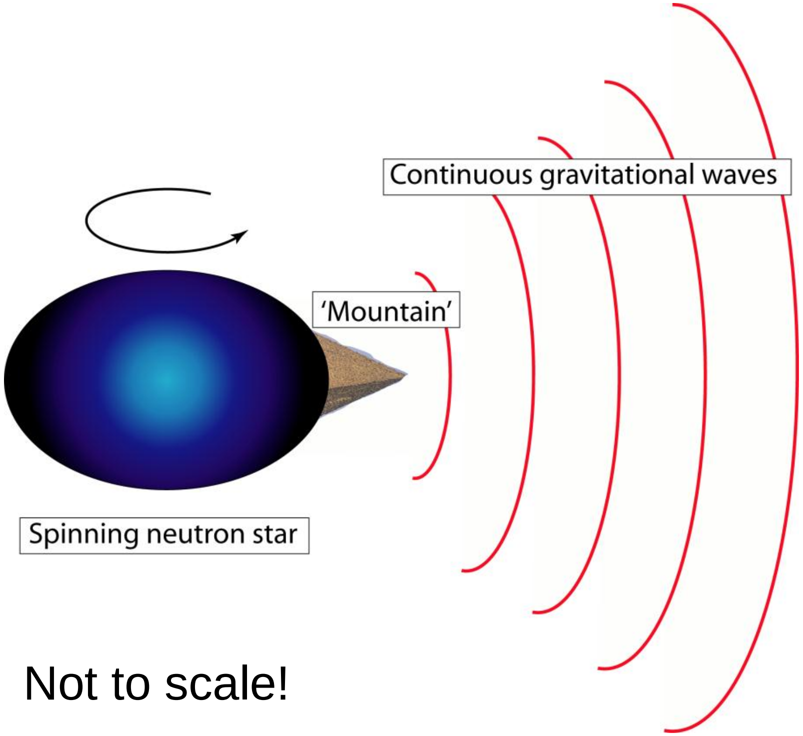
(Doppler shift parameters, antenna pattern modulation, whitening, data quality flags … any other linear transform necessary so data is ‘useable’)

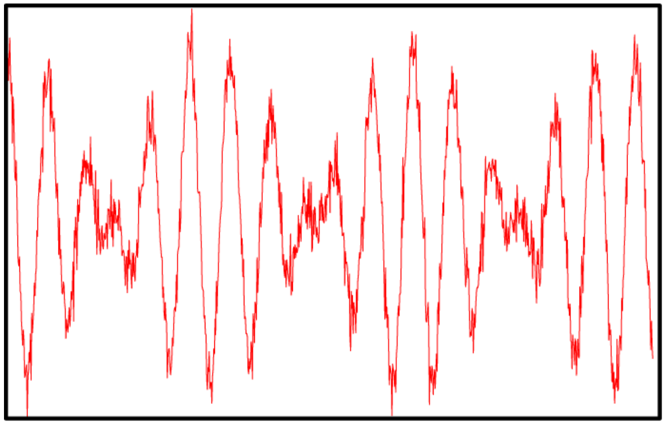
$\Psi =$

Phenomenon (GW)	Signal type	Sparse basis/frame
Rotating non-axisymmetric neutron stars	Periodic	Fourier domain
Binary compact object coalescense	Chirp/Gabor	Chirplet
Supernovae, other transients	Impulse	Time domain
Stochastic GW background	Correlation	Correlation space

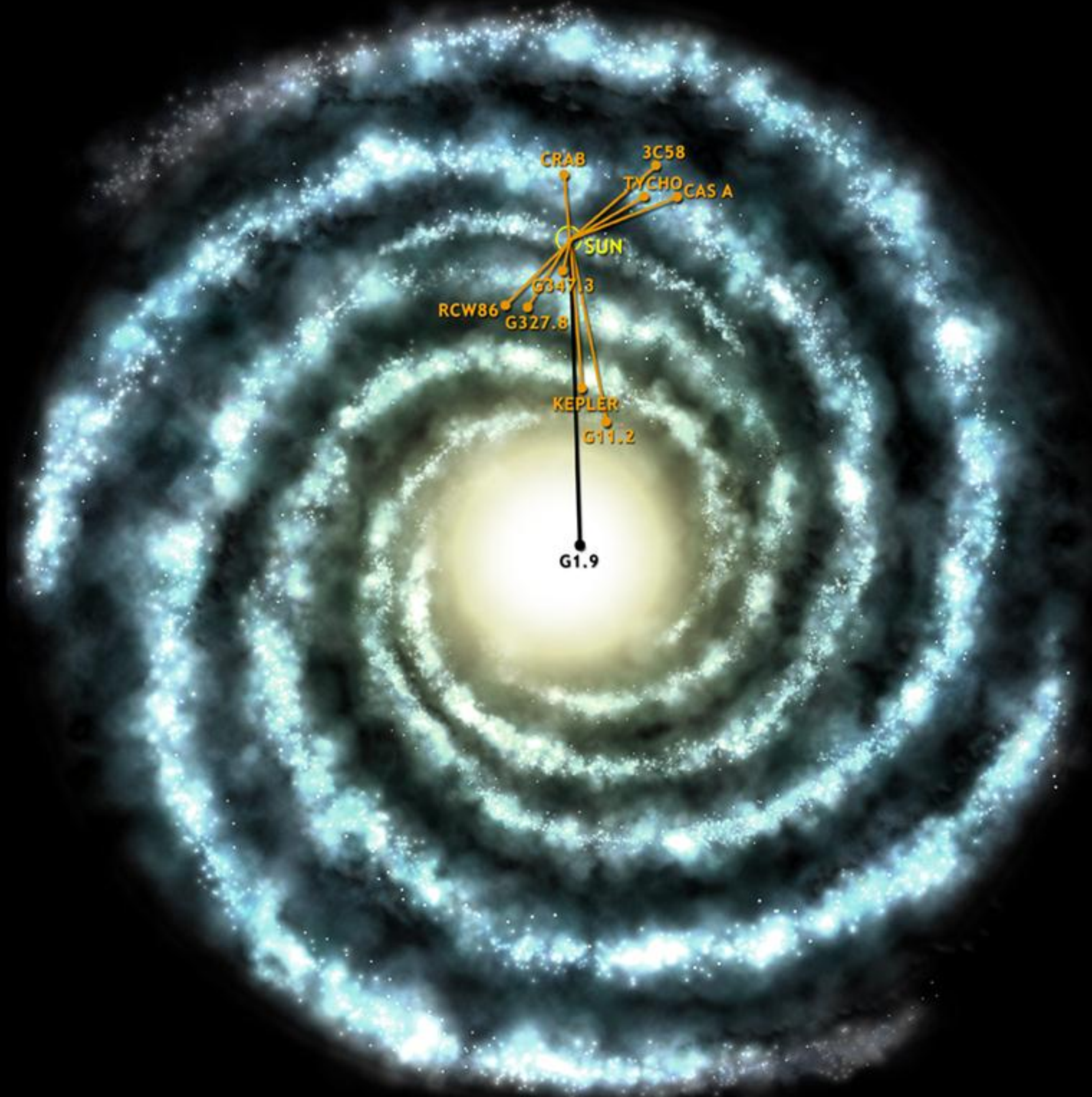
Case 1: Continuous GWs

- Rotating, non-axisymmetric neutron stars
- Signal model relatively well understood
- Low h_0, so average over long time

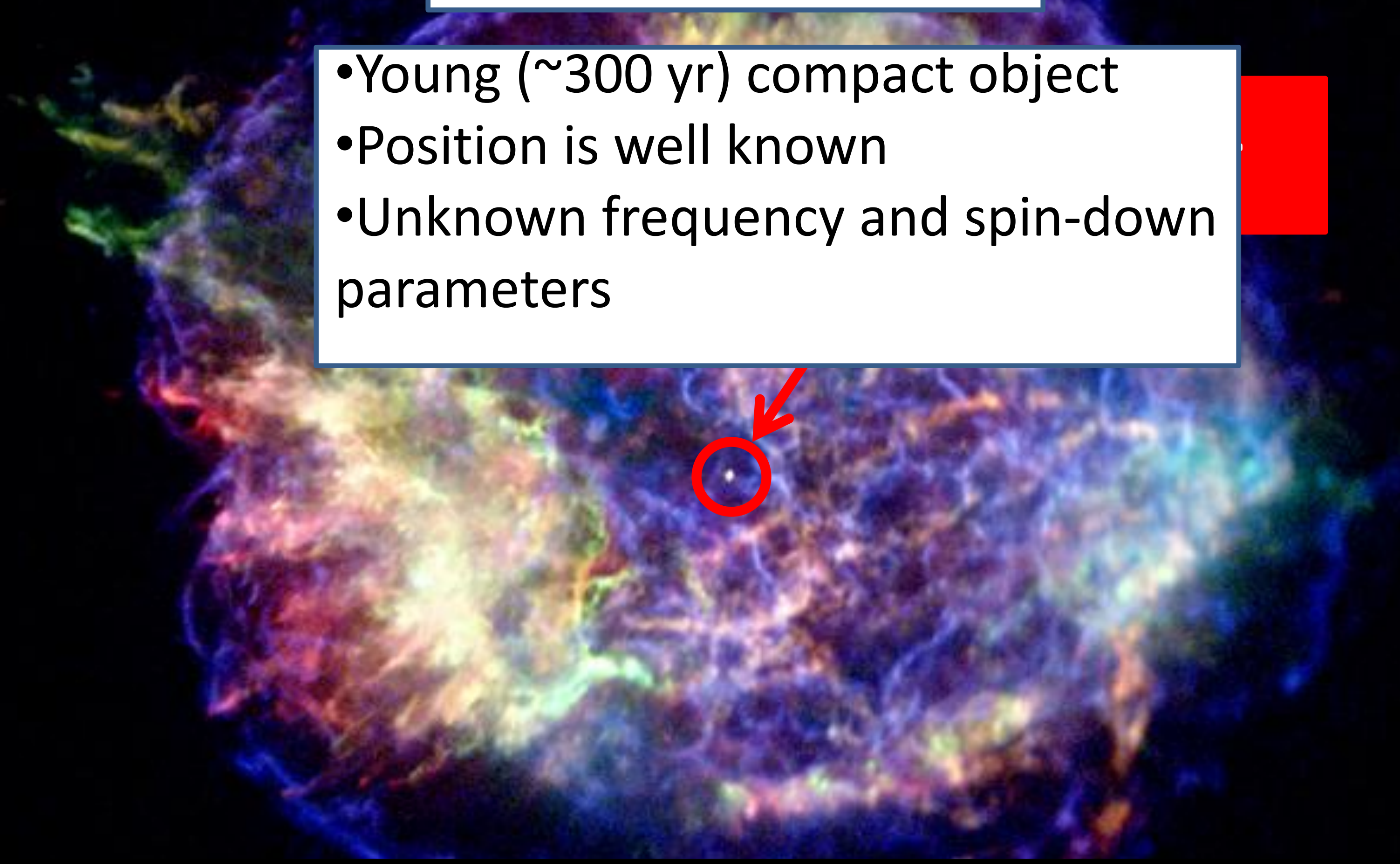




Not to scale!


CRAB
3C58
TYCHO
CAS A
SUN
G347.3
RCW86
G327.8
KEPLER
G11.2
G1.9

Cassiopeia A



- Young (~300 yr) compact object
- Position is well known
- Unknown frequency and spin-down parameters

Wette, K. *et al.:* "Searching for gravitational waves from Cassiopeia A with LIGO," *Class. Quantum Grav.*, **25**(235011):1-8 (2008)

The problem: many searches are computationally bound

e.g. Cas A search took 420,000 CPU hrs on Albert Einstein Institute's 32 Tflop Atlas* supercomputer (5,000 CPUs)

*Data from 2008, when search was performed

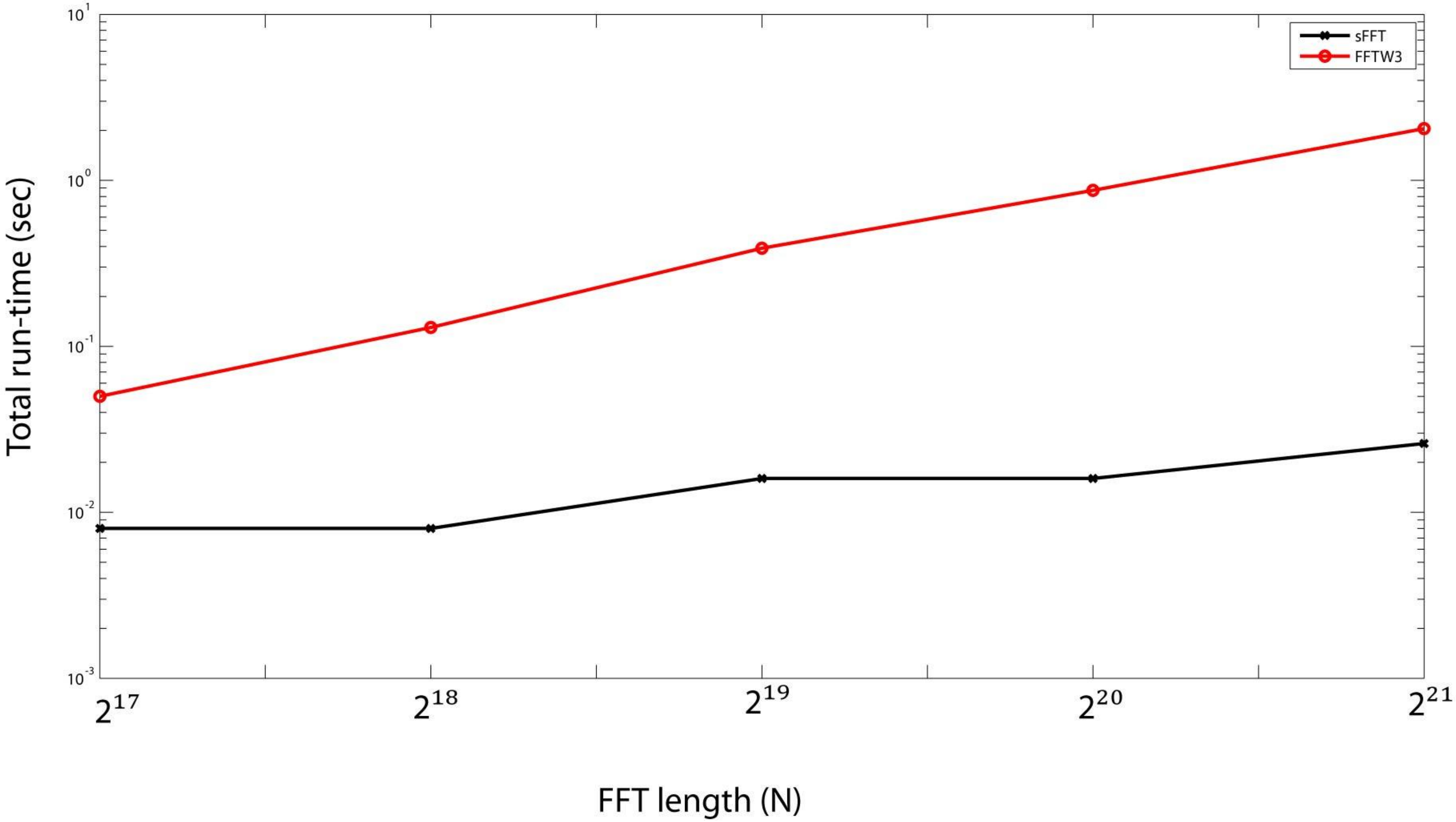
An improvement: resampling

Heterodyne data and downsample so the Nyquist frequency is a function of the search bandwidth

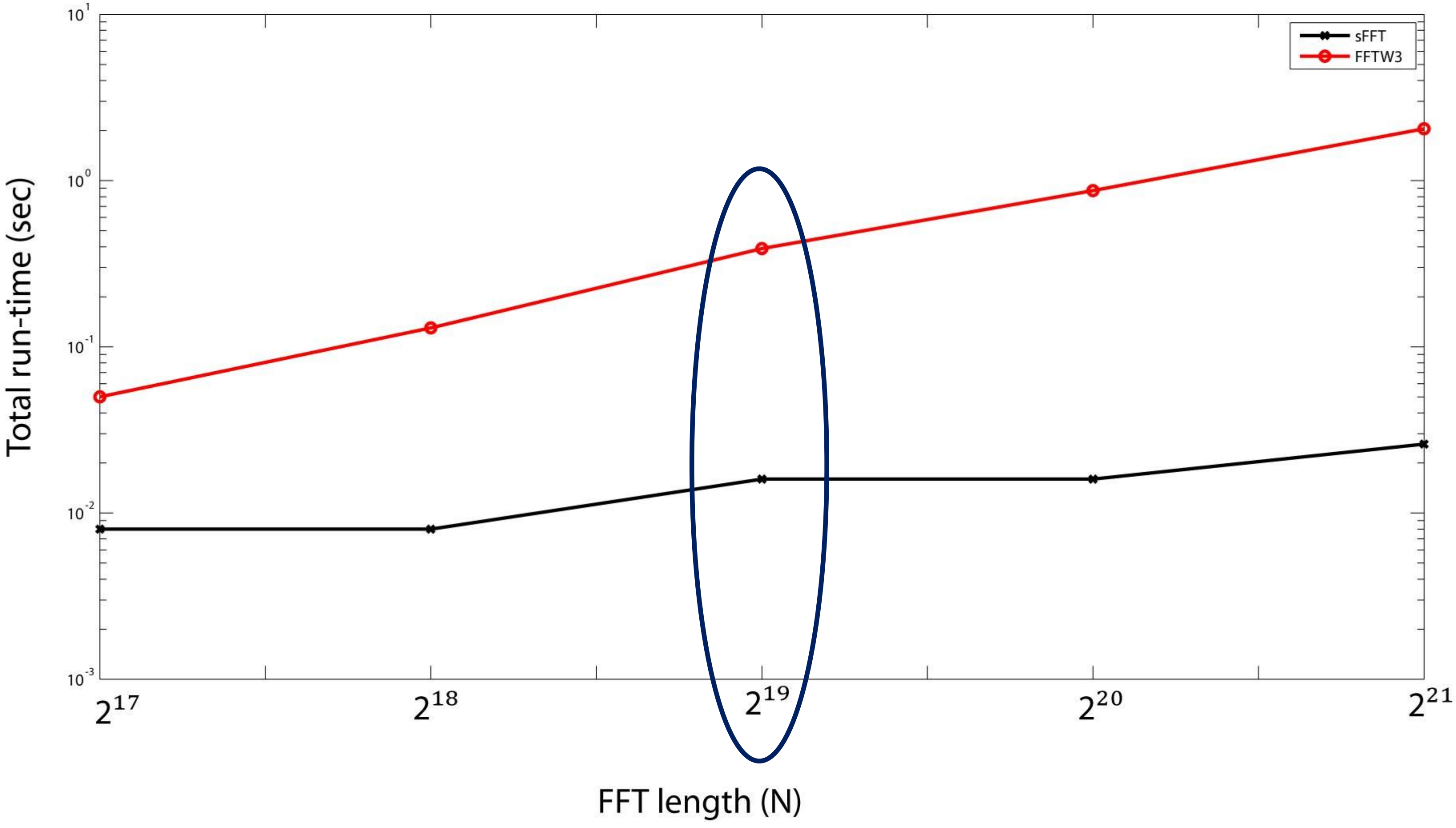
→ Significant speed-up: Cas A would take less than 1/10 the original cost.

P. Patel et al. , *Phys. Rev. D* **81**:084032:1–10 (2010)

Improvements from sFFT



Improvements from sFFT



Performance

- For a single 30 minute data stretch, searching between 100-300 Hz, gives N=720,000 (> 2^19)
- Randomised trails, for S=20, give run-times:

  sFFT: 14.8 ms

  FFTW3: 40.2 ms

  → Speed-up of over 250% !!!

- For $N{\sim}10^6$, this becomes > 467%

Will potentially make many (~ 1 dozen) currently borderline targets feasible; many more for aLIGO

E.g.

- G350.1-0.3
- DA 495 (G65.7+1.2)
- Some estimates for Vela Jr. (G266.2-1.2)

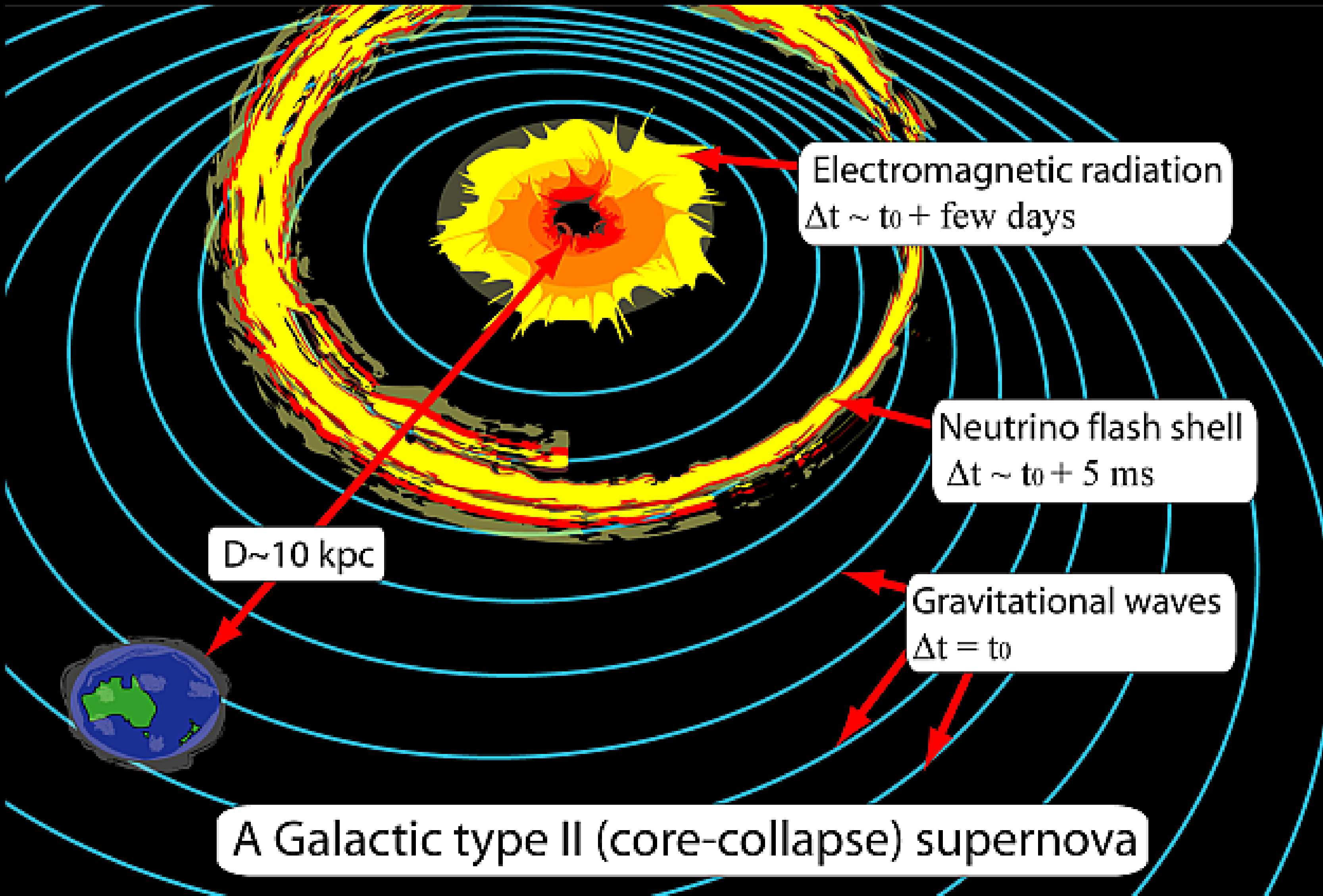
Case 2: GW burst sources

Transient, high amplitude events:

- Compact binary coalescense events
- Galactic core collapse supernovae

Highly energetic: produce range of emission species (EM, neutrino, p^+, p^-, e^-, e^+, GW radiation…)

Multi-messenger astronomy



EM Follow-up Programme

Locating and **O**bserving **O**ptical **C**ounterparts to **U**nmodelled **P**ulses

- Independent (EM) confirmation of GW burst events
- Triggered alerts from IFOs sent to telescopes with wide optical fields
- ANU's SkyMapper telescope part of the network
- Image analysis problem

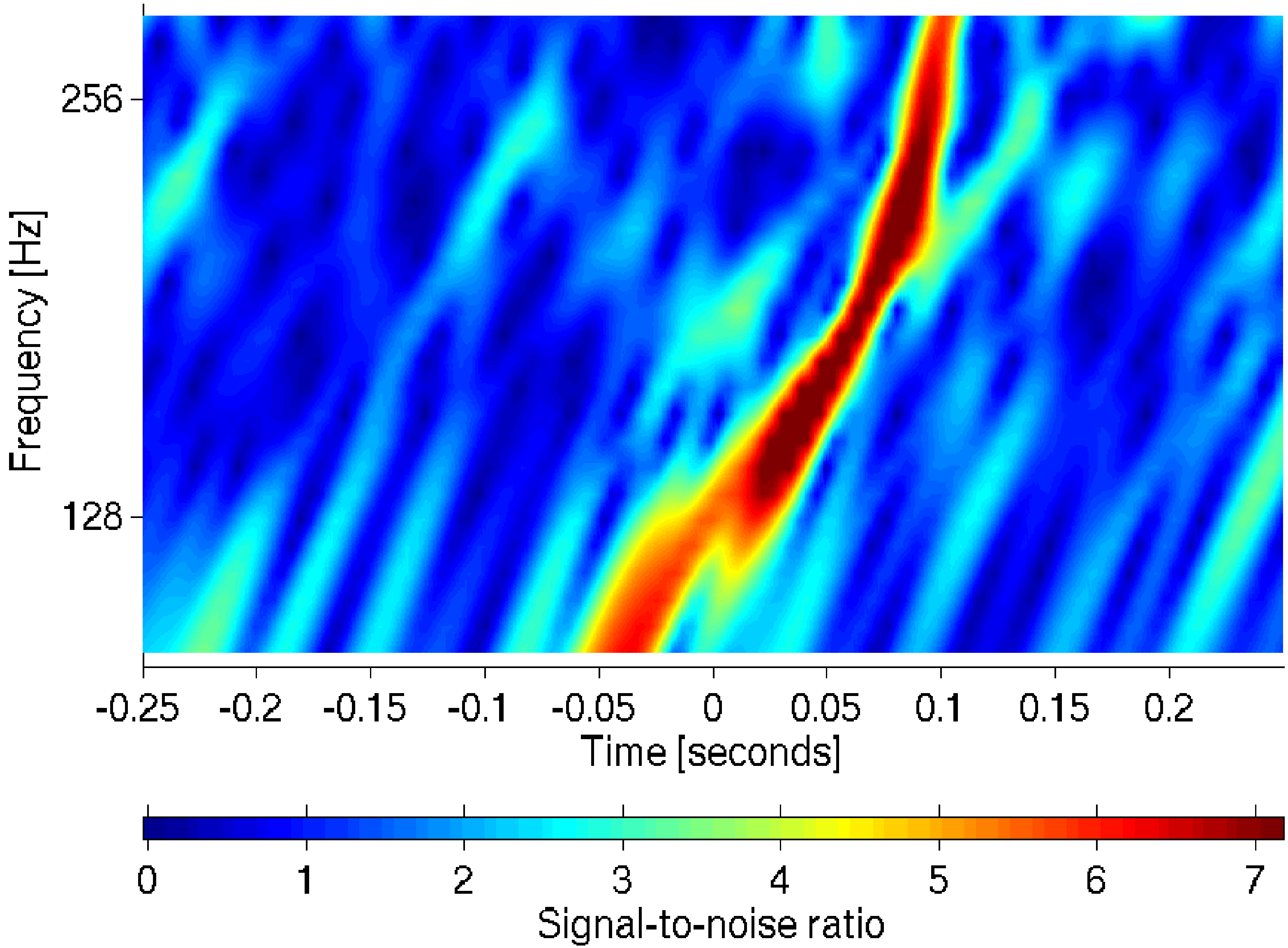
SkyMapper

Burst detection pipelines

- Omega, continuous wave-burst
- Look for excess energy in Gabor (time-frequency) plane
- Omega: Sine-Gaussian wavelets

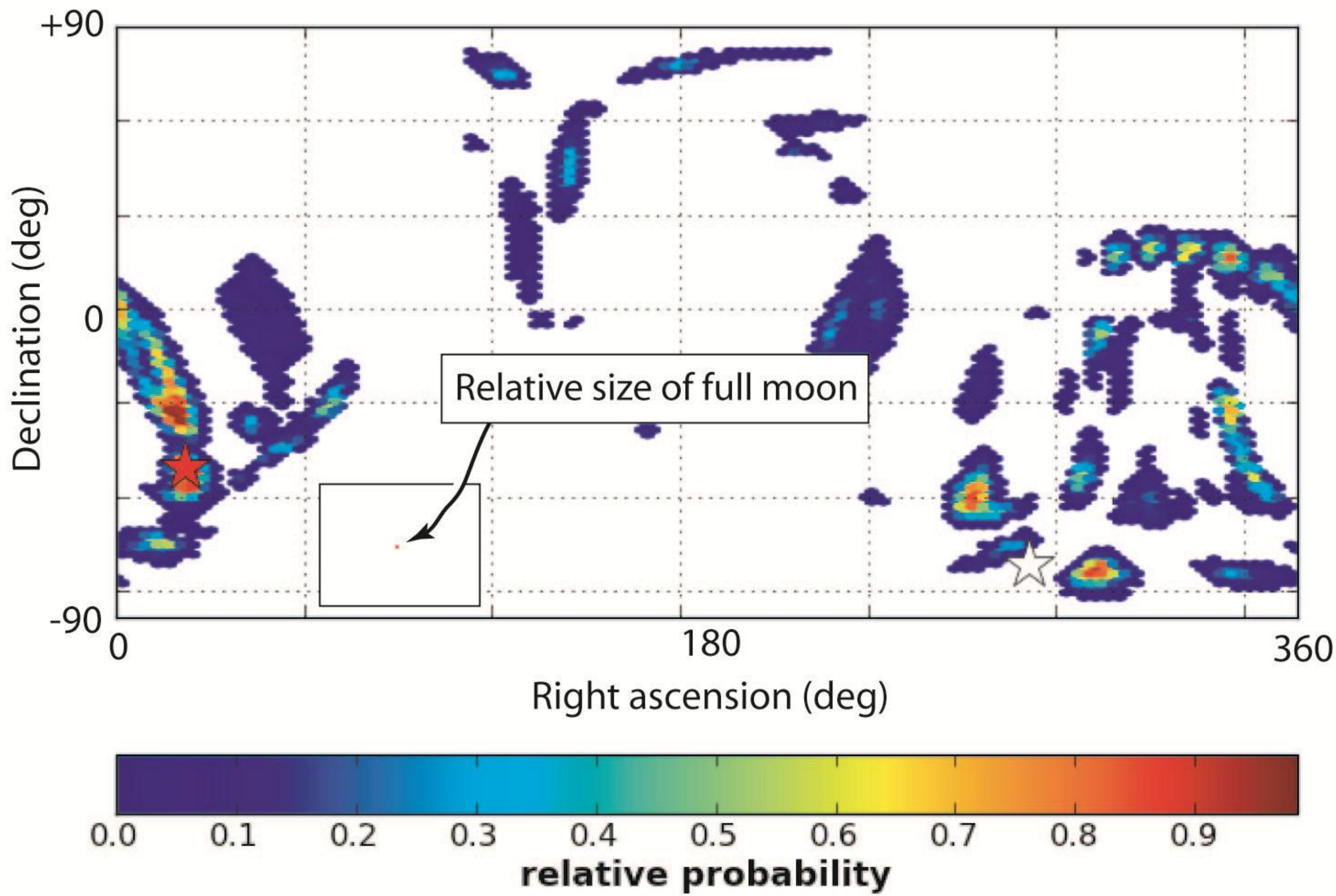
$$\psi(\tau) = A\ \exp\left(\frac{-(2\pi f)^2}{Q^2}(\tau - t)^2\right)\ \exp(2\pi i[f(\tau - t)])$$

$$A = \left(\frac{8\pi f^2}{Q^2}\right)^{1/4}$$



256
128
Frequency [Hz]
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Time [seconds]
0
1
2
3
4
5
6
7
Signal-to-noise ratio

The problem: poor position reconstruction



An improvement: chirplets

- Chirplet approach: add chirp rate parameter d

$$\psi(\tau) = A\ \exp\left(\frac{-(2\pi f)^2}{Q^2}(\tau - t)^2\right)\ \exp\left(2\pi i\left[f(\tau - t) + \frac{d}{2}(\tau - t)^2\right]\right)$$

- Covers 10 times more parameter space than Omega
- SNR enhancement of 45% (range increase of ~40%)
- High SNR signal is sparse in the Chirplet domain

É Chassande-Mottin *et al.*, *CQG* **27**:194017 (2010)

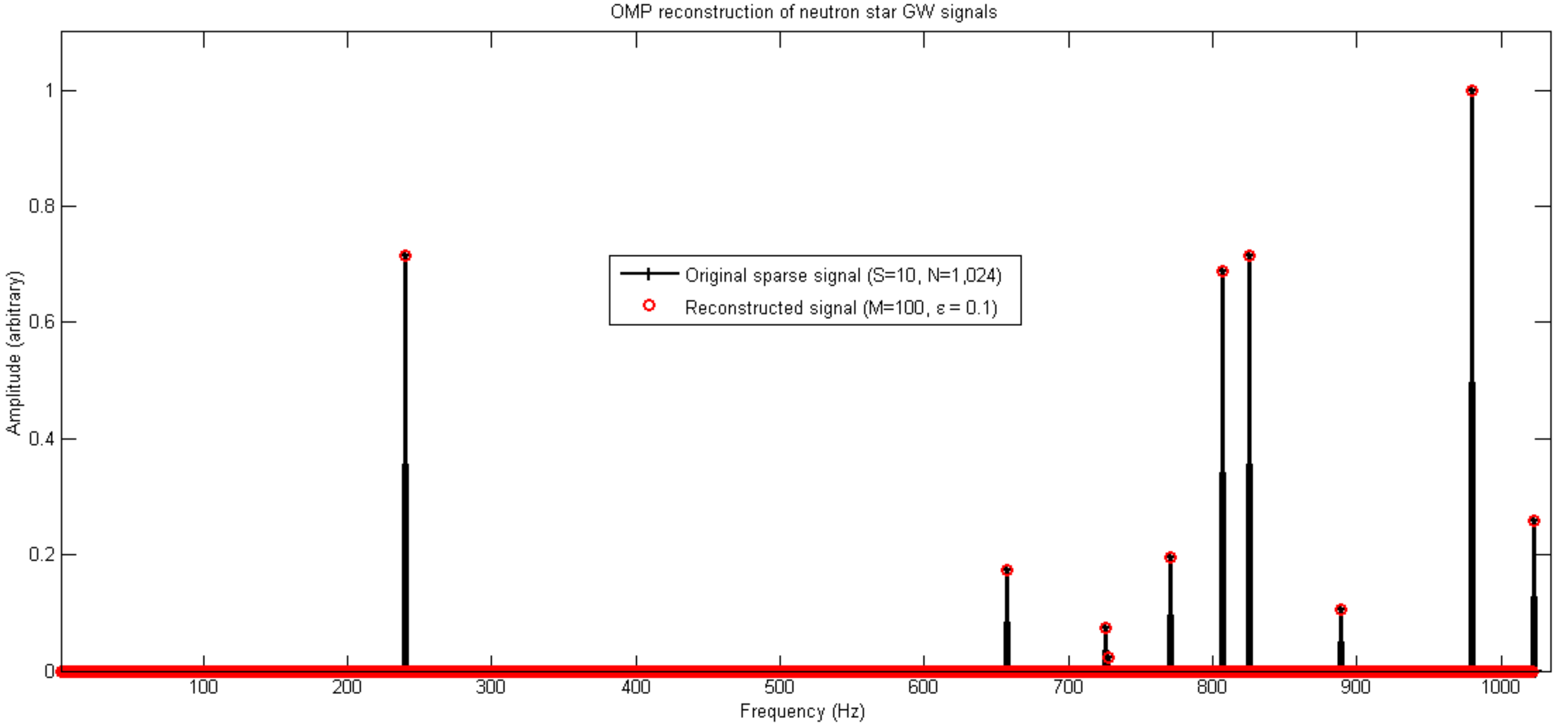
Nyquist limitations

- Nyquist limit in time-frequency plane (aliasing of chirplet templates), so reduced timing precision
- Nyquist limit: equivalent to saying that for a linear system with N unknown coefficients, require N equations to determine system.

The solution: compressive sampling in time-frequency plane

Patrick Flandrin and Pierre Borgnat: “Time-frequency energy distributions meet compressed sensing,” *IEEE Transactions on Signal Processing*, **58**(6):2974–2982 (2010)

Results on spectral lines



Under-sampling ratio: N/M ~ 10 times

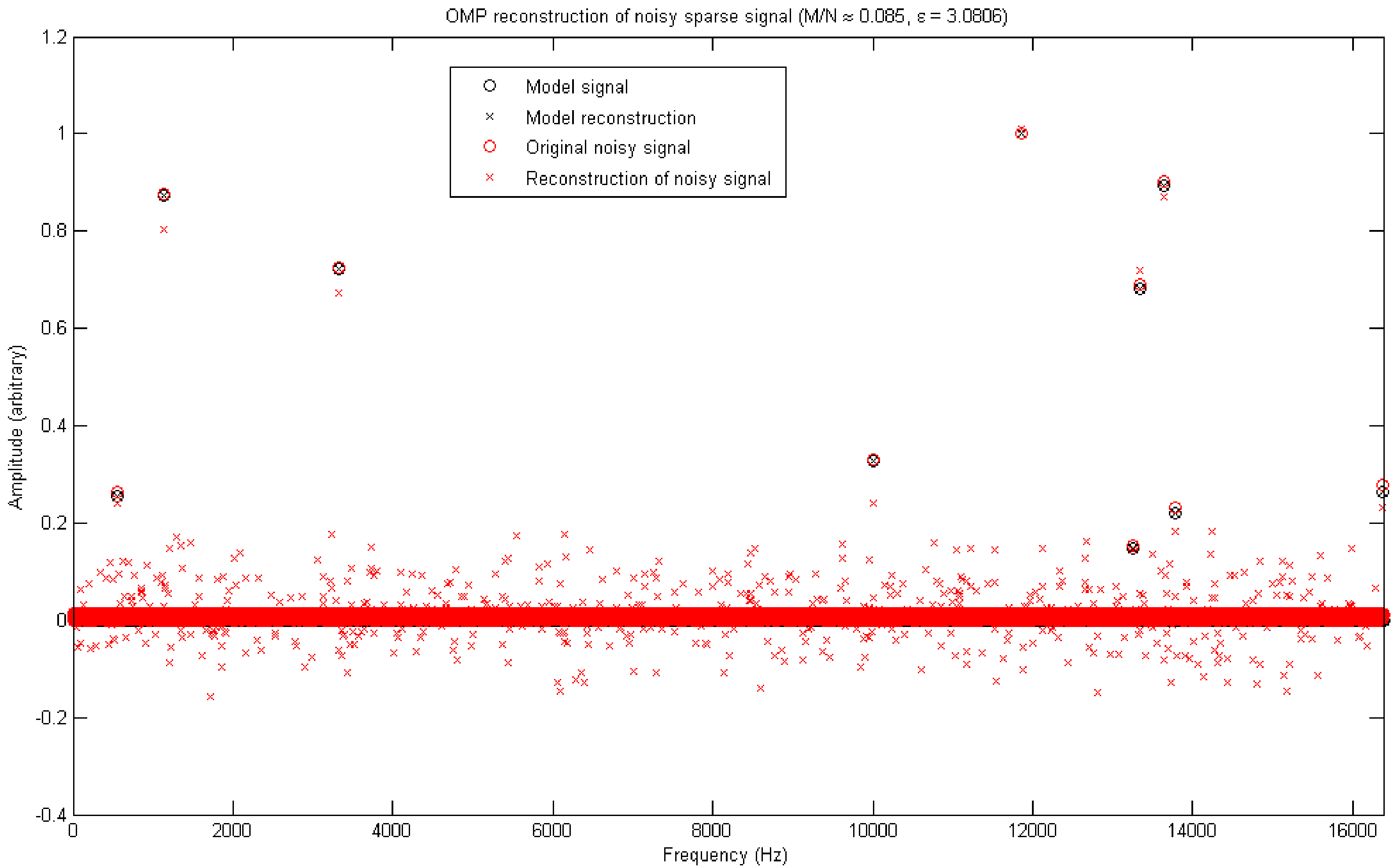
Noise

E.g. applied to CW detection statistic (the $\mathfrak{F}$-statistic). Assuming Gaussian noise, Chi-squared with 4 d.f.

$$y = \phi x + n \qquad \sum_{k=1}^{N} x^2 \sim \chi_4^2(\lambda)$$

$$\chi_4^2(x;\ \phi, y) = \sum_{k=1}^{4} \frac{n_k^2}{\sigma_k^2} \leq \epsilon^2$$

noting that p($\chi_4^2 \leq 9.49) = 0.95$, can get 95% C.I. from ϵ^2 = 9.49


OMP reconstruction of noisy sparse signal (M/N ≈ 0.085, ε = 3.0806)
Model signal
Model reconstruction
Original noisy signal
Reconstruction of noisy signal
Amplitude (arbitrary)
Frequency (Hz)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2000
4000
6000
8000
10000
12000
14000
16000

Issues

- Pure delta functions in time give maximum μ… Can't use CS
- LIGO/Virgo GW detectors most sensitive ~O(100) Hz---confined to compact region of Gabor plane
- Increased SNR---improve localisation of transient events in Gabor plane

Future work

- Faster OMP implementation: Field-Programmable Gate Arrays, ~3,000 times speed-up over conventional processors*
- Other GW data analysis applications
- CS applications in experimental GW detection methods (e.g. spectral line hunting)
- Noise studies

* R. Inta *et al*., *Int. J. of Reconfigurable Computing* **2012**:241439, 10 pp. (2012)

Thanks!

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Australian Government

Australian Research Council

Thanks for listening!

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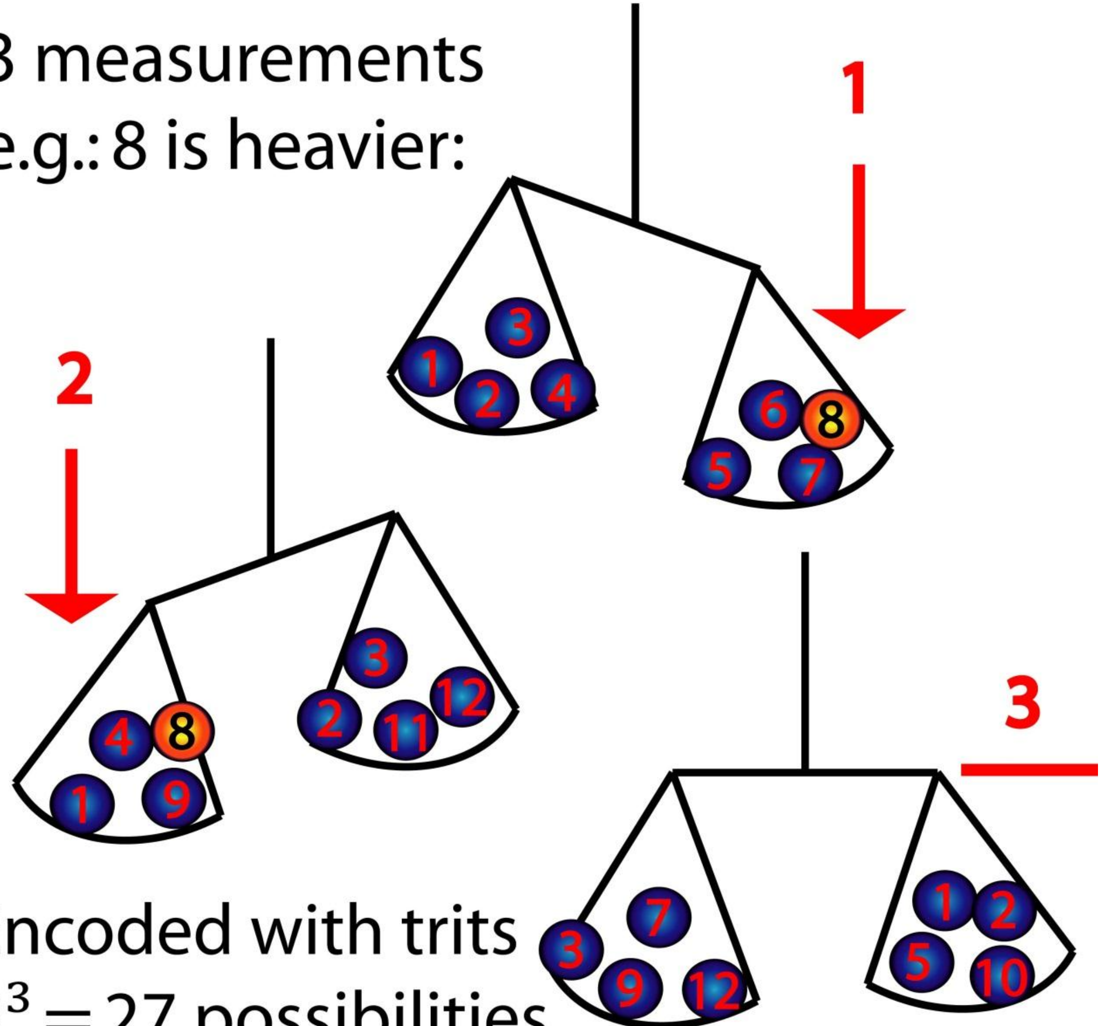
Solution to the 12-ball problem

Answer:
3 measurements
e.g.: 8 is heavier:



Encoded with trits
$3^3 = 27$ possibilities

http://www.primepuzzle.com/leeslatest/12_ball_solution.html (2006)