Now multiply both sides by $\sin\left(\frac{p\pi x}{L}\right)$, p integer, and integrate over x

$$\phi_0 \int_0^L \sin\left(\frac{p\pi x}{L}\right) dx = \frac{2\phi_0 L}{p\pi} \ (p \text{ odd}) = 0 \ (p \text{ even})$$

$$= \sum_{m,n} A_{mn} \Bigg(\underbrace{\int_0^L \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{p\pi x}{L}\right) dx}_{\frac{L}{2}\delta_{m,p}} \Bigg) \sin\left(\frac{n\pi y}{L}\right) \sinh(\gamma L)$$

So odd p only! for odd p

$$\frac{2\phi_0 L}{p\pi} = \sum_n A_{pn} \frac{L}{2} \sin\left(\frac{n\pi y}{L}\right) \sinh(\gamma L) \quad \text{or}$$

$$\frac{4\phi_0}{p\pi} = \sum_n A_{pn} \sin\left(\frac{n\pi y}{L}\right) \sinh(\gamma L)$$

→ Repeat for y, multiply both sides by $\sin\left(\frac{q\pi y}{L}\right)$, integrate over y, get....

$$\frac{16\phi_0}{pq\pi^2} = A_{pq} \sinh(\gamma L) \qquad \gamma = \frac{\pi}{L}\sqrt{p^2+q^2}$$

$$A_{pq} = \frac{16\phi_0}{pq\pi^2 \sinh(\gamma L)} \quad \text{and finally}$$

$$\phi(x,y,z) = \sum_{p,q} \frac{16\phi_0}{pq\pi^2 \sinh(\gamma L)} \sin\left(\frac{p\pi x}{L}\right) \sin\left(\frac{q\pi y}{L}\right) \sinh(\gamma z)$$

for p, q odd

Find $\phi\left(\frac{L}{2}, \frac{L}{2}, \frac{L}{2}\right)$ cube center

$$\phi = \sum_{p,q \text{ odd}} \frac{16\phi_0}{pq\pi^2 \sinh(\gamma L)} \underbrace{\sin\left(\frac{p\pi}{2}\right)} \sin\left(\frac{q\pi}{2}\right) \sinh\left(\gamma \frac{L}{2}\right)$$

→ $= +1$ if $p = 1, 5, 9 \ldots$, $= -1$ if $p = 3, 7, 11 \ldots$
$= (-1)^{\frac{p-1}{2}}$

for a particular p, q, the term in ϕ becomes

$$\frac{16\phi_0 \sinh(\gamma L/2)}{pq\pi^2 \sinh(\gamma L)} (-1)^{\frac{p+q-2}{2}} = \frac{16\phi_0}{2pq\pi^2 \cosh\left(\frac{\pi}{2}\sqrt{p^2+q^2}\right)} (-1)^{\frac{p+q}{2}-1}$$

$\sinh(2A) = 2\sinh A \cosh A$

p	q	$\frac{8\phi_0}{pq\pi^2 \cosh\left(\frac{\pi}{2}\sqrt{p^2+q^2}\right)}$
1	1	$+0.17337\,\phi_0$
1	3	$-0.003762\,\phi_0$
3	1	$-0.003762\,\phi_0$
3	3	$+0.000230\,\phi_0$
1	5	$+0.000108\,\phi_0$
5	1	$+0.000108\,\phi_0$

add, get $\boxed{\phi = 0.1663\,\phi_0}$ close enough