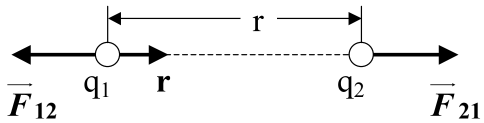
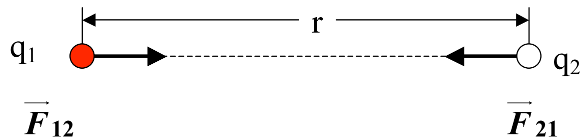
<u>23.3 Coulomb's Law</u>

Like charges



Unlike charges



The electric force exerted on q_2 due to a charge q_1:

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r^2}\hat{r}$$ (1785, Charles Coulomb)

For magnitude: $|F_{21}| = k_e|q_1||q_2| / r^2$

q_1, q_2: charges on the two particles (+ or -)

r: distance between two particles

$\hat{r}$: a unit vector directed from q_1 to q_2

k_e: Coulomb constant = 8.9875×10^9 N m^2/C^2

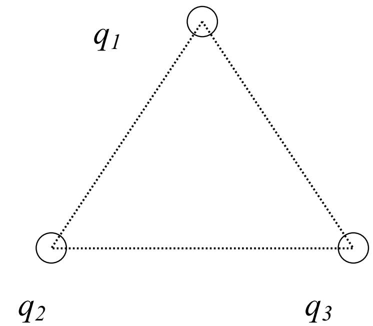
$$k_e = \frac{1}{4\pi\varepsilon_0}$$

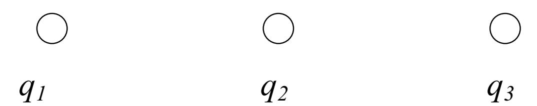
ε_0 = 8.8542×10^{-12} C^2/N m^2, permittivity of free space

Newton's third law: $\vec{F}_{21} = -\vec{F}_{12}$

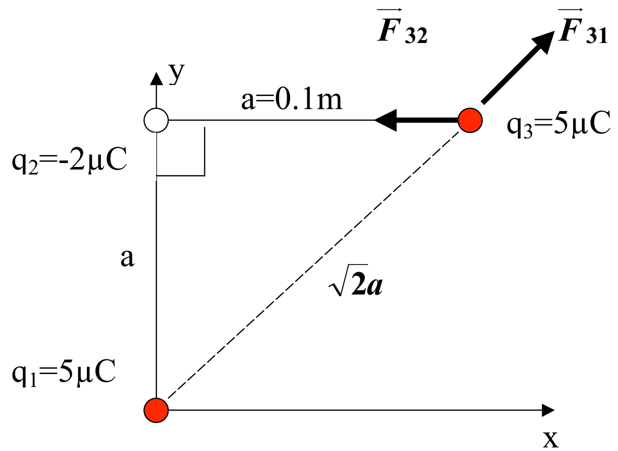
Multiple charges: $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots$

Examples:





Example: Find the Resultant Force on q_3



$$F_{32} = k_e \frac{|q_3||q_2|}{a^2}$$

$$= \left(8.99\times10^9 \frac{N\cdot m^2}{C^2}\right)\frac{(5\times10^{-6}C)(2\times10^{-6}C)}{(0.1\,m)^2} = 9.0\,N$$

$$F_{31} = k_e \frac{|q_3||q_1|}{(\sqrt{2}a)^2}$$

$$= \left(8.99\times10^9 \frac{N\cdot m^2}{C^2}\right)\frac{(5\times10^{-6}C)(5\times10^{-6}C)}{2(0.1\,m)^2} = 11\,N$$

$$F_x = F_{31x} + F_{32x} = 11\,N\,\cos 45^\circ - 9\,N = -1.1\,N$$

$$F_y = F_{31y} = 11\,N\,\sin 45^\circ = 7.9\,N$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-1.1N)^2 + (7.9N)^2} = 7.98N$$

23.4 The Electric Field

Electric field $\vec{E} \equiv \dfrac{\text{electric force } \vec{F} \text{ on a positive test charge } q_0}{\text{the magnitude of the test charge } q_0}$

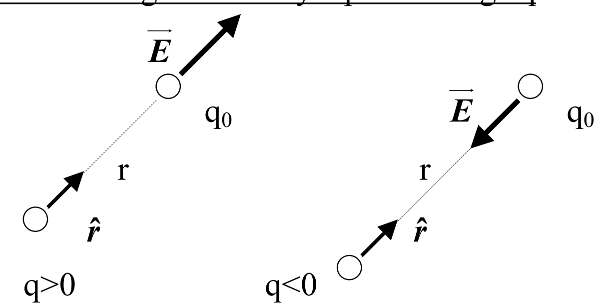
$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$

1) Unit: Newton/Coulomb (N/C)

2) Direction of $\vec{E}$ is the direction of $\vec{F}$ (test charge $q_0>0$)

3) $\vec{E}$ is the field produced by other charge, not by test charge.

The electric field generated by a point charge q



$$\vec{F} = k_e \frac{q q_0}{r^2}\hat{r}$$

$$\vec{E} \equiv \frac{\vec{F}}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

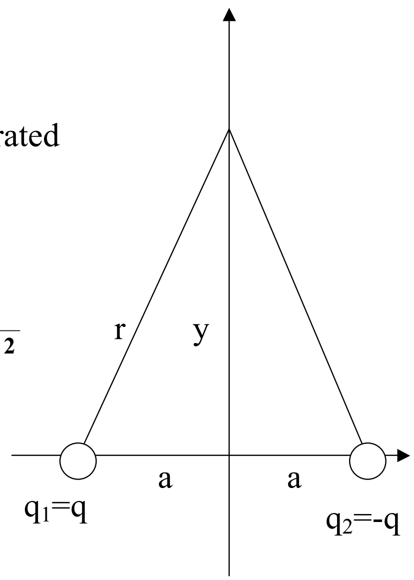
Positive charge: $\vec{E}$ point radially away from it

Negative charge: $\vec{E}$ point radially toward it

The electric field due to a group of point charges ($q_1, q_2, \cdots, q_n$)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

Ex. Electric Field of a dipole



Dipole: two charges q and –q separated by a distance 2a

$$\left|\vec{\boldsymbol{E}}_1\right| = \left|\vec{\boldsymbol{E}}_2\right| = \boldsymbol{k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}}$$

$$E = 2E_1 \cos\theta$$

$$= 2\left[k_e \frac{q}{y^2 + a^2}\right] \frac{a}{\sqrt{(y^2 + a^2)}} = k_e \frac{2aq}{(y^2 + a^2)^{3/2}}$$

If y >>a, $y^2 + a^2 \approx y^2$

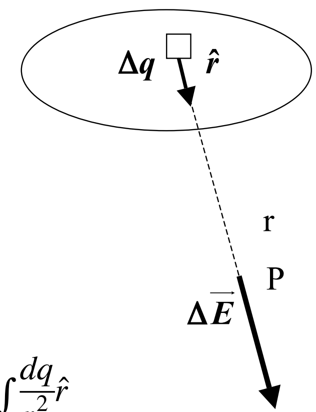
$$E \approx k_e \frac{2aq}{y^3} \propto \frac{1}{r^3}$$

(comparing point charge: $\boldsymbol{E \propto \frac{1}{r^2}}$)

23.5 Electric Field of a Continuous Charge Distribution=

For one element of charge $\boldsymbol{\Delta q}$

$$\boldsymbol{\Delta \vec{E} = k_e \frac{\Delta q}{r^2} \hat{r}}$$



Add all elements together:

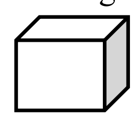
$$\sum_i \Delta \vec{E}_i \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

In the limit $\Delta q \rightarrow 0$

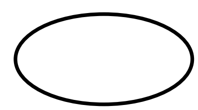
$$E = k_e \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i = k_e \int \frac{dq}{r^2} \hat{r}$$

Define Charge Density for Uniform Charge Distributions

Volume charge density $\rho \equiv \frac{Q}{V}$ C/m^3 $\boldsymbol{\Delta Q = \rho \Delta V}$

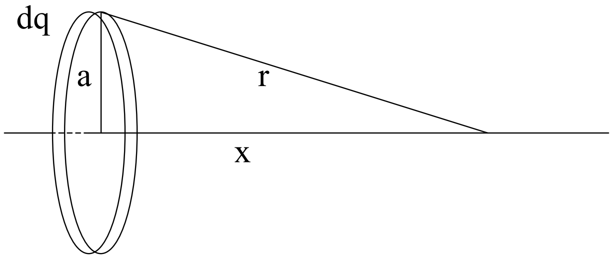


Surface charge density $\sigma \equiv \frac{Q}{A}$ C/m^2 $\boldsymbol{\Delta Q = \sigma \Delta A}$



Linear charge density $\lambda \equiv \frac{Q}{l}$ C/m $\boldsymbol{\Delta Q = \lambda \Delta l}$

The Electric Field of a Uniform Ring of Charge



$$dE = k_e \frac{dq}{r^2}$$

$$dE_x = dE\cos\theta, \qquad \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + a^2}}$$

$$dE_x = (k_e \frac{dq}{r^2})\frac{x}{r} = \frac{xk_e}{(x^2 + a^2)^{3/2}} dq$$

Notice that $\sum E_\perp = 0$

$$E = E_x = \int \frac{xk_e}{(x^2 + a^2)^{3/2}} dq$$

$$= \frac{xk_e}{(x^2 + a^2)^{3/2}} \int dq = \frac{xk_e}{(x^2 + a^2)^{3/2}} Q \qquad \text{(done!)}$$

------- However, a real physicist will not stop here -----

If x>>a, then $(x^2 + a^2) \to x^2$

$$E = \frac{xk_e}{(x^2 + a^2)^{3/2}} Q \to K_e \frac{Q}{x^2} \quad \text{(looks familiar? Why?)}$$

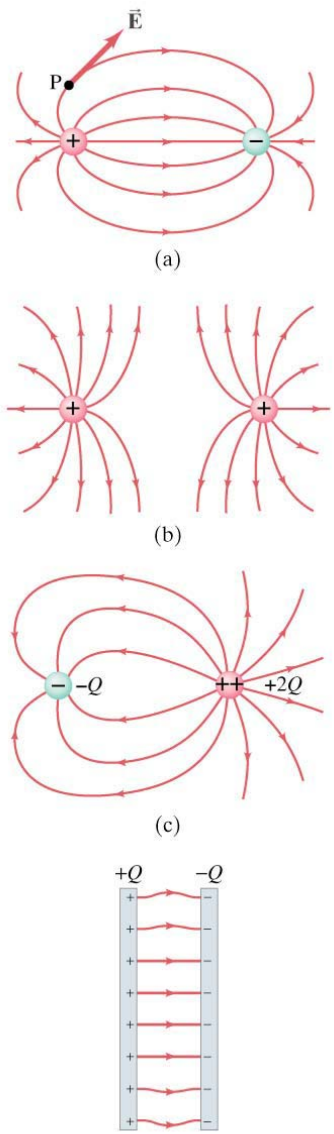
23.6 Electric Field Lines

To help visualize electric field patterns.

1) $\vec{E}$ is tangent to the electric field line at each point.

2) The number of line per unit area $\propto$ intensity of the field.

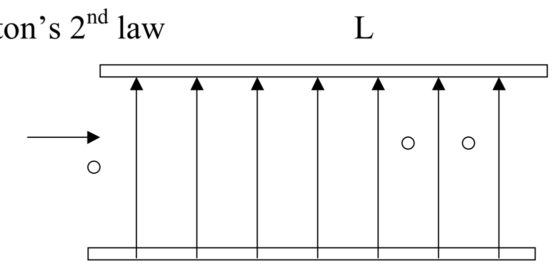
- ♦ They point away from Q > 0 and toward Q < 0.
- ♦ Number of lines leaving or approaching a charge $\propto$ |Q|
- ♦ When net charge = 0, all the lines begin on positive charge and terminated on negative charge.
- ♦ When net charge $\neq$ 0, lines may begin or terminate at infinity.
- ♦ No two lines can cross or tough (why?)



(a) (b) (c) (d)

23.7 Motion of Charged Particles in a Uniform Electric Field

$\vec{F} = q\vec{E} = m\vec{a}$ Newton's 2nd law



$\vec{a} = \dfrac{q\vec{E}}{m}$

$v_0 = 3.0\text{x}10^6$ m/s

$\vec{E} = 200\hat{j}$ N/C

L = 0.1 m

(a) Find the acceleration of the electron while in the electric field.

$$a = \frac{-eE}{m_e}\hat{j} = -\frac{(1.60\times10^{-19}\,\text{C})(200\text{ N/C})}{9.11\times10^{-31}\text{ kg}}\hat{j} = -3.51\times10^{13}\ \hat{j}\ \ \text{m/s}^2$$

(b) Find the time it takes the electron travel through the field.

$$t = \frac{L}{v_0} = -\frac{0.1\text{ m}}{3.0\times10^{6}\text{ m/s}} = 3.33\times10^{-8}\text{ s}$$

(c) Find the vertical displacement of the electron.

$$\Delta y = \tfrac{1}{2}at^2 = \tfrac{1}{2}(-3.51\times10^{13}\ \text{m/s}^2)(3.33\times10^{-8}\ \text{s})^2 = -0.0195\ m$$

(d) Find the speed of the electron as it emerges from the field.

$$v_x = v_0 = 3.0\times10^6\, m/s$$

$$v_y = at = (-3.51\times10^{13}\ \text{m/s}^2)(3.33\times10^{-8}\text{s}) = -1.17\times10^{6}\ \text{m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 3.22\times10^6\, m/s$$