Chapter 35 The Laws of Geometric Optics

35.1 The nature of light

**Light having a dual nature**: in some cases light acts like a wave and in others it acts like a particle.

**Wave**: interference, diffraction, Maxwell's EM wave model

**Particle**: reflection, refraction, photoelectric effect.

35.2 Measurement of the Speed of Light

\[ c = 2.9977 \times 10^8 \text{ m/s} \]

\[ \approx 3 \times 10^8 \text{ m/s} \]

**Example**: Assume Fizeau's wheel has 360 teeth and \( \omega = 27.5 \text{ rev/s} \). \( d = 7500 \text{ m} \). Light passing through opening A is blocked by tooth B on return. Find the speed of light.

**Solution**: The wheel turned \( 1/720 \text{ rev} \) while the light makes its round trip. The time is

\[ t = \frac{(1/720)\text{rev}}{27.5\text{rev/s}} = 5.05 \times 10^{-5} \text{ s} \]

The speed of light:

\[ c = \frac{2d}{t} = \frac{2(7500\text{m})}{5.05 \times 10^{-5} \text{ s}} = 2.97 \times 10^8 \text{ m/s} \]

35.3 The Ray Approximation in Geometric Optics

"Light travels in a straight line perpendicular to the wave front."

35.4 The Wave Under Reflection

**Reflection of Light**:

- **Smooth surface**
- **Rough surface**
Law of Reflection:
The angle of reflection
= the angle of incidence.
\[ \theta_r = \theta_i \]

Multiple Reflections:

The Wave Under Refraction

Refraction of light:
\[ \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \]

\( v_1 \) the speed of light in medium 1.
\( v_2 \) the speed of light in medium 2.

For example: in air \( v \approx 3 \times 10^8 \) m/s, in glass \( v \approx 2 \times 10^8 \) m/s.

Index of Refraction:
The speed of light in vacuum > the speed of light in any material.

Define:
\[ n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \]

- \( n \) is dimensionless.
- \( n \geq 1 \), \( n = 1 \) in vacuum.
- In a medium, the frequency of light does not change, but its wavelength does.
  \[ \lambda_1 = \frac{v}{f} \text{, and } \lambda_2 = \frac{v}{f} \]

\[ \frac{\lambda_1}{\lambda_2} = \frac{v_1 / f}{v_2 / f} = \frac{n_2}{n_1} \]

- In a medium, \( \lambda = \frac{v}{f} = \frac{v / c}{f / c} = \frac{\lambda_0}{n} \), \( \lambda_0 \) the wavelength of light in vacuum.

<table>
<thead>
<tr>
<th>Substance</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>2.419</td>
</tr>
<tr>
<td>Glass, crown</td>
<td>1.52</td>
</tr>
<tr>
<td>Ice</td>
<td>1.309</td>
</tr>
<tr>
<td>Silica</td>
<td>1.458</td>
</tr>
<tr>
<td>Water</td>
<td>1.333</td>
</tr>
<tr>
<td>Air</td>
<td>1.000293</td>
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</tbody>
</table>
Snell's Law: 
Substitute $v$ with $n$: 

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \frac{n_2}{n_1} \quad \text{or} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Summary: 

- $c = 2.9977 \times 10^8 \text{ m/s} \approx 3 \times 10^8 \text{ m/s}$
- $n = \frac{c}{\nu}$ (≥ 1)
- $\lambda_0 = \frac{c}{f}$
- $\lambda = \frac{\nu}{f}$

<table>
<thead>
<tr>
<th>Vacuum</th>
<th>Other Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light</td>
<td>$c$</td>
</tr>
<tr>
<td>Index of Refraction</td>
<td>$n = 1$</td>
</tr>
<tr>
<td>Wavelength</td>
<td>$\lambda_0 = c / f$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f$</td>
</tr>
</tbody>
</table>

Example: A light ray traveling through air is incident on a crown glass at an angle of 30° to the normal. Find the angle of refraction.

Solution: $\theta_i = 30^\circ$, $n_i = 1$, $n_2 = 1.52$.
Using Snell's law,

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{1}{1.52} \sin 30^\circ = 0.329$$
$$\theta_2 = \sin^{-1}(0.329) = 19.2^\circ$$

Example: Light of wavelength 589 nm in vacuum passes through a piece of silica ($n = 1.458$). Find the speed and the wavelength of light in silica.

Solution:

Speed

$$v_{\text{silica}} = \frac{c}{n} = \frac{3 \times 10^8 \text{ m/s}}{1.458} = 2.06 \times 10^8 \text{ m/s}$$

Frequency

$$f = \frac{c}{\lambda_0} = \frac{3 \times 10^8 \text{ m/s}}{589 \times 10^{-9} \text{ m}} = 5.09 \times 10^{14} \text{ Hz}$$

Wavelength

$$\lambda_{\text{silica}} = \frac{\nu}{f} = \frac{2.06 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 404 \text{ nm}$$

or

$$\lambda_{\text{silica}} = \frac{\lambda_0}{n} = \frac{589 \text{ nm}}{1.458} = 404 \text{ nm}$$
**Example:** A light beam passing through a flat glass. Show that the emerging beam is parallel to the incident beam.

**Solution:**

\[
\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1
\]

\[
\sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2 = \frac{n_2}{n_1} \left( \frac{n_1}{n_2} \sin \theta_1 \right) = \sin \theta_1
\]

Thus, \( \theta_3 = \theta_1 \),

the emerging beam and the incident beam are parallel.

**Example:** An underwater scuba diver sees the Sun at an apparent angle of 45° from the vertical. Where is the Sun?

**Solution:**

\[
\sin \theta_1 = n \sin \theta_2 = 1.333 \sin 45° = 0.9426
\]

\[
\theta_1 = \sin^{-1}(0.9426) = 70.1°
\]

\( 90°-70.1° = 19.9° \)

The Sun is 19.9° above the horizon.