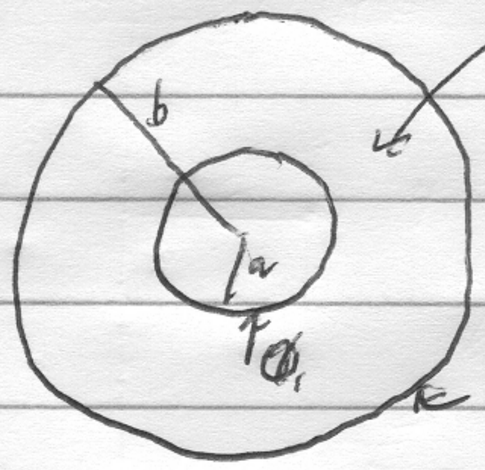
11-30



$\rho(r) = \rho_0 \left(r/a\right)^n$ ρ_0 const., $n > 0$

Poisson's eqn. $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = -\frac{\rho(r)}{\varepsilon_0}$

$$\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = -\frac{\rho_0}{\varepsilon_0 a^n} r^{n+2}$$

$$r^2 \frac{d\phi}{dr} = \frac{-\rho_0}{\varepsilon_0 a^n} \frac{1}{n+3} r^{n+3} + \alpha$$

$$\frac{d\phi}{dr} = -\frac{\rho_0}{\varepsilon_0 a^n (n+3)} r^{n+1} + \frac{\alpha}{r^2}$$

integrate again $\phi(r) = -\frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)} r^{n+2} - \frac{\alpha}{r} + \beta$

at $\underline{r=a}$, $\phi = \phi_1$

$$\phi_1 = -\frac{\rho_0 a^2}{\varepsilon_0 (n+2)(n+3)} - \frac{\alpha}{a} + \beta$$

so $\beta = \phi_1 + \frac{\rho_0 a^2}{\varepsilon_0 (n+2)(n+3)} + \frac{\alpha}{a}$

at $\underline{r=b}$ $\phi = \phi_2$

$$\phi_2 = -\frac{\rho_0 b^{n+2}}{\varepsilon_0 a^n (n+2)(n+3)} - \frac{\alpha}{b} + \beta$$

working cont.

$$\phi_2 = -\frac{\rho_0 b^{n+2}}{\varepsilon_0 a^n (n+2)(n+3)} - \frac{\alpha}{b} + \phi_1 + \frac{\rho_0 a^{2+n}}{\varepsilon_0 a^n (n+2)(n+3)} + \frac{\alpha}{a}$$

$$\left(\frac{1}{a} - \frac{1}{b}\right)\alpha = \phi_2 - \phi_1 + \frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)}\left(b^{n+2} - a^{n+2}\right)$$

$$\alpha = \frac{ab}{b-a}\left[\phi_2 - \phi_1 + \frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)}\left(b^{n+2} - a^{n+2}\right)\right]$$

$$\beta = \phi_1 + \frac{\rho_0 a^{2+n}}{\varepsilon_0 (n+2)(n+3)} + \frac{b}{b-a}\left[\phi_2 - \phi_1 + \frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)}\left(b^{n+2} - a^{n+2}\right)\right]$$

So, simply

$$\phi(r) = -\frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)} r^{n+2} - \frac{ab}{b-a}\left[\phi_2 - \phi_1 + \frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)}\left(b^{n+2} - a^{n+2}\right)\right]\frac{1}{r}$$

$$+ \phi_1 + \frac{\rho_0 a^{2+n}}{\varepsilon_0 (n+2)(n+3)} + \frac{b}{b-a}\left[\phi_2 - \phi_1 + \frac{\rho_0}{\varepsilon_0 a^n (n+2)(n+3)}\left(b^{n+2} - a^{n+2}\right)\right]$$