Midterm Solutions 3305 Fall 13

1. $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}')\,d\tau'}{R}$ assumes $\phi = 0$ at ∞ for all charge elements. This can't be true for elements at $r \to \infty$. Those elements give ∞ ϕ everywhere.

2. $\nabla\cdot(u\vec{A}) = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\cdot(uA_x\hat{x} + uA_y\hat{y} + uA_z\hat{z})$

$$= \frac{\partial}{\partial x}(uA_x) + \frac{\partial}{\partial y}(uA_y) + \frac{\partial}{\partial z}(uA_z)$$

$$= \frac{\partial u}{\partial x}A_x + u\frac{\partial A_x}{\partial x} + \frac{\partial u}{\partial y}A_y + u\frac{\partial A_y}{\partial y} + \frac{\partial u}{\partial z}A_z + u\frac{\partial A_z}{\partial z}$$

terms 1, 3, 5 give $\vec{A}\cdot\nabla u$

" 2, 4, 6 give $u(\nabla\cdot\vec{A})$

3) Sphere, radius a, $\rho(\vec{r}') = A(1 - r/a)$ $r < a$, 0 for $r > a$.

$$Q = 4\pi A\int_0^a (1 - r/a)r^2\,dr = 4\pi A\left[\int_0^a r^2\,dr - \frac{1}{a}\int_0^a r^3\,dr\right]$$

$$Q = 4\pi A\left[\frac{1}{3}a^3 - \frac{1}{4}a^3\right] = \underline{\frac{\pi A}{3}a^3} \qquad \pi A = \frac{3Q}{a^3}$$

$\underline{r < a}$

$\cancel{q_{in} = \frac{\pi A}{3}r^3 \qquad E(4\pi r^2) = \frac{1}{\varepsilon_0}\frac{\pi A}{3}r^3}$ oops

$$q_{in} = 4\pi A\left[\int_0^r r^2\,dr - \frac{1}{a}\int_0^r r^3\,dr\right] = 4\pi A\left[r^3/3 - \frac{1}{4a}r^4\right]$$

$$E(4\pi r^2) = 4\pi A\left[\frac{1}{3}r^3 - \frac{1}{4a}r^4\right]/\varepsilon_0$$

$$\underline{\underline{E = \frac{A}{\varepsilon_0}\left[\frac{1}{3}r - \frac{1}{4a}r^2\right]}}$$

in terms of Q $\quad E = \frac{3Q}{\pi a^3\varepsilon_0}\left[\frac{1}{3}r - \frac{1}{4a}r^2\right]$

$\underline{r > a}$ $\quad q_{in} = Q = \frac{\pi A a^3}{3}$

$$E(4\pi r^2) = \frac{\pi A a^3}{3\varepsilon_0} \qquad \underline{\underline{E = \frac{Aa^3}{12\varepsilon_0}\frac{1}{r^2}}}$$

in terms of Q

$$E = \frac{3Q}{\pi a^3}\frac{a^3}{12\varepsilon_0}\frac{1}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} \checkmark$$