Compressive sampling techniques for improving the localisation of gravitational wave burst events

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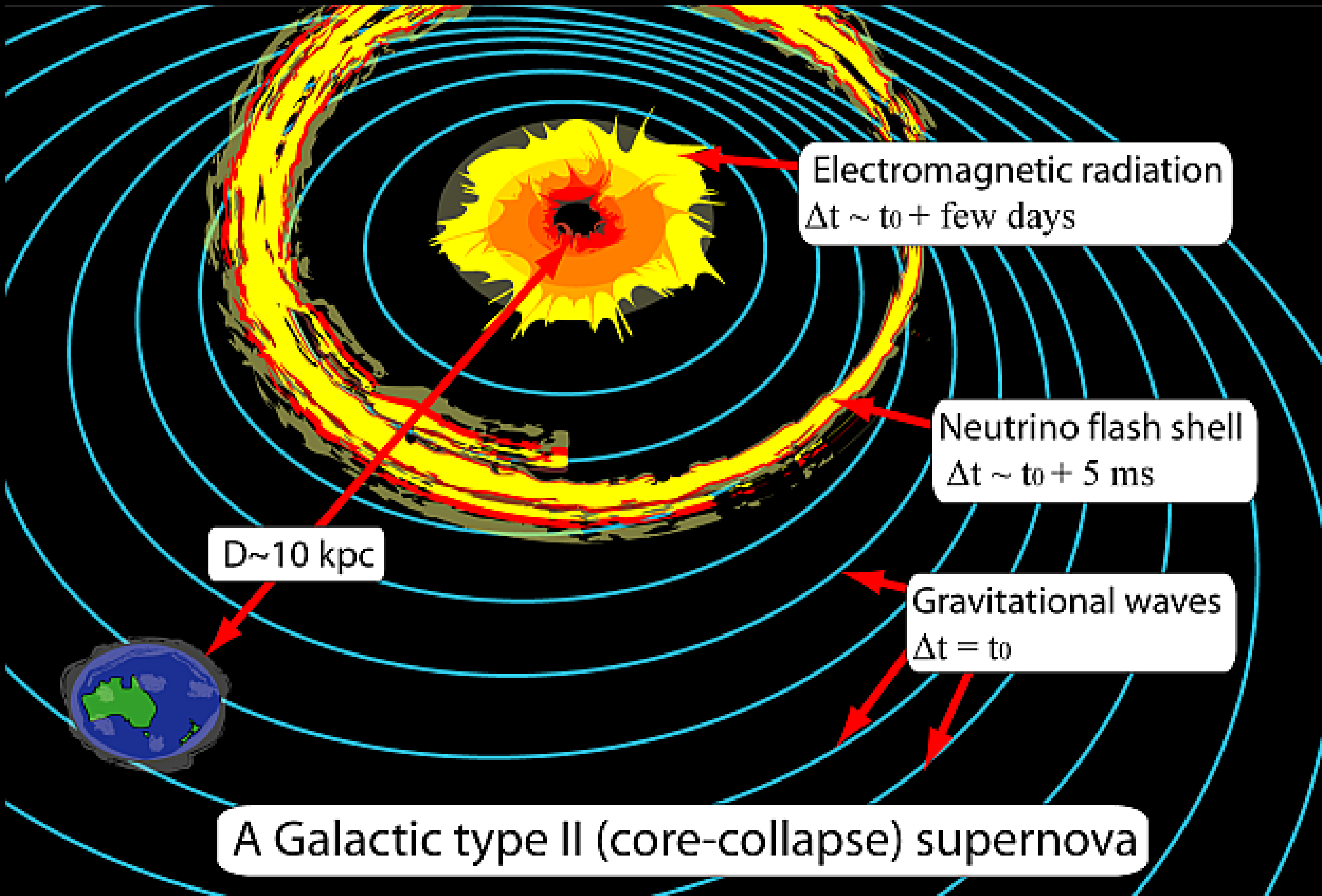
GW burst sources

Transient, high amplitude events:

- Compact binary coalescense events
- Galactic core collapse supernovae

Highly energetic: produce range of emission species (EM, neutrino, p^+, p^-, e^-, e^+, GW radiation…)

Multi-messenger astronomy



EM Follow-up Programme

Locating and Observing Optical Counterparts to Unmodelled Pulses

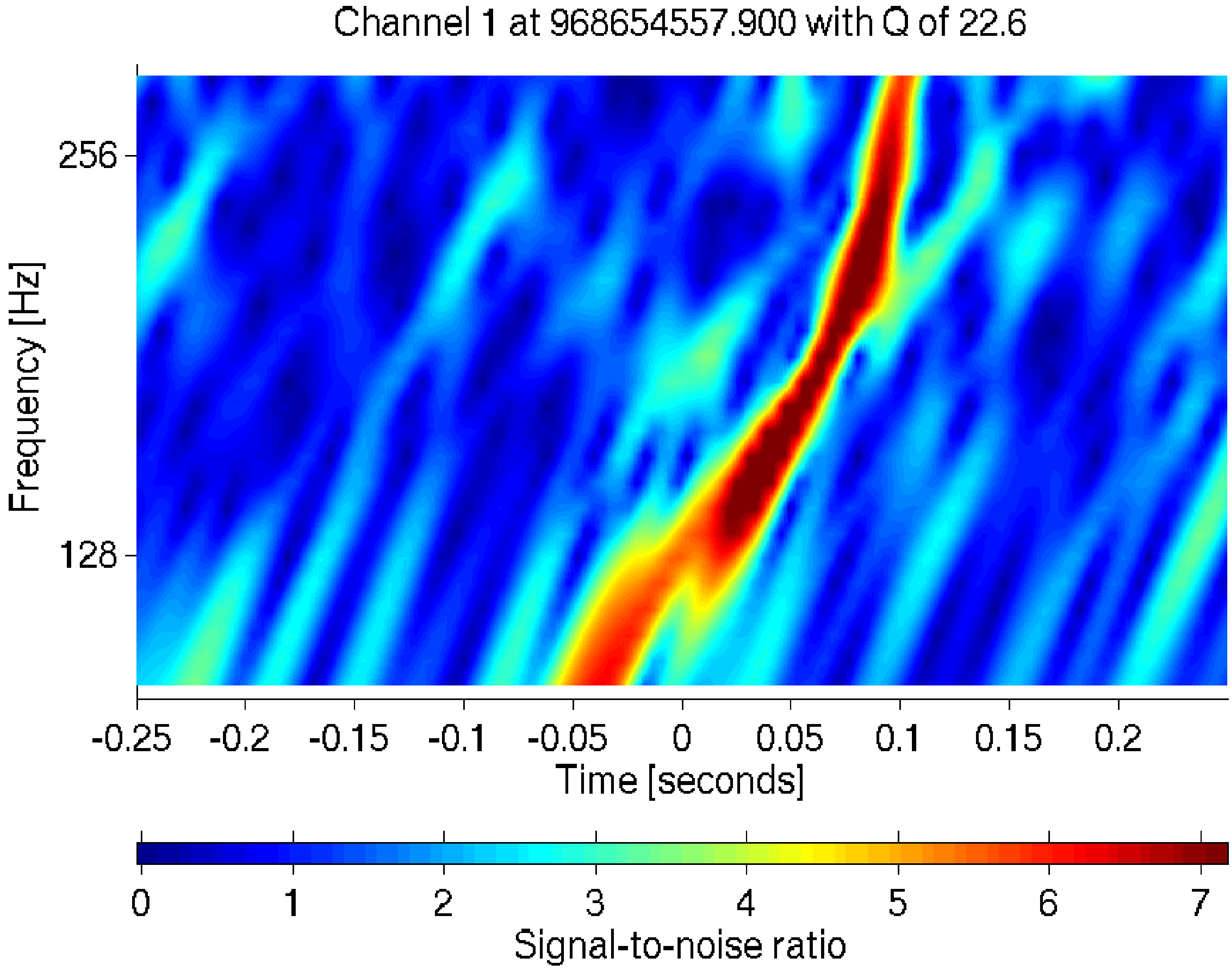
- Independent (EM) confirmation of GW burst events
- Triggered alerts from IFOs sent to telescopes with wide optical fields
- Alerts handled by LUMIN
- ANU's SkyMapper telescope part of the network
- Image analysis problem

Burst detection pipelines

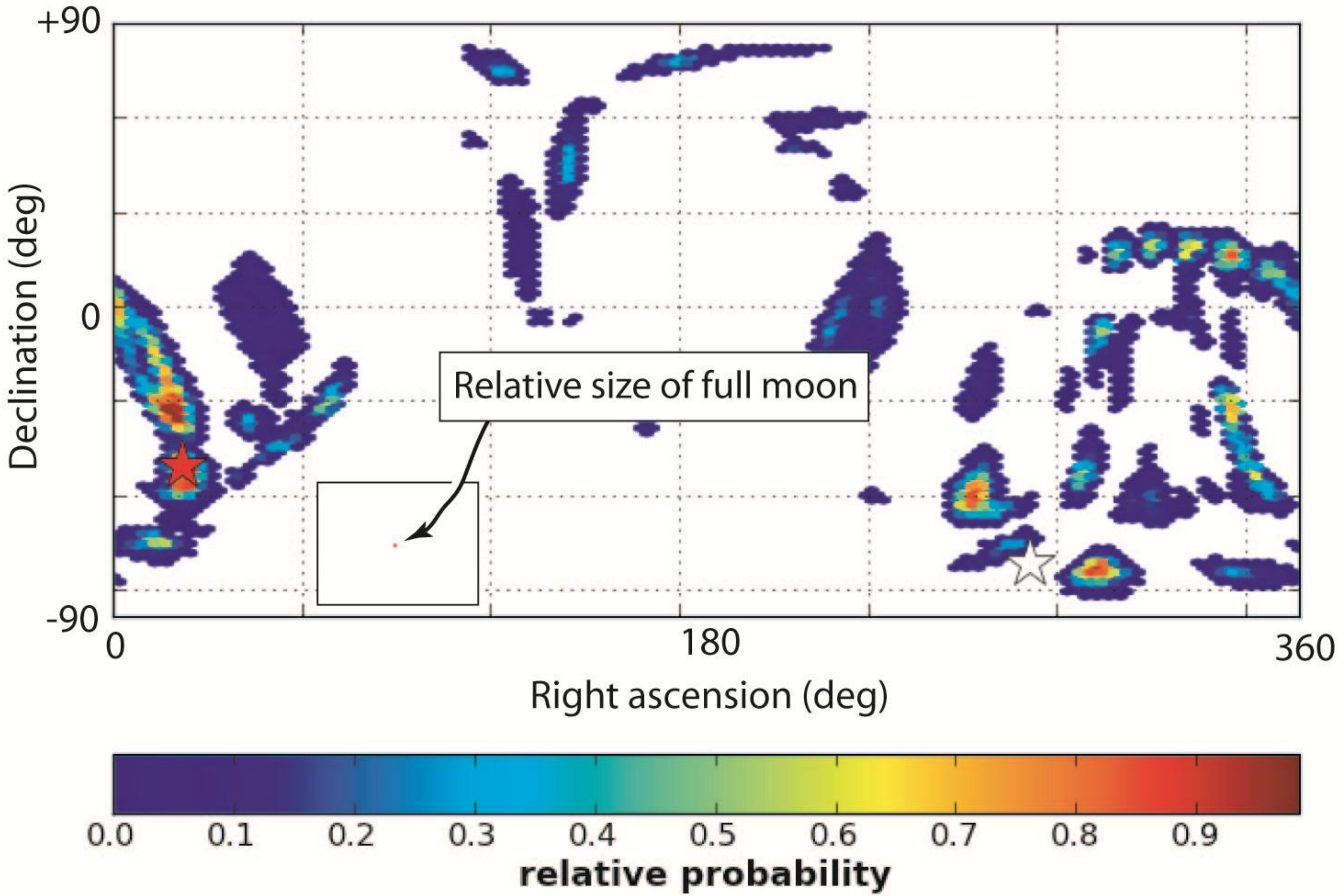
- Omega, continuous wave-burst
- Look for excess energy in Gabor (time-frequency) plane
- Omega: Sine-Gaussian wavelets

$$\psi(\tau) = A \exp\left(\frac{-(2\pi f)^2}{Q^2}(\tau - t)^2\right) \exp(2\pi i[f(\tau - t)])$$

$$A = \left(\frac{8\pi f^2}{Q^2}\right)^{1/4}$$


Channel 1 at 968654557.900 with Q of 22.6
256
128
Frequency [Hz]
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Time [seconds]
0
1
2
3
4
5
6
7
Signal-to-noise ratio

LIGO-Virgo: poor position reconstruction



Improvements to localisation

- Chirplet approach: add chirp rate parameter d

$$\psi(\tau) = A\ \exp\left(\frac{-(2\pi f)^2}{Q^2}(\tau - t)^2\right)\ \exp\left(2\pi i\left[f(\tau - t) + \frac{d}{2}(\tau - t)^2\right]\right)$$

- Covers 10 times more parameter space than Omega
- SNR enhancement of 45% (range increase of ~40%)
- High SNR signal is sparse in the Chirplet domain

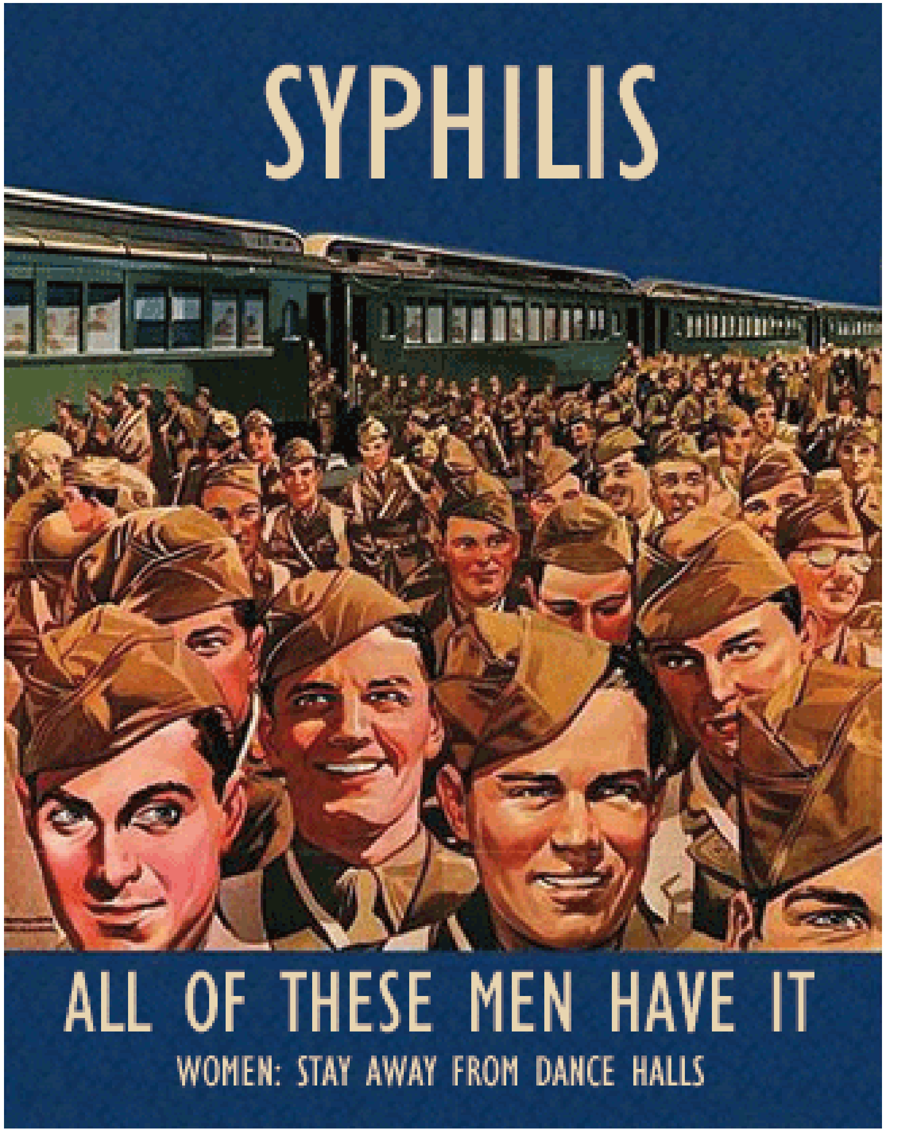
É Chassande-Mottin *et al., CQG* **27**:194017 (2010)

Nyquist limitations

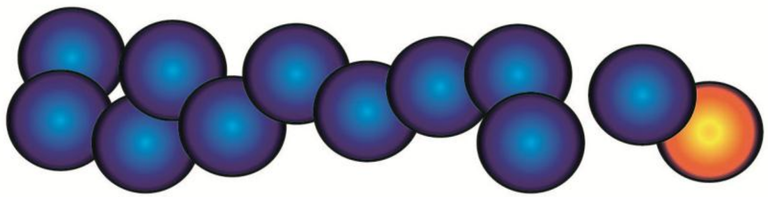
- Nyquist limit in time-frequency plane (aliasing of chirplet templates), so reduced timing precision
- Nyquist limit: equivalent to saying that for a linear system with N unknown coefficients, require N equations to determine system.

Sparse methods

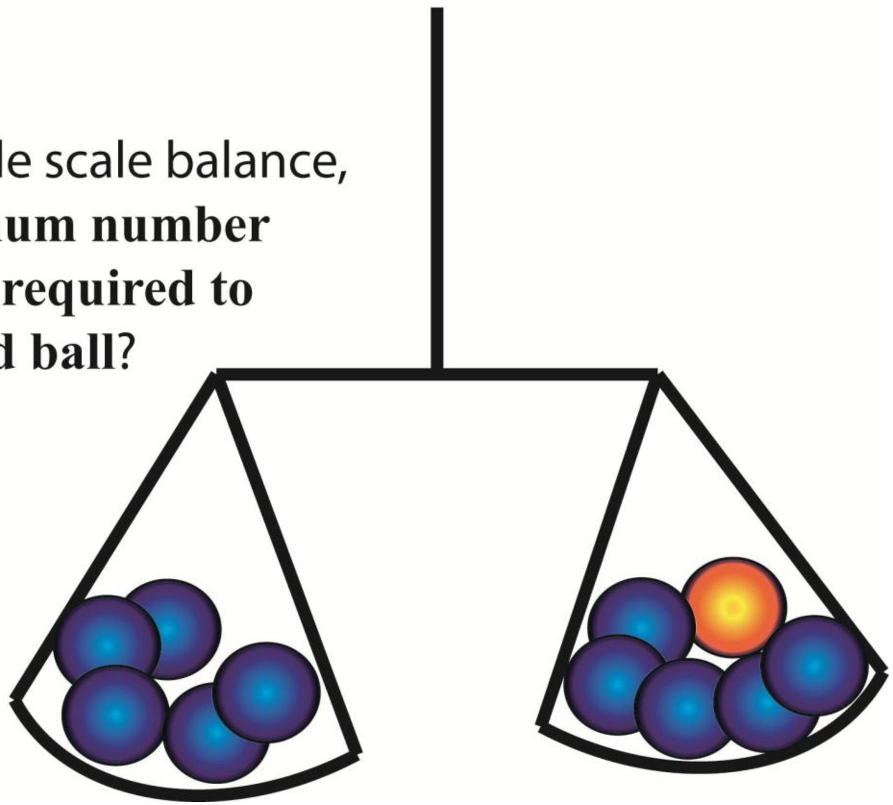
- Syphilis in WWII
- Cylons in `Battlestar Galactica'
- Twelve Ball Problem



You're given 12 identical balls, except exactly **one is heavier/lighter than the others.**



Using only a simple scale balance, what is the **minimum number of measurements required to determine the odd ball?**



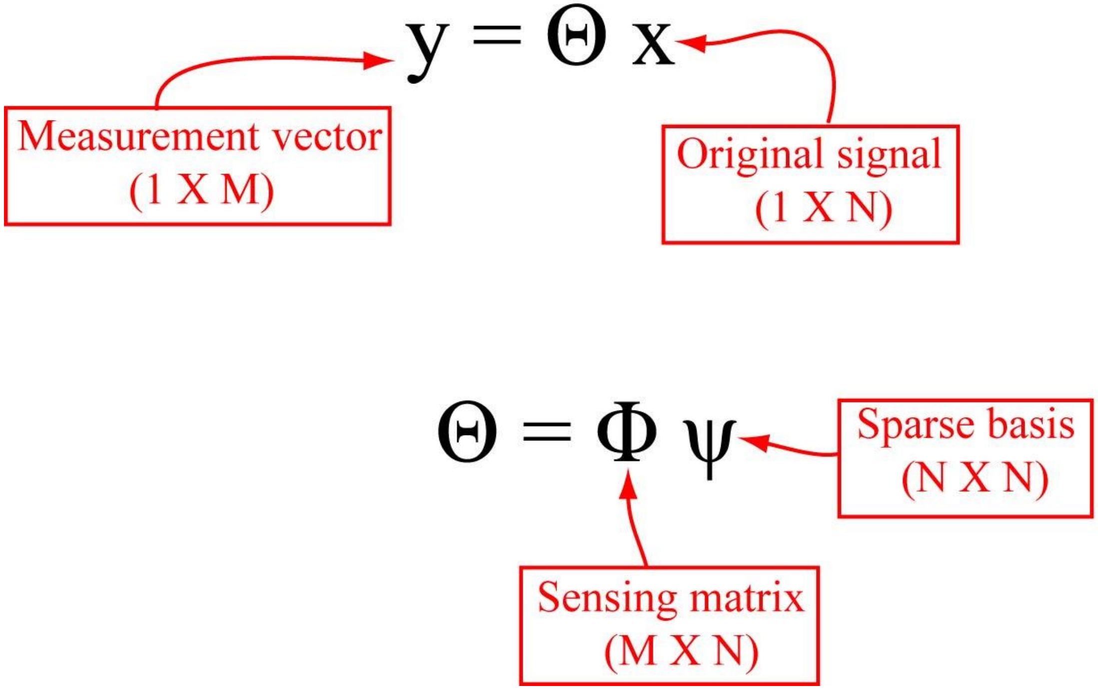
Compressive sampling (CS)

- Unified framework for encoding/reconstruction of sparse signals
- Applications from radar, Herschel space observatory, to a single pixel camera

Image: M. F. Duarte et al., Rice University

E. Candès, J. Romberg and T. Tao: *IEEE Trans. on Information Theory* **52**(2):489–509 (2006)

CS framework



Determined system: M = N
CS (‘under-determined’): M << N

To solve for x, impose sparsity:

$$\text{find } \|x\|_0 \quad \text{s.t.} \quad \|y - \Theta\, x\|_2 = 0$$

...but this is a combinatorial problem!
(Need to guess up to ${}^{N}C_{M}$ coefficients)

Instead, almost as good (esp. for large N):

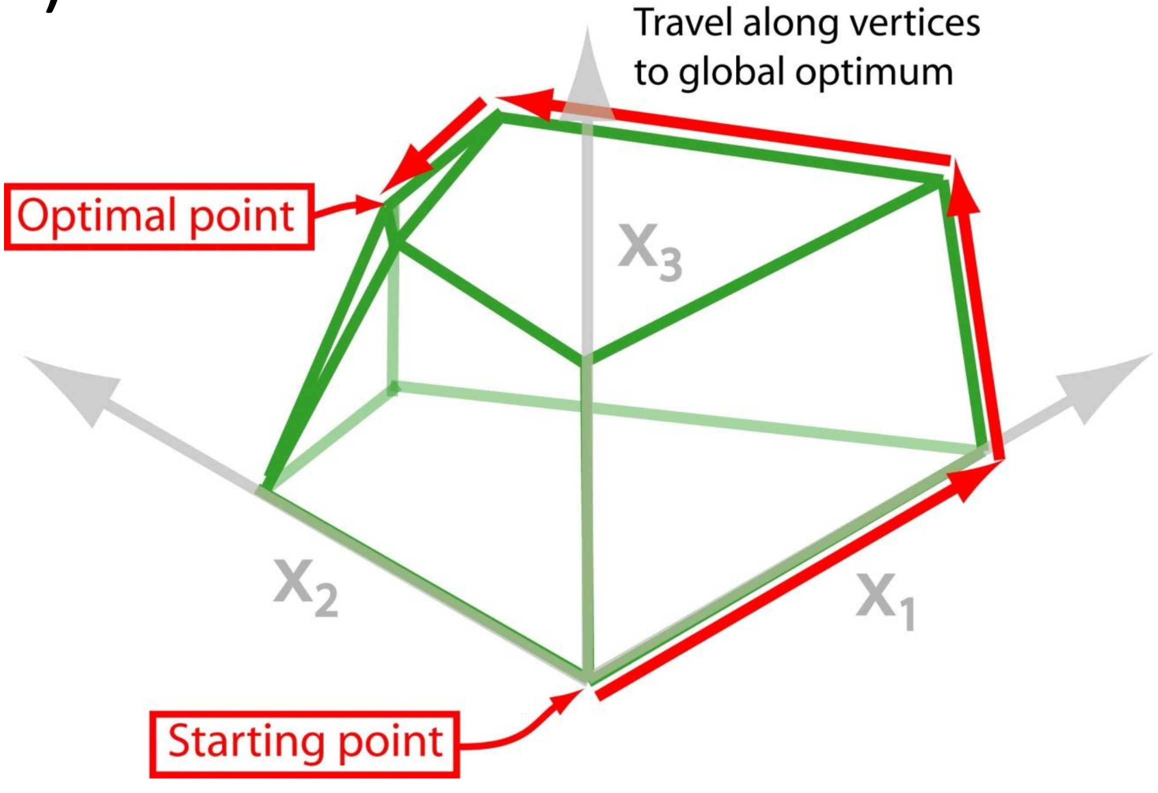
$$\text{find } \|x\|_1 \quad \text{s.t.} \quad \|y - \Theta\, x\|_2 = 0$$

Compressible/noisy signals

Find: $\|x\|_1$ s.t. $\|y - \Theta x\|_2 \leq \epsilon$

Linear Program

- Optimisation reconstruction methods
- E.g. Simplex method (linear constraints give convex polytope)



Orthogonal Matching Pursuit

- Recursive Greedy algorithm
- Very similar to CLEAN algorithm used in radio astronomy
- Easy to implement noise-based criteria

CS results

Can determine minimum number of samples required for perfect reconstruction:

$$M > C\, S\, \mu^2 \ \log(N)$$

Where *C* is small (~0.5) , and the mutual coherence:

$$\mu \ \stackrel{\text{def}}{=}\ \sqrt{N} \max_{i,j<N} | < \varphi_i, \psi_j > |$$

Measures how ‘spread out’ the signal is in the sampling domain

Mutual coherence

Guarantees sampled low dimensional subspace sufficiently covers sparse basis (Restricted Isometry)

e.g. Delta functions in Fourier domain are minimally coherent with time domain:

$$\mu = \sqrt{N} \max_k \int \delta(f - f_k) e^{i2\pi f t} dt = \sqrt{N} \frac{1}{\sqrt{N}} = 1$$

E. Candès and J. Romberg, *Inverse Problems*, 23:969–985 (2007)

CS applications

Require:

1. Sparse/compressible representation
2. Low mutual coherence

GW applications

Interpret GW data in CS framework:

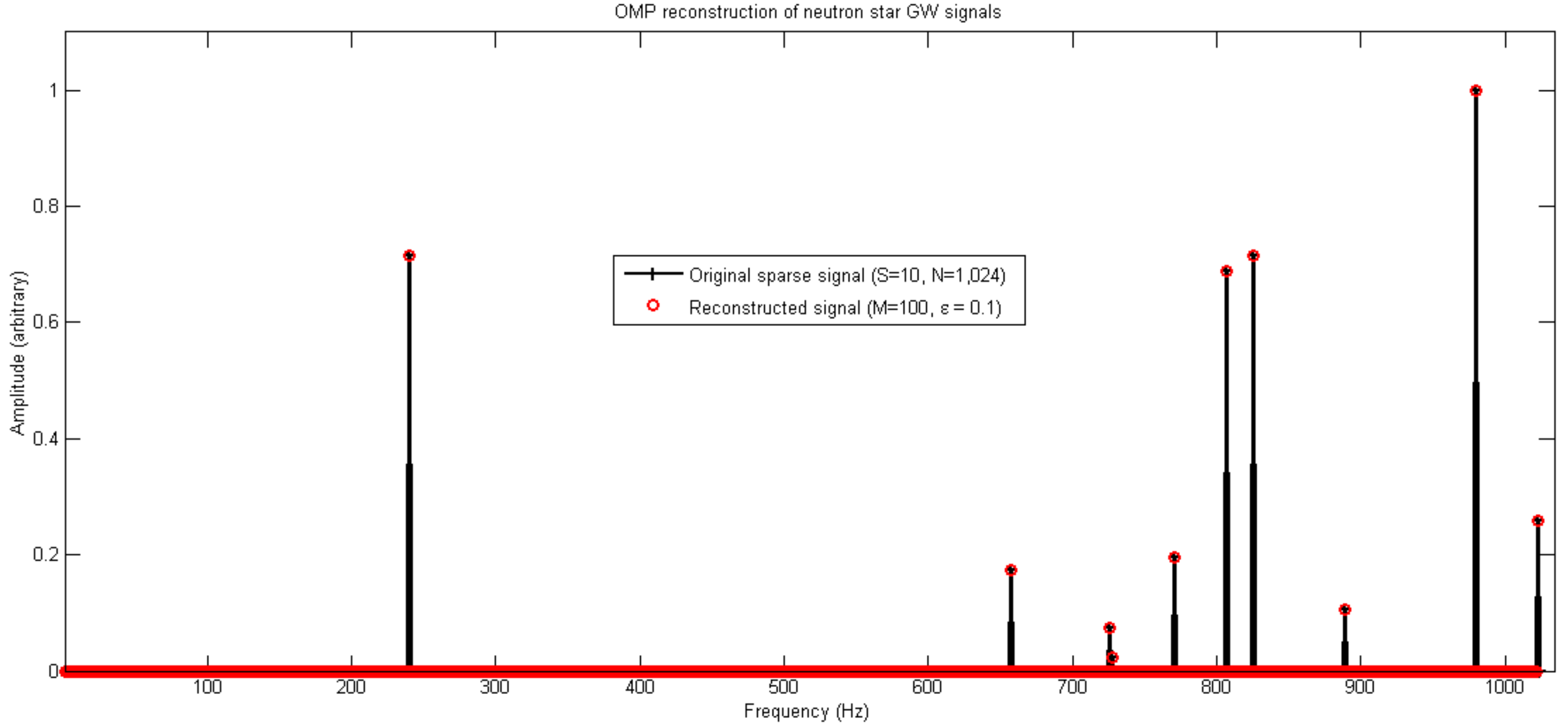
$$\Phi = \ D \times A \times W \times \cdots \qquad \text{(time domain)}$$

(Doppler shift parameters, antenna pattern modulation, whitening, data quality flags … any other linear transform necessary so data's 'useable')

$\Psi =$

Phenomenon (GW)	Signal type	Sparse basis/frame
Rotating non-axisymmetric neutron stars	Periodic	Fourier domain
Binary compact object coalescense	Chirp/Gabor	Chirplet
Supernovae, other transients	Impulse	Time domain
Stochastic GW background	Correlation	Correlation space

Results on spectral lines



Under-sampling ratio: N/M ~ 10 times

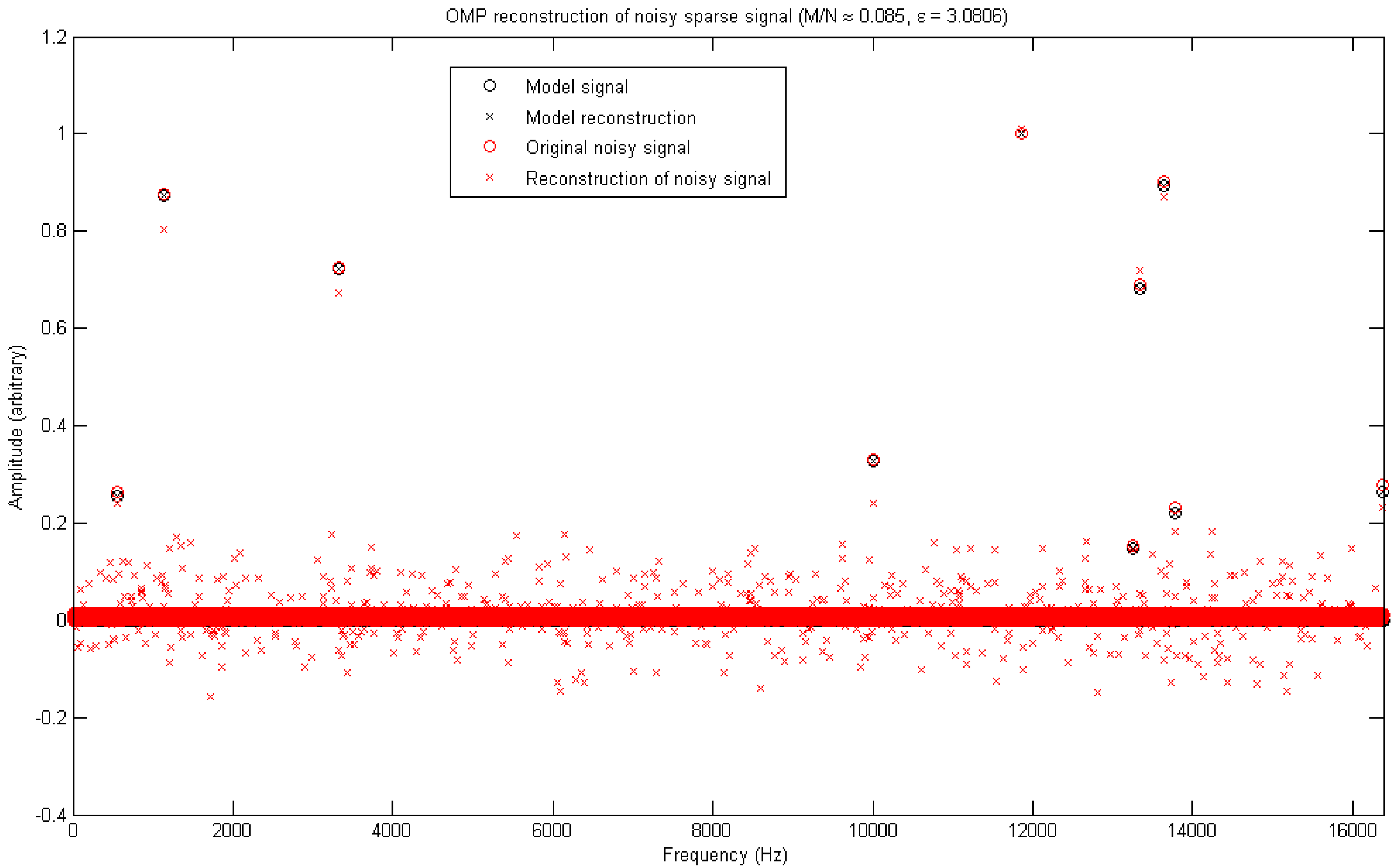
Noise

E.g. applied to CW detection statistic (the $\mathfrak{F}$-statistic). Assuming Gaussian noise, Chi-squared with 4 d.f.

$$y = \phi x + n \qquad \sum_{k=1}^{N} x^2 \sim \chi_4^2(\lambda)$$

$$\chi_4^2(x;\ \phi, y) = \sum_{k=1}^{4} \frac{n_k^2}{\sigma_k^2} \leq \epsilon^2$$

noting that p($\chi_4^2 \leq 9.49) = 0.95$, can get 95% C.I. from ϵ^2 = 9.49



OMP reconstruction of noisy sparse signal (M/N ≈ 0.085, ε = 3.0806)
Model signal
Model reconstruction
Original noisy signal
Reconstruction of noisy signal
Amplitude (arbitrary)
Frequency (Hz)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2000
4000
6000
8000
10000
12000
14000
16000

GW burst detection

- Pure delta functions in time give maximum μ… Can't use CS?!?
- LIGO/Virgo GW detectors most sensitive ~O(100) Hz---confined to compact region of Gabor plane
- Increased SNR---improve localisation of transient events in Gabor plane

P. Flandrin and P. Borgnat: *IEEE Transactions on Signal Processing*, **58**(6):2974–2982 (2010)

Breaking News: The 'FFFT'

- Sparse Fast Fourier Transform
- Implements non-recursive OMP
- Computational complexity: $O(S \log(N))$

Compare to FFT [$O(N \log(N))$]:
Speed-up is $O(N/S)$, so for $N=10^6$, S=100 :

FFFT is 10,000 times as fast as FFT!

Hassanieh, H., Indyk, P., Katabi, D., and Price, E.: "Nearly Optimal Sparse Fourier Transform," *arXiv* **1201.2501v1** (12 Jan 2012)

Future work

- Faster OMP implementation: Field-Programmable Gate Arrays, ~3,000 times speed-up over conventional processors
- Other GW data analysis applications
- CS applications in experimental GW detection methods (e.g. spectral line hunting)

Thanks!

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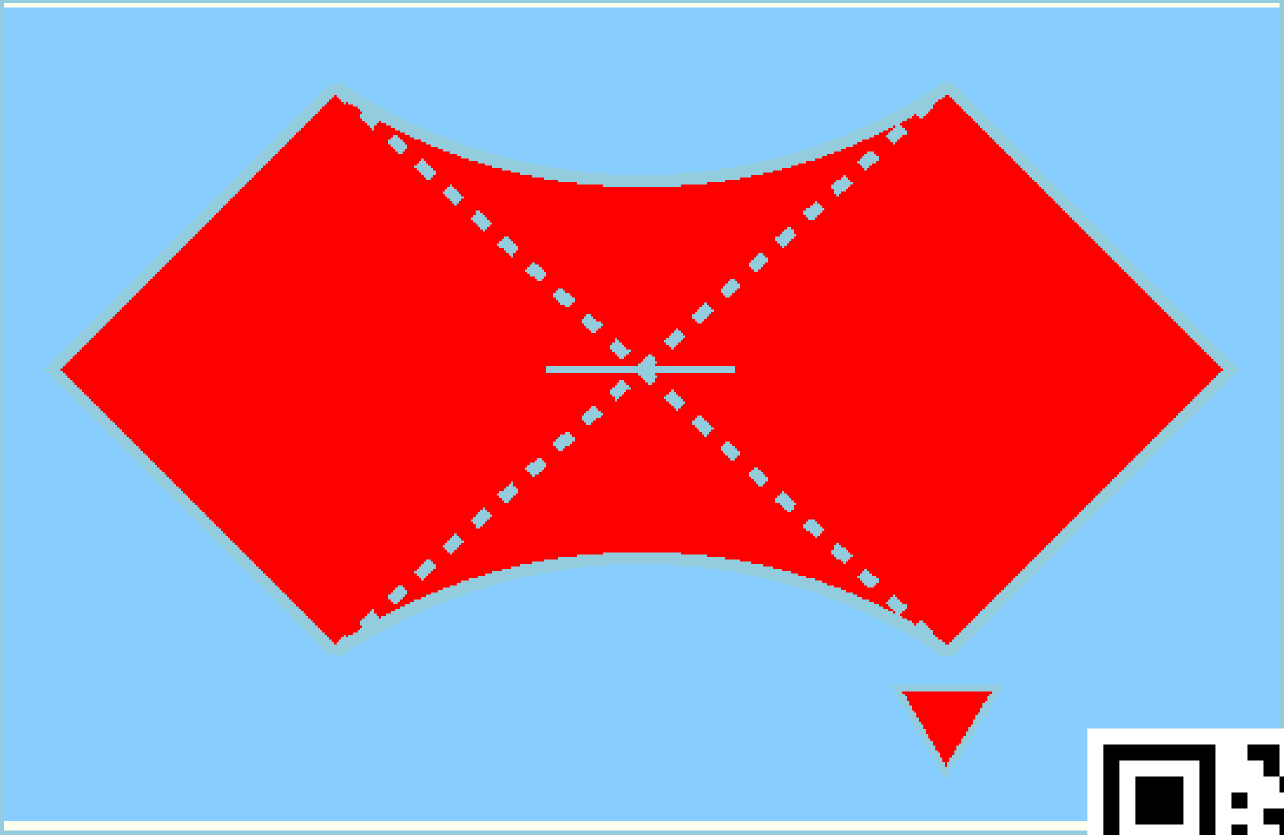
Thanks for listening!



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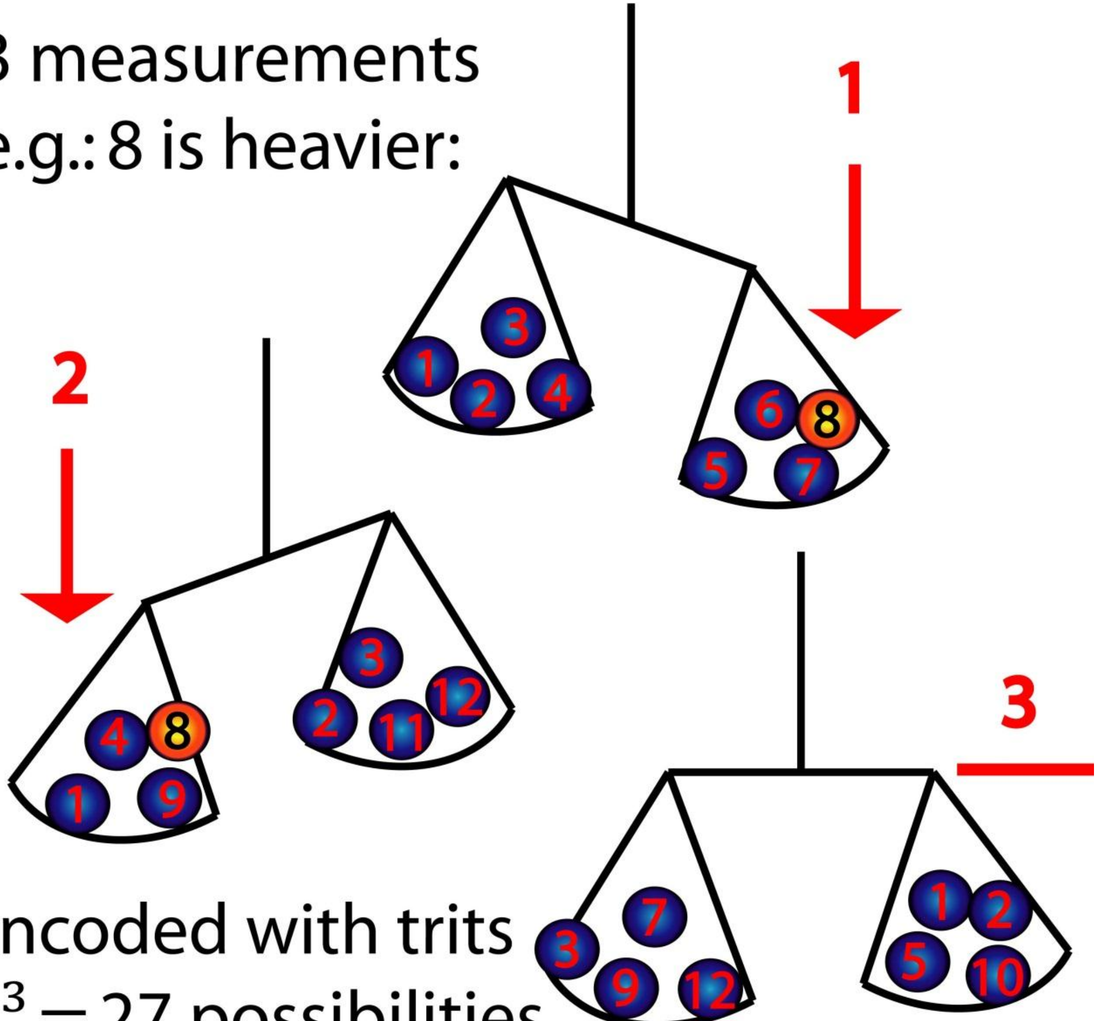
Solution to the 12-ball problem

Answer:
3 measurements
e.g.: 8 is heavier:



Encoded with trits
$3^3 = 27$ possibilities

http://www.primepuzzle.com/leeslatest/12_ball_solution.html (2006)