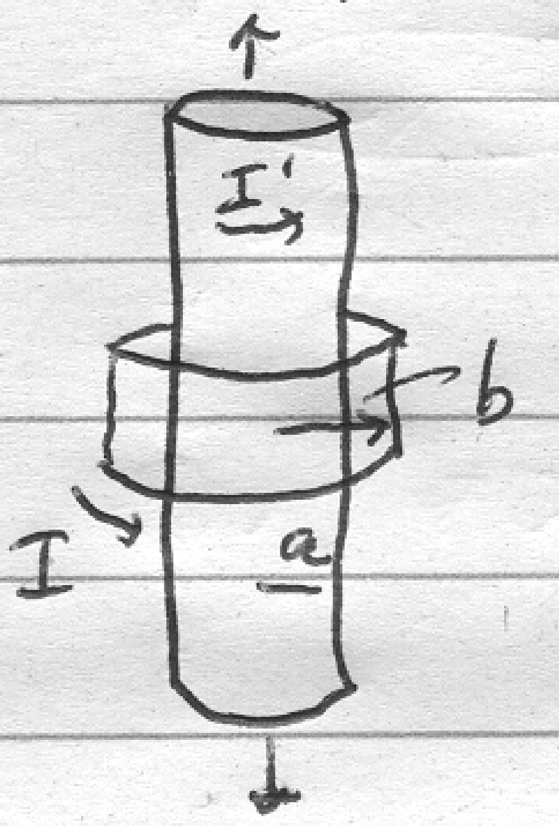
6\. b. $\vec{B} = \nabla \times \vec{A}$, so we would need the ρ dependence of $\vec{A}$ to find B_z.

7\.



no field between a + b!

a) $\Phi_m = B_{solenoid}\, \pi a^2$

$= \mu_0 n_1 I' \pi a^2$ for 1 turn

for whole coil

$\Phi_{m,tot} = \mu_0 N n_1 \pi a^2 I'$

b) $M = \frac{\Phi_{m,tot}}{I'} = \mu_0 N n_1 \pi a^2$

c) $\vec{F} = I I' \nabla M$. But $\Phi_{m,tot}$, and so M, does not change if we change radius of coil, so $\nabla M = 0$, $\vec{F} = 0$.

OR ... just note that $\vec{B}_{solenoid} = 0$ where coil is, so no force on coil!

8\. $\vec{B} = \alpha \rho^2 \hat{\varphi}$ for $\rho < a$, $\vec{B} = 0$ for $\rho > a$.

a) $\nabla \times \vec{B} = \mu_0 \vec{J} = -\hat{\rho}\left(\frac{\partial B_\varphi}{\partial z}\right) + \hat{z}\frac{1}{\rho}\frac{d}{d\rho}(\rho B_\varphi) = 3\alpha\rho\hat{z}$

so $\underline{\vec{J} = 3\alpha\rho/\mu_0}$, $\rho < a$, $\vec{J} = 0$ for $\rho > a$. Since $\vec{B} = 0$ for $\rho > a$, there must be a $\vec{K}$ at a so that $I_{tot} = 0$ through an Amperian loop with $\rho > a$. (C)

$I_{tot} = 0 = \oint_{C'} \vec{K}\cdot d\vec{s} + \int_{S'} \vec{J}\cdot d\vec{a} \Rightarrow K(2\pi a) = -\int_0^a J\, da$

$\vec{K} = -\frac{1}{2\pi a}\left(\frac{3\alpha}{\mu_0}\right)\int_0^{2\pi}\int_0^a \rho\, \rho\, d\rho\, d\varphi\, \hat{z}$, $\underline{\vec{K} = -\frac{\alpha a^2}{\mu_0}\hat{z}}$

b) $U_m = \frac{1}{2\mu_0}\int_V B^2\, d\tau = \frac{\alpha^2}{2\mu_0}\int_0^{2\pi}\int_0^{L}\int_0^{a} \rho^4\, \rho\, d\rho\, dz\, d\varphi$

$= \frac{L\pi\alpha^2}{6\mu_0} a^6$ oops $L = \ell$

$$\underline{U_m = \frac{\ell \pi \alpha^2 a^6}{6\mu_0}}$$