This print-out should have 17 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. The due time is Central time.

AP B 1993 MC 2

05:04, trigonometry, multiple choice, > 1 min, fixed.

001 (part 1 of 1) 10 points

A ball falls straight down through the air under the influence of gravity. There is a retarding force $\vec{F}$ on the ball with magnitude given by $F = b\,v$, where v is the speed of the ball and b is a positive constant.

The magnitude of the acceleration $\vec{a}$ of the ball at any time is equal to which of the following?

1. $\|\vec{a}\| = g - b$

2. $\|\vec{a}\| = g - \frac{b\,v}{m}$ **correct**

3. $\|\vec{a}\| = g + \frac{b\,v}{m}$

4. $\|\vec{a}\| = \frac{g}{b}$

5. $\|\vec{a}\| = \frac{b\,v}{m}$

Explanation:

There are two forces acting (in opposite directions) on the ball, the gravity $m\,g$ and the retarding force $F = b\,v$.

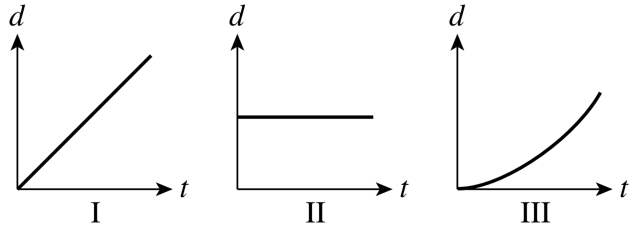
$$m\,a = m\,g - b\,v$$
$$a = g - \frac{b\,v}{m}\,.$$

AP B 1998 MC 43 44

05:04, trigonometry, multiple choice, < 1 min, fixed.

002 (part 1 of 2) 10 points

Three objects can only move along a straight, level path. The graphs below show the position d of each of the objects plotted as a function of time t.



The magnitude of the velocity $\|\vec{v}\|$ of the object is increasing in which of the cases?

1. II only

2. III only **correct**

3. I and II only

4. I and III only

5. I, II, and III

Explanation:

Case I: The object moves at constant speed.
Case II: The object remains at rest.
Case III: The speed of the object increases with time.

Thus, the magnitude of the momentum of the object increases only in case III.

003 (part 2 of 2) 10 points

The sum of the forces $\sum F_i = 0$ on the object is zero in which of the cases?

1. II only

2. III only

3. I and II only **correct**

4. I and III only

5. I, II, and III

Explanation:

When the sum of forces on the object is zero, the acceleration of object is zero by Newton's 2nd law.

In cases I and II, the velocity of the object doesn't change with the time. The sum of the forces on the object is zero.

In case III, the object is accelerating. Thus, the sum of forces on the object is not zero.

Comparing Accelerations 02

05:04, trigonometry, numeric, > 1 min, normal.

004 (part 1 of 1) 10 points

Suppose a cart is being moved by a certain net force.

If the net force is increased by a factor of 2, by what factor does its acceleration change?

Correct answer: 2 .

Explanation:

Basic Concepts:

$$F_{net} = m\, a_{net}$$

Solution: The new force is $n\, F$, so multiply by n on both sides:

$$n\, F_{net} = n\, (m\, a_{net}) = m\, (n\, a_{net})$$

Since the mass doesn't change, the acceleration must change by a factor of n also.

Elevator Acceleration 02

05:04, trigonometry, numeric, > 1 min, normal.

005 (part 1 of 1) 10 points

A person weighing 0.8 kN rides in an elevator that has a downward acceleration of 1.5 m/s^2.

The acceleration of gravity is 9.8 m/s^2 .

What is the magnitude of the force of the elevator floor on the person?

Correct answer: 0.677551 kN.

Explanation:

Basic Concepts:

$$\sum \vec{F} = m\, \vec{a}$$

$$\vec{\mathcal{W}} = m\, \vec{g}$$

Solution: Since $\mathcal{W} = m\, g$,

$$\begin{aligned} F_{net} = ma &= \mathcal{W} - f \\ f &= \mathcal{W} - ma \\ &= \mathcal{W}\left(1 - \frac{a}{g}\right) \\ &= (0.8\text{ kN})\left(1 - \frac{1.5\text{ m/s}^2}{9.8\text{ m/s}^2}\right) \\ &= 0.677551\text{ kN}\,. \end{aligned}$$

Force on a Bullet 02

05:04, trigonometry, numeric, > 1 min, normal.

006 (part 1 of 1) 10 points

A(n) 8 g bullet leaves the muzzle of a rifle with a speed of 400 m/s.

What constant force is exerted on the bullet while it is traveling down the 0.7 m length of the barrel of the rifle?

Correct answer: 914.286 N.

Explanation:

Average acceleration can be found from

$$v_f^2 = v_o^2 + 2\, a\, \ell$$

Since $v_o = 0$, we have

$$a = \frac{v^2}{2\, \ell}$$

Thus

$$\begin{aligned} F = m\, a = m\, \frac{v^2}{2\, \ell} \\ &= \frac{(8\text{ g})(400\text{ m/s})^2}{2\,(0.7\text{ m})} \cdot \frac{1\text{ kg}}{1000\text{ g}} \\ &= 914.286\text{ N} \end{aligned}$$

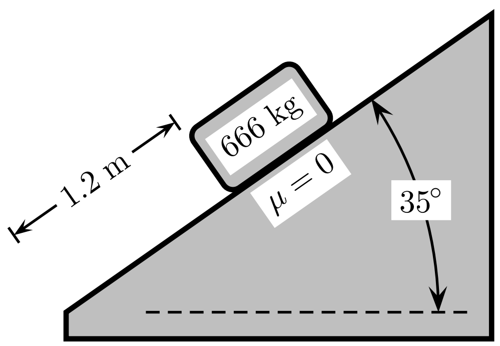
Pushed Down an Incline

05:04, trigonometry, numeric, > 1 min, normal.

007 (part 1 of 1) 10 points

A 666 kg block is pushed on the slope of a 35° frictionless inclined plane to give it an initial speed of 50 cm/s along the slope. The block is 1.2 m from the bottom of the incline.

The acceleration of gravity is 9.8 m/s^2 .

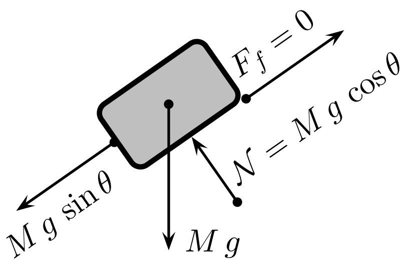


What is the speed of the block at the bottom of the plane?
Correct answer: 3.70682 m/s.
Explanation:

$$\text{Let}: \quad m = 666 \text{ kg}\,, \quad \theta = 35^\circ\,, \quad \text{and} \quad v_0 = 50 \text{ cm/s} = 0.5 \text{ m/s}\,.$$

Note: It doesn't make any difference whether the block is initially pushed up or down the slope.

Consider the free body diagram for the block



We therefore have $a_x = g\sin\theta$. The final speed is

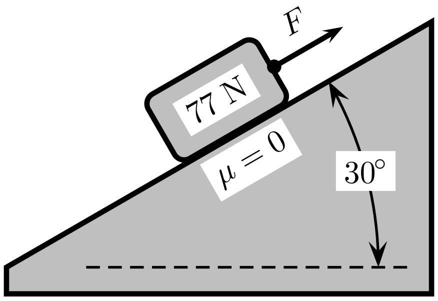
$$\begin{aligned} v^2 &= v_0^2 + 2\,a_x\,(x - x_0) \\ &= v_0^2 + 2\,g\sin\theta(x - x_0) \\ &= (0.5 \text{ m/s})^2 \\ &\quad + 2\,(9.8 \text{ m/s}^2)\sin 35^\circ\;(1.2 \text{ m}) \\ &= 13.7405 \text{ m}^2/\text{s}^2\,, \quad \text{so} \\ v &= \sqrt{13.7405 \text{ m}^2/\text{s}^2} \\ &= 3.70682 \text{ m/s}\,. \end{aligned}$$

Forces on a Sled

05:06, trigonometry, numeric, > 1 min, normal.

008 (part 1 of 2) 10 points

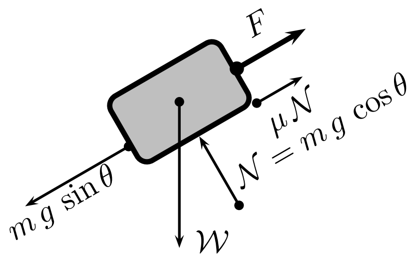
A child holds a sled on a frictionless, snow-covered hill, inclined at an angle of 30°.



If the sled weighs 77 N, find the force exerted on the rope by the child.
Correct answer: 38.5 N.
Explanation:

$$\text{Given}: \quad \mathcal{W} = 77 \text{ N}\,, \quad \theta = 30^\circ\,, \quad \text{and} \quad \mu = 0\,.$$

Consider the free body diagram for the block



Basic Concepts: If we "tilt" our world, and consider the forces parallel to the hill,

$$F_{net} = \sum F_{up} - \sum F_{down} = 0$$

then the forces perpendicular to the hill,

$$F_{net} = \sum F_{out} - \sum F_{in} = 0$$

Solution: Consider the free body diagram for the sled: The weight of the sled has components $\mathcal{W}\sin\theta$ acting down the hill and $\mathcal{W}\cos\theta$ acting straight into the hill.

The system is in equilibrium, so for forces parallel to the hill,

$$F_{net} = T - \mathcal{W}\sin\theta = 0$$

$$\begin{aligned} \Longrightarrow T &= \mathcal{W}\sin\theta \\ &= (77 \text{ N})\sin 30^\circ \\ &= 38.5 \text{ N} \end{aligned}$$

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009 (part 2 of 2) 10 points

What force is exerted on the sled by the hill?
Correct answer: 66.684 N.
Explanation:

For forces perpendicular to the hill,

$$F_{net} = \mathcal{N} - \mathcal{W}\cos\theta = 0$$

$$\begin{aligned}\Longrightarrow \mathcal{N} &= \mathcal{W}\cos\theta \\ &= (77\text{ N})\cos 30^\circ \\ &= 66.684\text{ N}\end{aligned}$$

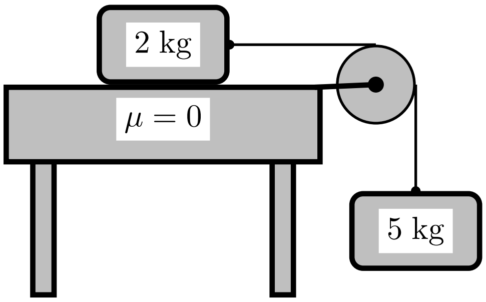
Accelerated System 05

05:07, trigonometry, numeric, > 1 min, normal.

010 (part 1 of 2) 10 points

Mass of 2 kg lies on a frictionless table, pulled by another mass of 5 kg under the influence of Earth's gravity.

The acceleration of gravity is 9.8 m/s^2 .

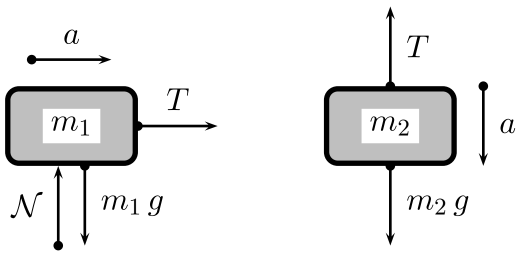


What is the magnitude of the net external force (gravitational) acting on the two masses?

Correct answer: 49 N.

Explanation:

$$\begin{aligned}\text{Given :}\quad m_1 &= 2\text{ kg}\,,\\ m_2 &= 5\text{ kg}\,,\quad\text{and}\\ \mu &= 0\,.\end{aligned}$$



The net force on the system is simply the weight of m_2.

$$\begin{aligned}F_{net} &= m_2\,g\\ &= (5\text{ kg})\,(9.8\text{ m/s}^2)\\ &= 49\text{ N}\,,\end{aligned}$$

where the gravitational force on mass m_1 is balanced by the normal force of the table.

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011 (part 2 of 2) 10 points

What is the magnitude of the acceleration a of the two masses?

Correct answer: 7 m/s^2.

Explanation:

From Newton's second law,

$$f_{net} = m_2\,g = (m_1 + m_2)\,a\,.$$

Solving for a,

$$\begin{aligned}a &= \frac{m_2\,g}{m_1+m_2}\\ &= \frac{(5\text{ kg})\,(9.8\text{ m/s}^2)}{(2\text{ kg})+(5\text{ kg})}\\ &= 7\text{ m/s}^2\,.\end{aligned}$$

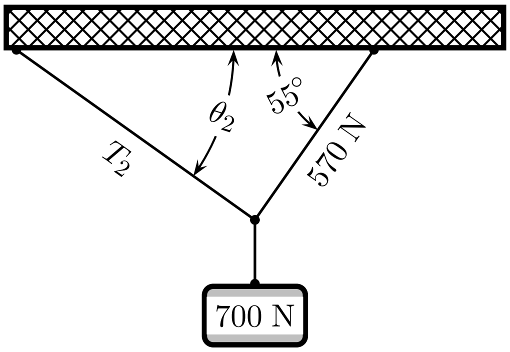
Hanging Weight 04

05:07, trigonometry, numeric, > 1 min, wording-variable.

012 (part 1 of 2) 10 points

Consider the 700 N weight held by two cables shown below. The left-hand cable had tension T_2 and makes an angle of θ_2 with the ceiling. The right-hand cable had tension 570 N and makes an angle of 55° with the ceiling.

The right-hand cable makes an angle of 55° with the ceiling and has a tension of 570 N .

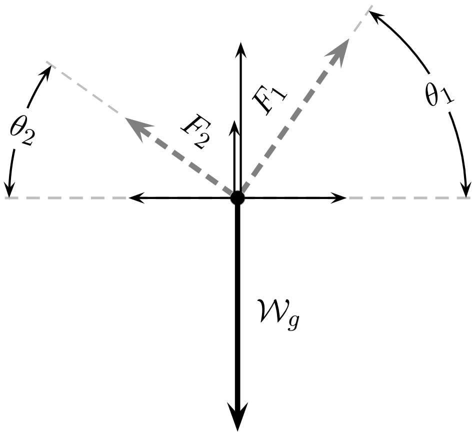


a) What is the tension T_2 in the left-hand cable slanted at an angle of θ_2 with respect to the wall?

Correct answer: 401.518 N.

Explanation:

Observe the free-body diagram below.



Note: The sum of the x- and y-components of F_1, F_2, and $\mathcal{W}_g$ are equal to zero.

$$\begin{aligned}\text{Given :}\quad \mathcal{W}_g &= 700\ \text{N}\,,\\ F_1 &= 570\ \text{N}\,,\\ \theta_1 &= 55^\circ\,,\quad \text{and}\\ \theta_2 &= 90^\circ - \theta\,.\end{aligned}$$

Basic Concept: Vertically and Horizontally, we have

$$\begin{aligned} F^x_{net} &= F^x_1 - F^x_2 = 0\\ &= F_1\cos\theta_1 - F_2\cos\theta_2 = 0 \qquad (1)\\ F^y_{net} &= F^y_1 + F^y_2 - \mathcal{W}_g = 0\\ &= F_1\sin\theta_1 + F_2\sin\theta_2 - \mathcal{W}_g = 0 \quad (2)\end{aligned}$$

Solution: Using Eqs. 1 and 2, we have

$$\begin{aligned} F^x_2 &= F_1\cos\theta_1 \qquad (1)\\ &= (570\ \text{N})\cos 55^\circ\\ &= 326.939\ \text{N}\,,\quad \text{and}\end{aligned}$$

$$\begin{aligned} F^y_2 &= F_3 - F_1\sin\theta_1 \qquad (2)\\ &= 700\ \text{N} - (570\ \text{N})\sin 55^\circ\\ &= 700\ \text{N} - 466.917\ \text{N}\\ &= 233.083\ \text{N}\,,\quad \text{so}\end{aligned}$$

$$\begin{aligned} F_2 &= \sqrt{(F^x_2)^2 + (F^y_2)^2}\\ &= \sqrt{(326.939\ \text{N})^2 + (233.083\ \text{N})^2}\\ &= 401.518\ \text{N}\,.\end{aligned}$$

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013 (part 2 of 2) 10 points

b) What is the angle θ_2 which the left-hand cable makes with respect to the ceiling?

Correct answer: 35.4861 °.

Explanation:

Using Eq. 2, we have

$$\begin{aligned}\theta_2 &= \arctan\left(\frac{F^y_1}{F^x_1}\right)\\ &= \arctan\left(\frac{466.917\ \text{N}}{326.939\ \text{N}}\right)\\ &= 35.4861^\circ\,.\end{aligned}$$

Pulleys 03

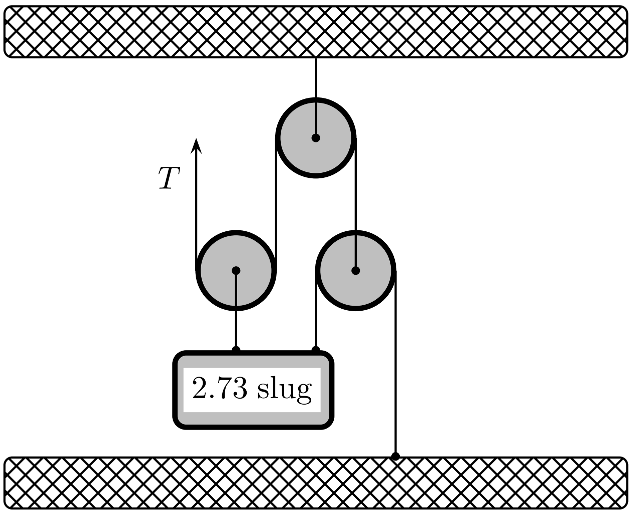
05:07, trigonometry, numeric, > 1 min, normal.

014 (part 1 of 1) 10 points

Assume: All pulleys are massless and frictionless. *Note*: lb $\equiv$ slug $\dfrac{\text{ft}}{\text{s}^2}$.

Given: The acceleration of gravity is $g = 32\ \text{ft/s}^2$.

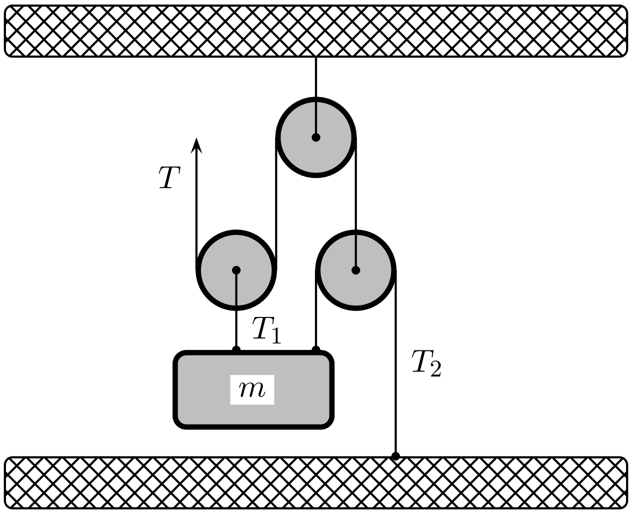
The system is in equilibrium. The suspended mass is 2.73 slug.



Find the tension T.

Correct answer: 34.944 lb.

Explanation:



At the pulley on the left-hand side:

$$T_1 = 2\,T$$

At the pulley on the right-hand side:

$$T = 2\,T_2$$

$$T_2 = \frac{T}{2}$$

Thus at the suspended mass:

$$T_1 + T_2 = 2\,T + \frac{T}{2} = m\,g$$

Multiplying by 2,

$$4\,T + T = 2\,m\,g$$

$$T = \frac{2\,m\,g}{5}$$

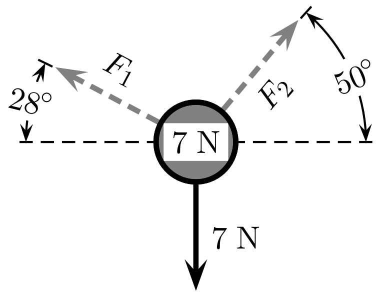
Note: Since $T_1 = 4\,T_2$, to balance the mass, the connection to the mass is not symmetrical; *i.e.*, T_1 holds $\frac{4}{5}$ of the weight while T_2 holds only $\frac{1}{5}$ of the weight.

Static Equilibrium 03

05:07, trigonometry, numeric, > 1 min, wording-variable.

015 (part 1 of 2) 10 points

The 7 N weight is in equilibrium under the influence of the three forces acting on it. The F_1 force acts from above on the left at an angle of 28° with the horizontal. The F_2 force acts from above on the right at an angle of 50° with the horizontal. The 7 N force acts straight down.



Note: F_1 and F_2 are not to scale.

What is the magnitude of the force F_2, as shown in the figure above?

Correct answer: 6.31871 N.

Explanation:

Note: Standard angular measurements are from the positive x-axis in a counter-clockwise direction.

$$\begin{aligned} \text{Given}: \quad \alpha &= 28^\circ\,, \\ \alpha_1 &= 180^\circ - \alpha\,, \\ \alpha_1 &= 152^\circ\,, \\ \alpha_2 &= 50^\circ\,, \\ F_3 &= 7\text{ N}\,, \quad \text{and} \\ \alpha_3 &= 270^\circ\,. \end{aligned}$$

Basic Concepts:

$$\begin{aligned} \sum F^x &= 0 \\ F_1^x + F_2^x + F_3^x &= 0 \\ F_1\cos\alpha_1 + F_2\cos\alpha_2 + F_3\cos\alpha_3 &= 0 \\ F_1\cos\alpha_1 &= -F_2\cos\alpha_2 && (1) \\ F_1 &= -F_2\,\frac{\cos\alpha_2}{\cos\alpha_1}\,, \quad \text{and} && (2) \\ \sum F^y &= 0 \\ F_1^y + F_2^y + F_3^y &= 0 \\ F_1\sin\alpha_1 + F_2\sin\alpha_2 - F_3 &= 0 && (3) \\ F_1 &= -\frac{F_2\sin\alpha_2 - F_3}{\sin\alpha_1}\,, \quad \text{since} && (4) \\ F_3\sin\alpha_3 &= F_3\sin 270^\circ = -F_3\,, \quad \text{and} \\ F_3\cos\alpha_3 &= F_3\cos 270^\circ = 0\,. \end{aligned}$$

Solution: Setting F_1 in Eqs. 2 and 4 equal, we have

$$F_2\,\frac{\cos\alpha_2}{\cos\alpha_1} = F_2\,\frac{\sin\alpha_2}{\sin\alpha_1} - \frac{F_3}{\sin\alpha_1}$$

$$F_2\left(\frac{\sin\alpha_2}{\sin\alpha_1}-\frac{\cos\alpha_2}{\cos\alpha_1}\right)=\frac{F_3}{\sin\alpha_1}$$

$$\begin{aligned}F_2 &= \frac{\dfrac{F_3}{\sin\alpha_1}}{\dfrac{\sin\alpha_2}{\sin\alpha_1}-\dfrac{\cos\alpha_2}{\cos\alpha_1}}\\ &= \frac{F_3}{\sin\alpha_2-\cos\alpha_2\tan\alpha_1}\\ &= \frac{7\text{ N}}{\sin 50^\circ-\cos 50^\circ\tan 152^\circ} \qquad (5)\\ &= 6.31871\text{ N}\,.\end{aligned}$$

Alternate Solution: Using Eqs. 1 and 3, we have

$$F_1\cos\alpha_2 = -F_2\cos\alpha_1 \qquad (1)$$

$$F_1\sin\alpha_2 = F_3 - F_2\sin\alpha_1 \qquad (3)$$

Dividing Eq. 3 by Eq. 1, we have

$$\begin{aligned}\frac{F_1\sin\alpha_1}{F_1\cos\alpha_1} &= \frac{F_3-F_2\sin\alpha_2}{-F_2\cos\alpha_2}\\ \tan\alpha_1 &= \frac{F_3-F_2\sin\alpha_2}{-F_2\cos\alpha_2}\\ -F_2\cos\alpha_2\tan\alpha_1 &= F_3-F_2\sin\alpha_2\\ F_2 &= \frac{F_3}{\sin\alpha_2-\cos\alpha_2\tan\alpha_1}\,,\end{aligned}$$

which is the same as Eq. 5.

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016 (part 2 of 2) 10 points

What is the magnitude of the force F_1, as shown in the figure above?

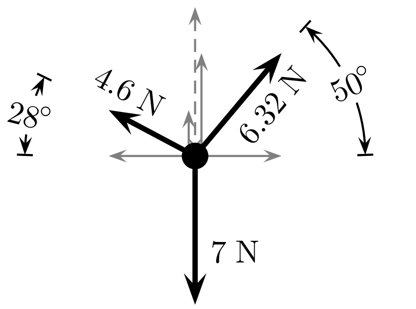
Correct answer: 4.60004 N.

Explanation:

Using Eq. 1, we have

$$\begin{aligned}F_1 &= -F_2\left(\frac{\cos\alpha_2}{\cos\alpha_1}\right)\\ &= -(6.31871\text{ N})\left(\frac{\cos 50^\circ}{\cos 152^\circ}\right)\\ &= 4.60004\text{ N}\end{aligned}$$

Observe the free-body diagram below.



The vertical dashed vector is the sum of $F_1\sin\alpha_1 = 2.15959$ N and $F_2\sin\alpha_2 = 4.84041$ N and is equal in length to $F_3^y = 7$ N.

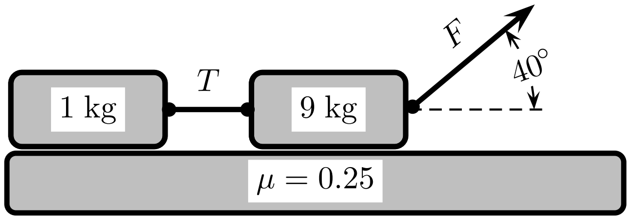
Pulling Two Blocks 02

05:08, trigonometry, numeric, > 1 min, wording-variable.

017 (part 1 of 1) 10 points

Two blocks connected by a string are pulled across a rough horizontal surface by a force applied to one of the blocks, as shown.

The acceleration of gravity is 9.8 m/s^2.



If each block has an acceleration of 2 m/s^2 to the right, what is the magnitude of the applied force?

Correct answer: 48.0177 N.

Explanation:

$$\text{Given :}\quad \begin{aligned} M_1 &= 1\text{ kg}\,,\\ M_2 &= 9\text{ kg}\,,\\ \theta &= 40^\circ\,,\\ a &= 2\text{ m/s}^2\,,\quad\text{and}\\ \mu &= 0.25\,.\end{aligned}$$

For the mass M_1, $N_1 = W_1$, so

$$F_{net} = M_1\,a = T - \mu\,M_1\,g\,. \qquad (1)$$

For the mass M_2,

$$\begin{aligned}N_2 + F\sin\theta &= W_2 = M_2\,g\\ N_2 &= M_2\,g - F\sin\theta\,,\quad\text{so}\qquad (2)\\ F_{net} &= M_2\,a\\ &= F\cos\theta - T\\ &\quad - \mu\,(M_2\,g - F\sin\theta)\,. \qquad (3)\end{aligned}$$

Adding Eqs. 1 and 3, we have

$$\begin{aligned}(M_1 + M_2)\, a = F \cos\theta - \mu\, M_1\, g - \mu\, M_2\, g \\ + \mu\, F \sin\theta\,.\end{aligned}$$

$$\begin{aligned}F &= \frac{(M_1 + M_2)\,(a + \mu\, g)}{\cos\theta + \mu \sin\theta} \\ &= \frac{1\ \mathrm{kg} + 9\ \mathrm{kg}}{\cos 40^\circ + (0.25) \sin 40^\circ} \\ &\quad \times \left[2\ \mathrm{m/s^2} + (0.25)\left(9.8\ \mathrm{m/s^2}\right)\right] \\ &= \boxed{48.0177\ \mathrm{N}}\,.\end{aligned}$$

Using Eq. 2, we have

$$\begin{aligned}N_2 &= M_2\, g - F \sin\theta \\ &= (9\ \mathrm{kg})\,(9.8\ \mathrm{m/s^2}) - (48.0177\ \mathrm{N}) \sin(40^\circ) \\ &= 57.3348\ \mathrm{N}\,.\end{aligned}$$