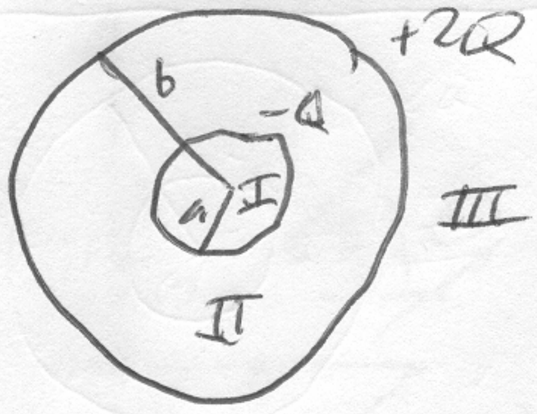
4.



in III $\quad \Phi_{III} = \dfrac{Q}{4\pi\varepsilon_0 r}$

at $r = b \quad \Phi_{III} = \Phi_{II}(b) = \dfrac{Q}{4\pi\varepsilon_0 b}$

in II $\quad \vec{E} = -\dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{r^2}\hat{r}$

integrate in to r

$$\Delta\Phi_{II}(r) = -\int_b^r \vec{E}\cdot d\vec{s} \qquad d\vec{s} = dr\,\hat{r}$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_b^r \frac{1}{r^2}dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\Big|_b^r = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{r}\right]$$

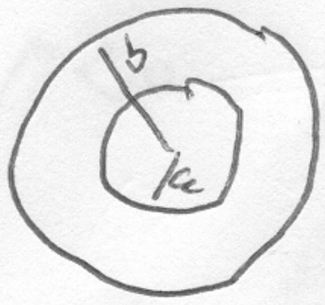
to find Φ_{II} must add Φ_{II} at b

$$\underline{\Phi_{II} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{2}{b} - \frac{1}{r}\right]}$$

in I $\quad \Phi_I = \text{const} = \Phi_I(a)$ $\quad$ E at $b+\varepsilon$ $Q/4\pi\varepsilon_0 b^2$

$$\underline{\Phi_I = \frac{Q}{4\pi\varepsilon_0}\left[\frac{2}{b} - \frac{1}{a}\right]}$$

5.



Disc of radius b along z axis, for $z > 0$

$$\vec{E}(z) = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{b^2+z^2}}\right]\hat{z} \qquad (3\text{-}15)$$

Superpose with a disc of $\sigma' = -\sigma$ and radius a.

$$\vec{E}_{actual} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{b^2+z^2}} - \left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)\right]\hat{z}$$

$$= \underline{\frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right]\hat{z}}$$

$z \gg a, b$

$$\vec{E} \sim \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{2}\frac{a^2}{z^2} - \left(1 - \frac{1}{2}\frac{b^2}{z^2}\right)\right]$$

$$\vec{E} \simeq \frac{\sigma}{4\varepsilon_0}\frac{b^2-a^2}{z^2} \qquad \text{now } Q = \pi\sigma(b^2-a^2)$$

So $\vec{E} \simeq \underline{\dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{z^2}}$ point charge!