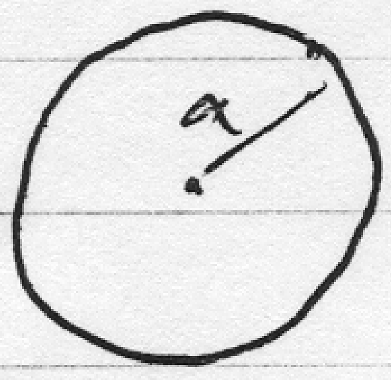
5. <u>W-11-26</u> Dielectric cylinder in a previously uniform field.



Use b.c.s at $\rho = a$:

$$\epsilon \frac{\partial \phi}{\partial \rho}\Big|_{in} = \epsilon_0 \frac{\partial \phi}{\partial \rho}\Big|_{out}, \quad \phi \text{ continuous}$$

at $\rho = \infty$, $\phi \to -E_0 \rho \cos\varphi \quad (\vec{E} = E_0 \hat{x})$

$\boxed{\text{outside}}$

$$\phi_{out} = \alpha + \beta \ln \rho + \sum_{m=1}^{\infty} (A_m \rho^m + B_m \rho^{-m}) \times (C_m \sin m\varphi + D_m \cos m\varphi)$$

set $\underline{\alpha = 0}$ (fixes reference point of potential at origin)

$\underline{\beta = 0}$ since no net charge on cylinder

$\underline{\rho = \infty \text{ b.c.}} \Rightarrow$ all $A_m = 0$ except $A_1 = -E_0$ and $C_m = 0$ by symmetry

$$\phi_{out} = -E_0 \rho \cos\varphi + \sum_{m=1}^{\infty} B_m \rho^{-m} \cos m\varphi$$

$\boxed{\text{inside}}$

$$\phi_{in} = \sum_{m=1}^{\infty} A_m \rho^m \cos m\varphi$$

$$\epsilon \frac{\partial \phi_{in}}{\partial \rho}\Big|_{\rho=a} = \epsilon_0 \frac{\partial \phi_{out}}{\partial \rho}\Big|_{\rho=a}$$

$$\epsilon \sum_{m=1}^{\infty} m A_m a^{m-1} \cos m\varphi = \epsilon_0 \left[-E_0 \cos\varphi + \sum_{m=1}^{\infty} (-m) B_m a^{-m-1} \cos m\varphi\right]$$

$\cos m\varphi$ are orthogonal; can equate coefficients

$\underline{m=1}$ $\quad \epsilon A_1 = -\epsilon_0 E_0 - \epsilon_0 B_1/a^2 \Rightarrow B_1 = -E_0 a^2 - \frac{\epsilon}{\epsilon_0} a^2 A_1$

$\underline{m>1}$ $\quad \epsilon m A_m a^{m-1} = \epsilon_0 (-m) B_m a^{-m-1}$

$$B_m = -\frac{\epsilon}{\epsilon_0} a^{2m} A_m$$

$\underline{\phi_{in} = \phi_{out} \text{ at } \rho = a}$

$$\Rightarrow \sum_{m=1}^{\infty} A_m a^m \cos m\varphi = -E_0 a \cos\varphi + \sum_{m=1}^{\infty} B_m a^{-m} \cos m\varphi$$