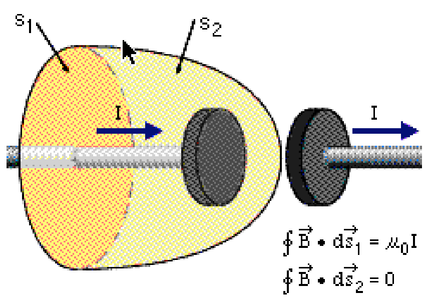
Chapter 34 Electromagnetic Waves

34.1 Displacement Current and the General Form of Ampere's Law

Review Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

The line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_0 I$. I is the total steady current passing through any surface bounded by the closed path.

Problem with capacitors:
Current pass through $S_1 = I$
Current pass through $S_2 = 0$
This contradicts the Ampere's Law!



Reason: The current is discontinuous in a capacitor.

Solution: Maxwell added a "displacement current" in the Ampere's Law. Which makes the generalized Ampere's law valid in all cases.

Displacement Current: $I_d \equiv \varepsilon_0 \dfrac{d\Phi_E}{dt}$

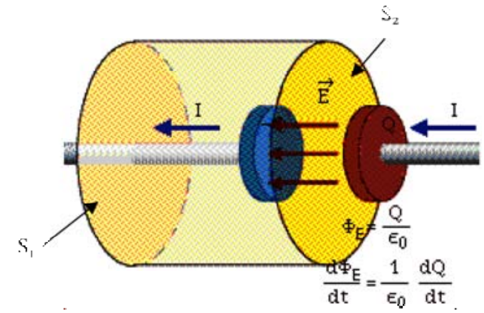
Φ_E is the flux of the electric field. $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Generalized Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Displacement current is zero, if current is continuous and steady. Because electric field flux is constant.
- As the capacitor is being charged (or discharged), no actual current passing through the plates. But the changing electric field between the plates may be thought of as a sort of current which is equivalent to the current in a wire.

Example: Displacement current in a capacitor.



The electric flux through S_2:

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = EA$$

The electric field:

$$E = Q / \varepsilon_0 A$$

Therefore,

$$\Phi_E = EA = Q / \varepsilon_0$$

The displacement current:

$$I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt}$$

I_d is identical to the current I through S_1

Magnetic field are produced both by conduction current and by changing electric fields.

34.2 Maxwell's Equations and Hertz's Discoveries

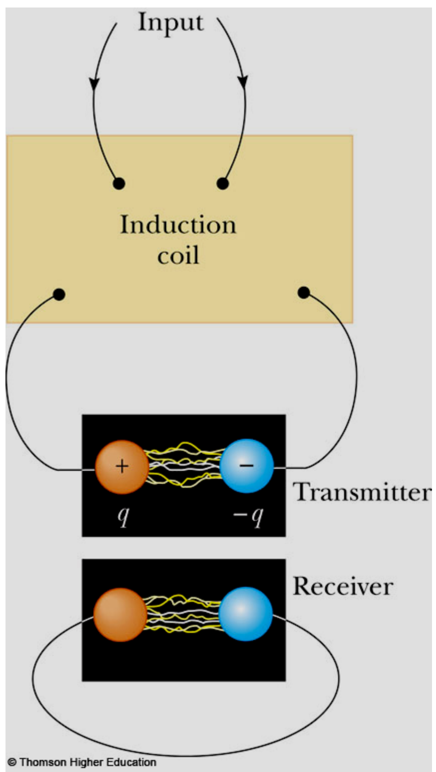
Maxwell's Equations

$\oint \vec{E} \cdot d\vec{A} = \dfrac{Q}{\varepsilon_0}$	(Gauss's law)
$\oint \vec{B} \cdot d\vec{A} = 0$	(Gauss's law in magnetism)
$\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$	(Faraday's law)
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \varepsilon_0 \mu_0 \dfrac{d\Phi_E}{dt}$	(Ampere-Maxwell law)

Force on a charged particle:

$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ (Lorentz force)

Maxwell's equations, together with the Lorentz force law give a complete description of all *classical* electromagnetic interactions.



- Maxwell introduced the concept of displacement current: a time varying electric field produces a magnetic field, just as a time-varying magnetic field produces an electric field.

- Maxwell's equations also predicted the existence of electromagnetic (EM) waves that propagate through space with the speed of light. Light is a form of electromagnetic radiation.

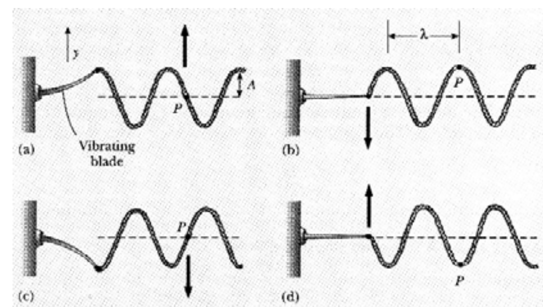
- Heinrich Hertz first generated and detected electromagnetic waves.

$$\omega = \frac{1}{\sqrt{LC}}$$

speed = 3×10^8 m/s

34.3 Plane Electromagnetic Waves

- Review the properties of sinusoidal waves.



General wave equation:

$$\boxed{\frac{\partial^2 f}{\partial^2 x} = \left(\frac{1}{v^2}\right)\frac{\partial^2 f}{\partial^2 t}}$$

Wave function:

$$\boxed{y = A\cos(kx - \omega t)}$$

A: amplitude

k: angular wave number

$$\boxed{k \equiv \frac{2\pi}{\lambda}}$$

ω: angular frequency $\boxed{\omega = 2\pi f = \dfrac{2\pi}{T}}$

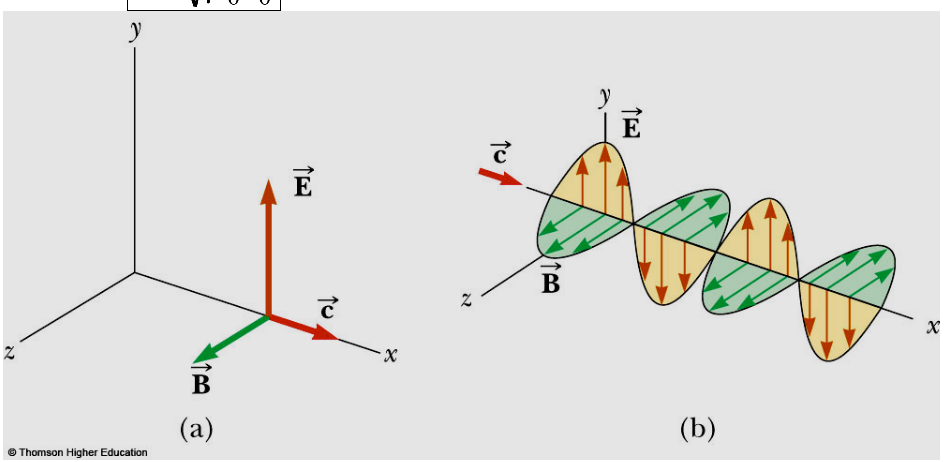
Speed of wave: $\boxed{v = \dfrac{\lambda}{T} = \lambda f}$

From Maxwell's 3rd and 4th equations (see textbook), it can be shown:

$$\boxed{\frac{\partial^2 E}{\partial^2 x} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial^2 t}}, \quad \text{and} \quad \boxed{\frac{\partial^2 B}{\partial^2 x} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial^2 t}}$$

- These are wave equations. The speed of the EM wave

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} = 2.99792 \times 10^8 \text{ m/s}$$



- The electric and magnetic components of plane EM waves ⊥ to each other, and also ⊥ to the direction of wave propagation (Transverse waves).

$$\boxed{E = E_{\max}\cos(kx - \omega t)}, \text{ and } \boxed{B = B_{\max}\cos(kx - \omega t)}$$

- From the relation (see textbook): $\dfrac{\partial E}{\partial x} = -\dfrac{\partial B}{\partial t}$

  We have $-kE_{\max}\sin(kx - \omega t) = -\omega B_{\max}\sin(kx - \omega t)$

$$kE_{\max} = \omega B_{\max} \text{ or } \frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c$$

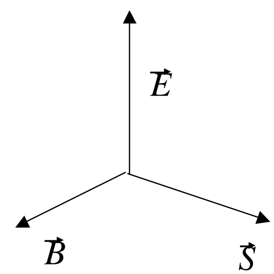
Thus, $\boxed{\dfrac{E_{\max}}{B_{\max}} = \dfrac{E}{B} = c}$

34.3 Energy Carried by Electromagnetic Waves

- Electromagnetic waves carry energy.
- The rate of flow of energy crossing a unit area:

  Poynting vector $\boxed{\vec{S} \equiv \dfrac{1}{\mu_0}\vec{E} \times \vec{B}}$

  Units: J/s·m^2 =W/m^2



- Magnitude of $\vec{S}$: $S = \dfrac{EB}{\mu_0}$

  Using B = E/c,

  $$S = \frac{E(t)^2}{\mu_0 c} = \frac{c}{\mu_0}B(t)^2$$

  Note that S is a function of time also.

- Wave intensity I: the time average of S over one or more cycles.

$$I = S_{av} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c}{2\mu_0}B_{\max}^2 = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{\mu_0 c} = \frac{c}{\mu_0}B_{rms}^2$$

- Energy densities:

$$u_B = \frac{B^2}{2\mu_0}, \quad \text{(magnetic field)}$$

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 \quad \text{(electric field)}$$

Using B=E/c, $u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0\varepsilon_0 E^2}{2\mu_0} = \frac{\varepsilon_0 E^2}{2} = u_E$

Thus, $$u_B = u_E = \frac{B^2}{2\mu_0} = \frac{\varepsilon_0 E^2}{2}$$

For an EM wave, the instantaneous magnetic-field energy density = the instantaneous electric-field energy density

Total instantaneous energy density u: $$u = u_B + u_E = \frac{B^2}{\mu_0} = \varepsilon_0 E^2$$

- $$I = S_{av} = cu_{av}$$

  The intensity of an EM wave
  = the average energy density × the speed of light.

Example: A point source of EM radiation has an average power output of 800W.

(a) Calculate $E_{\max}$ and $B_{\max}$ at 3.5 m from the source.

$$I = \frac{P(source)}{4\pi r^2}, \quad \text{also } I = \frac{E_{\max}^2}{2\mu_0 c}$$

Solving for $E_{\max}$ gives:

$$E_{\max} = \sqrt{\frac{\mu_0 c P(source)}{2\pi r^2}}$$

$$= \sqrt{\frac{(4\pi\times 10^{-7} n/A^2)(3\times 10^8 m/s)(800W)}{2\pi(3.5m)^2}}$$

$$= 62.6 V/m$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{62.6V/m}{3\times 10^8 m/s} = 2.09\times 10^{-7} T$$

(b) Calculate the average energy density 3.5m from the source.

$$u_{av} = \varepsilon_0 E_{\max}^2/2$$

$$= 8.85\times 10^{-12} C^2/Nm^2)(62.6V/m)^2/2$$

$$= 1.73\times 10^{-8} J/m^3$$

34.5 Momentum and Radiation Pressure

- EM waves have linear momentum.
- A pressure is exerted on a surface when an EM wave strike it.

Total momentum delivered to a surface:

$$\boxed{p = \frac{U}{c}} \qquad \text{(complete absorption, blackbody)}$$

$$\boxed{p = \frac{2U}{c}} \qquad \text{(complete reflection, mirror)}$$

where U is the total energy of incident wave during time t.

Total radiation pressure (force/area):

$$\boxed{P = \frac{S}{c}} \qquad \text{(complete absorption, blackbody)}$$

$$\boxed{P = \frac{2S}{c}} \qquad \text{(complete reflection, mirror)}$$

Example: Solar Energy: The sum delivers about 1000W/m^2 of EM flux to the earth surface.

(a) Calculate the total power incident on a roof of dimensions 8 m × 20 m.

The Poynting vector S = 1000 W/m^2

Assume the sunlight is incident normal to the roof:

Power = SA = (1000 W/m^2)(8m × 20m)

= 1.6 × 10^{5}W = 160 KW

(b) Calculate the radiation pressure and radiation force on the roof. Assuming the roof is a perfect absorber.

Pressure: $P = \frac{S}{c} = \frac{1000 W/m^2}{3 \times 10^8 m/s} = 3.33 \times 10^{-6} N/m^2$

Force: F = PA = (3.33× 10^{-6} N/m^2)(160m^2)= 5.33× 10^{-4} N

(c) How much solar energy is incident on the roof in 1 h?

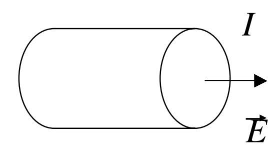
Energy = Power × time

= (1.6 × 10^{5}W)(3600s)

= 5.76 × 10^8 J

Example: A long, straight wire of resistance *R*, radius *a*, and length *l* carries a constant current *I*. Calculate the Poynting vector on the surface of the wire.

The electric field: $E = V/l = IR/l$



The magnetic field on the surface:

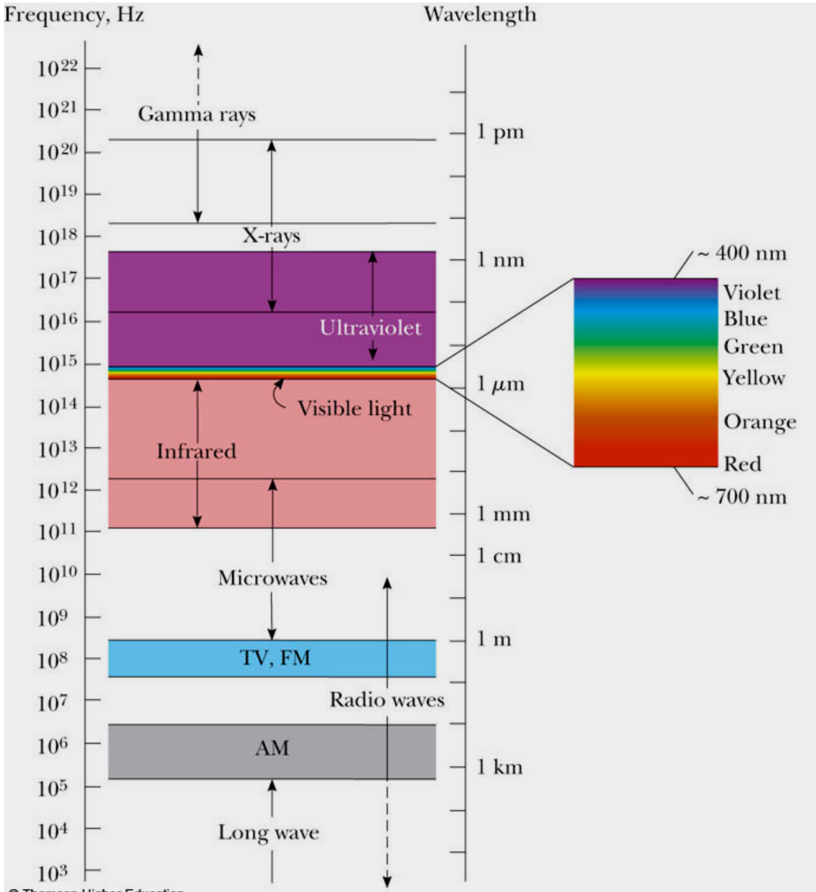
$B = \mu_0 I / 2\pi a$

The Poynting vector $\vec{S}$ is directed radially inward:

$$S = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\left(\frac{RI}{l}\right)\frac{\mu_0 I}{2\pi a} = \frac{I^2 R}{2\pi a l} = \frac{I^2 R}{A}$$

or $AS = I^2 R$

- The rate at which EM energy flows into the wire, *SA*, = the rate of energy dissipated as joule heat, I^2R.



Frequency, Hz
Wavelength
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
Gamma rays
X-rays
Ultraviolet
Visible light
Infrared
Microwaves
TV, FM
Radio waves
AM
Long wave
1 pm
1 nm
1 μm
1 mm
1 cm
1 m
1 km
~ 400 nm
Violet
Blue
Green
Yellow
Orange
Red
~ 700 nm