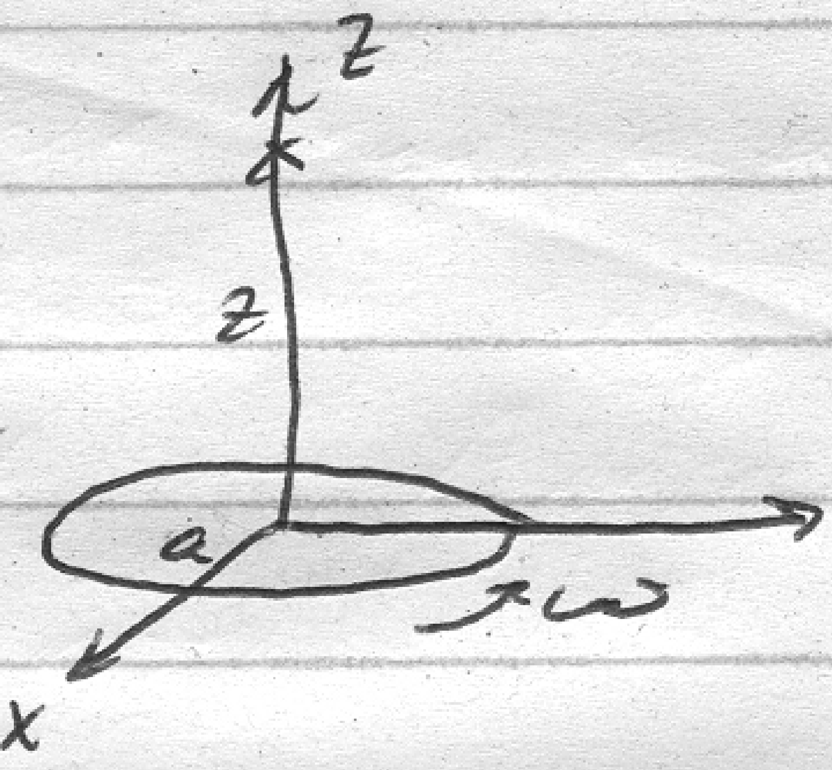
5.



a) $\vec{K} = \sigma\vec{v} = \sigma\omega\rho\,\hat{\varphi}$

Split it into bazillions of infinitesimally thick rings, thickness $d\rho$. The current for a ring is $dI = K\,d\rho = \sigma\omega\rho\,d\rho$

For $z \gg a$, eqn 14-20 gives

$$\vec{B} \simeq \frac{\mu_0 I a^2}{2z^3}\hat{z}$$

so $d\vec{B} \simeq \frac{\mu_0 (dI) a^2}{2z^3}\hat{z} = \frac{\mu_0 \sigma\omega\rho^2}{2z^3}\rho\,d\rho\,\hat{z}$

and $\vec{B} \simeq \frac{\mu_0\sigma\omega}{2z^3}\int_0^a \rho^3 d\rho\,\hat{z} = \underline{\frac{\mu_0\sigma\omega a^4}{8z^3}\hat{z}}$

b) $d\vec{m} = \pi\rho^2 (dI)\hat{z} = \sigma\omega\pi\rho^3 d\rho\,\hat{z}$

so $\vec{m} = \pi\sigma\omega\int_0^a \rho^3 d\rho\,\hat{z} = \frac{\pi\sigma\omega a^4}{4}\hat{z}$

Note: gives $\vec{B} = \frac{\mu_0 m}{2\pi z^3}\hat{z}$, agrees with (19-24)

c) On x axis, from (19-24), if $x \gg a$,

$$\vec{B} = -\frac{\mu_0 m}{4\pi x^3}\hat{z} = -\frac{\mu_0\pi\sigma\omega a^4}{16\pi x^3}\hat{z} = \underline{-\frac{\mu_0\sigma\omega a^4}{16x^3}\hat{z}}$$

6. a (16-18) $\vec{A} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}\,da}{R}$

$da = \rho\,d\rho\,d\varphi \quad R = \sqrt{\rho^2 + z^2}, \quad \vec{K} = \sigma\omega\rho\,\hat{\varphi}$

so $\vec{A} = \frac{\mu_0\sigma\omega}{4\pi}\int_0^{2\pi}\int_0^a \frac{\rho^2\,d\rho\,d\varphi\,\hat{\varphi}}{\sqrt{\rho^2 + z^2}}$

but $\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$

When we insert $\hat{\varphi}$, the φ integrals vanish!

so $\underline{\vec{A} = 0}$ on axis.