Chapter 28 Direct Current circuits

28.1 Electromotive Force (emf)

Sources of emf: A battery or any other device that provides electrical energy. “charge pump”

\[ I_r: \text{internal resistance} \]

\[ R: \text{potential on } R, \text{ or terminal voltage of the battery} \]

\[ V = \varepsilon - Ir \] or \[ \varepsilon = IR + Ir \]

- \( \varepsilon \) is the open-circuit voltage. (if \( I = 0 \), \( V = \varepsilon = V_{\text{max}} \))
- If \( r \) is small (a good battery), \( V \approx \varepsilon \).
- If \( r \) is large (a bad battery), \( V \ll \varepsilon \).
- Power dissipation \( I \varepsilon = I^2 R + I^2 r \)

Example: A battery has an emf of 12 V, and internal resistance of 0.05 \( \Omega \). Its terminals are connected to a load resistance of 3 \( \Omega \).

(a) Find the current.

\[ I = \frac{\varepsilon}{R + r} = \frac{12V}{3.05\Omega} = 3.93A \]

(b) Find the terminal voltage:

\[ V = \varepsilon - Ir = 12V (3.93A) (0.05\Omega) = 11.8V \]

Find the equivalent resistance

\[ \frac{1}{14\Omega} = \frac{1}{12\Omega} + \frac{1}{2\Omega} + \frac{1}{6\Omega} + \frac{1}{3\Omega} \]
28.2 Resistors in Series and in Parallel

In Series:

The current is the same through each resistor in series connection:

\[ I = I_1 = I_2 \]

The total voltage = sum of voltages of each resistor:

\[ V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2) = IR_{eq} \]

Generally, \[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

In Parallel:

The voltage (potential difference) across each resistor in a parallel circuit is the same:

\[ V = V_1 = V_2 \]

The total current = sum of currents passing through each resistor in a parallel circuit

\[ I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}} \]

Generally, \[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]
28.3 Kirchhoff’s Rules

(1) **Junction Rule:** "The sum of the current entering any junction must equal the sum the currents leaving that junction."

\[ I_{in} = I_{out} \]

\[ I_1 = I_2 + I_3 \]

"Conservation of Charge"

(2) **Loop Rule:** "The sum of potential differences across all the elements around any closed circuit loop must be zero." --> “Conservation of Energy"

\[
\begin{align*}
\Delta V &= V_b - V_a = -IR \\
\Delta V &= V_b - V_a = +IR \\
\Delta V &= V_b - V_a = +\epsilon \\
\Delta V &= V_b - V_a = -\epsilon
\end{align*}
\]

**Strategy to solve complex circuits:**
- Label all quantities and assign directions of the currents.
- Apply the junction rule (Kirchhoff’s 1st rule).
- Apply Kirchhoff’s loop rule to loops. Pay attention to signs.
- Solve the equations.

**Example:** Find the current \( I_1, I_2, \) and \( I_3 \)

(1) Assign directions of currents. Just make a reasonable guess.

(2) Junction rule:

\[ I_1 + I_2 = I_3 \]

(3) Identify loops (3):

Apply Kirchhoff’s 2nd rule 3-1= 2 times.

Lower loop: \[ 10 \text{ V} - (6 \text{ } \Omega)I_1 - (2 \text{ } \Omega)I_3 = 0 \]

Upper loop: \[ -14 \text{ V} - 10 \text{ V} + (6 \text{ } \Omega)I_1 - (4 \text{ } \Omega)I_2 = 0 \]

(4) 3 unknowns, and 3 independent equations.

\( I_1 = 2 \text{ A}, I_2 = -3 \text{ A}, \) and \( I_3 = -1 \text{ A} \)

Negative value indicates that the actual direction of current is opposite to the labeled direction.
Water in the bucket  ⇒ Charge in the capacitor
Water flow in the pipe  ⇒ Electric current

\[ I(t) = I_0 \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ W_{\text{max}} = \frac{2}{3} \rho V R^2 \]

\[ q(t) = \frac{\varepsilon}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ I_0 = \frac{\varepsilon}{R} \]

\[ I(t) = I_0 \left( 1 - e^{-\frac{t}{\tau}} \right) \]
28.4 RC Circuits

Charging a capacitor:

- $t < 0$: C is NOT charged and $I = 0$
- $t = 0$: The switch is closed.
- $t > 0$: C begins to charge.

When $t > 0$, at any given moment, Kirchhoff’s loop rule gives:

\[
\varepsilon - IR - \frac{q}{C} = 0
\]

(1) Initial Current:

At $t = 0$, $q = 0$. No potential drop on C, all the potential drop is on $R$.

Therefore, $I_0 = \frac{\varepsilon}{R}$ (current at $t = 0$)

(2) When $t \to \infty$, C is charged to its maximum $q_{\text{max}}$, and the current is 0. All the potential drop is on C. No potential drop on $R$.

$q_{\text{max}} = C\varepsilon$ (maximum charge)

(3) Between the above two limiting cases, the charge is governed by eq:

\[
\frac{dq}{dt} = \frac{\varepsilon - q}{R - RC}
\]

The solution:

\[
q(t) = C\varepsilon[1 - e^{-t/RC}] = q_{\text{max}}[1 - e^{-t/RC}]
\]

\[I(t) = \frac{\varepsilon}{R} e^{-t/RC} = I_0 e^{-t/RC}\]

Time constant: $\tau = RC$, has a unit of seconds.

At $t = \tau$, $I = e^{-1}I_0 = 0.368I_0$, $q = q_{\text{max}}[1 - e^{-1}] = 0.632q_{\text{max}} = 0.632C\varepsilon$

The work done by the battery during the charging:

\[W_{\text{battery}} = Q_{\text{max}} \varepsilon = C\varepsilon^2\]

When C is fully charge, the energy stored in C:

\[U = U = \frac{1}{2} q_{\text{max}} \varepsilon = \frac{1}{2} C\varepsilon^2 = \frac{1}{2} W_{\text{battery}}\]

The other half of energy goes into joule heat in the resistor.
Discharging a Capacitor

Kirchhoff’s loop rule:

\[ \frac{q}{c} - IR = 0 \]

\[ IR = \frac{q}{c} \]

(1) Initial Current:
At \( t = 0 \), \( q = Q \)
\[ I_0 = \frac{Q}{RC} \] (current at \( t = 0 \))

(2) When \( t \to \infty \), \( I \to 0 \), \( q \to 0 \)
All the energy become heat.

(3) Generally, \( \frac{dq}{dt} = -\frac{q}{RC} \)
Solve it:
\[ I(t) = I_0 e^{-t/RC} \]
\[ q(t) = Q e^{-t/RC} \]

Example: Charging a capacitor, \( \varepsilon = 12 \text{ V} \),
\( C = 5 \mu \text{F}, R = 8 \times 10^5 \Omega \).
(a) Find time constant
\[ \tau = RC = (8 \times 10^5 \Omega)(5 \times 10^{-6} \text{ F}) = 4.0 \text{ s} \]
(b) Find the \( q_{\text{max}} \)
\[ Q = C \varepsilon = (5 \times 10^{-6} \text{ F})(12 \text{ V}) = 60 \times 10^{-6} \text{ C} = 60 \mu \text{C} \]
(c) Find the maximum current
\[ I_0 = \varepsilon/R = 12 \text{ V}/8 \times 10^5 \Omega = 15 \times 10^{-6} \text{ A} = 15 \mu \text{A} \]
(d) \( I(t) \) and \( q(t) \)
\[ q(t) = 60.0 [1 - e^{-t/4}] \mu \text{C} \]
\[ I(t) = 15.0 e^{-t/4} \mu \text{A} \]
(e) Calculate \( q(t) \) and \( I(t) \) at \( t = 1 \) time constant
Let \( t = 1 \tau = 4 \text{ s} \)
\[ q(t) = 60.0 [1 - e^{-1}] \mu \text{C} = 60.0 \times 0.632 \mu \text{C} = 37.9 \mu \text{C} \]
\[ I(t) = 15.0 e^{-1} \mu \text{A} = 15.0 \times 0.368 \mu \text{A} = 5.52 \mu \text{A} \]
(f) After how many time constants is the current one tenth of its initial value?
Let \[ I(t) = \frac{I_0}{10} = I_0 e^{-t/\tau} \]
\[ \frac{1}{10} = e^{-t/\tau} \]
\[ -\ln 10 = -\frac{t}{\tau} \to t = \tau \ln 10 = 2.30 \tau \]