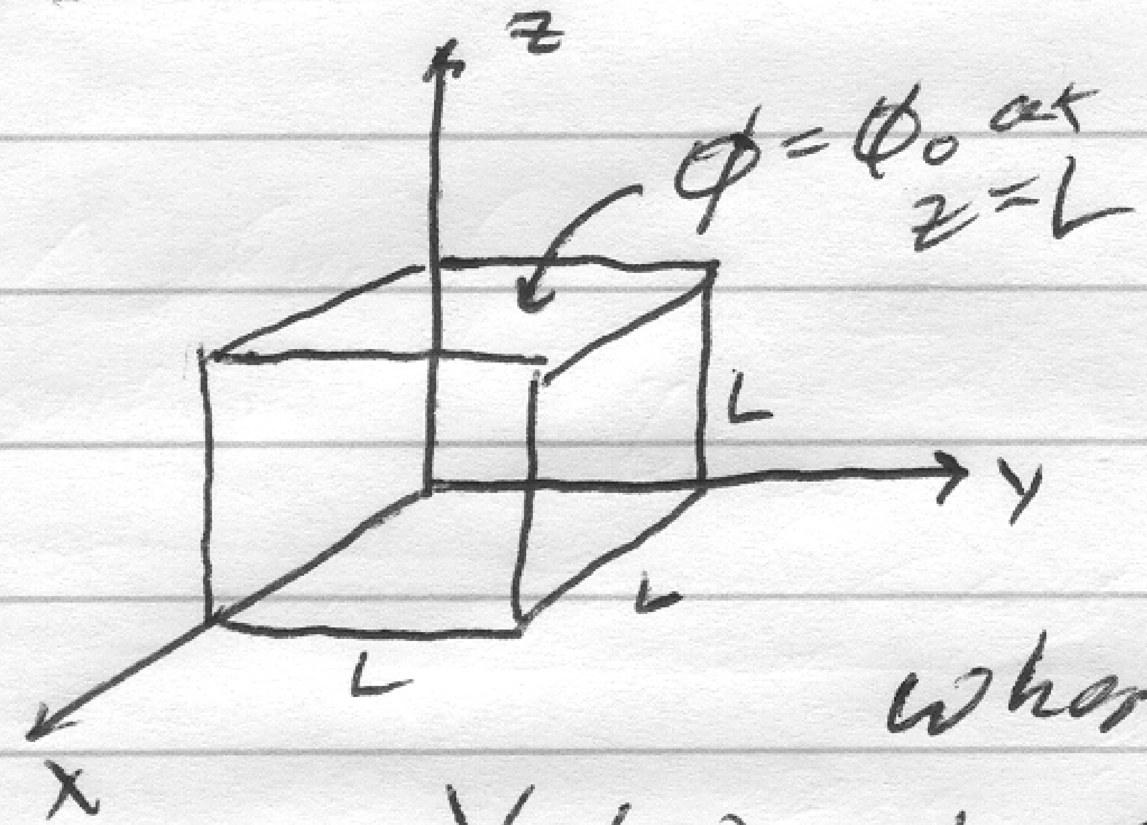
W 11-18



Other 5 sides, $\phi = 0$

$$\phi(x,y,z) = \sum_{\alpha\beta\gamma} X_\alpha(x)\, Y_\beta(y)\, Z_\gamma(z)$$

where $X_\alpha(x) = a_1 e^{\alpha x} + a_2 e^{-\alpha x}$

$Y_\beta(y) = b_1 e^{\beta y} + b_2 e^{-\beta y}$, $Z(z) = c_1 e^{\gamma z} + c_2 e^{-\gamma z}$

in x direction $\phi = 0$ at $x = 0$ and $x = L$, so must have oscillatory solutions; same for y!

Rewrite $X_\alpha(x) = a_1 \sin|\alpha| x + a_2 \cos|\alpha| x$ where α is imaginary $\alpha = i|\alpha|$. Since $\phi = 0$ at $X = 0$, no cosines allowed. Since $\phi = 0$ at $X = L$, $\sin|\alpha| L = 0$ or $|\alpha| L = m\pi$ or $|\alpha| = \frac{m\pi}{L}$ (m integer).

Thus $X_m(x) = a_m \sin\left(\frac{m\pi x}{L}\right)$.

Same arguments apply for y, giving $Y_n(y) = b_n \sin\left(\frac{n\pi y}{L}\right)$, n integer.

Now $\gamma^2 = -\alpha^2 + \beta^2 \Rightarrow \gamma = \frac{\pi}{L}\sqrt{m^2+n^2}$ real

and $\phi = 0$ at $z = 0$ so

$Z_\gamma(z) = C_{mn} \sinh(\gamma z)$ since $C_1 + C_2 = 0$
$C_2 = -C_1$

So the solution reduces to, combining the constants into A_{mn},

$$\phi(x,y,z) = \sum_{m,n} A_{mn} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) \sinh(\gamma z)$$

Last b.c.! $\phi = \phi_0$ at $z = L$

$$\phi(x,y,L) = \phi_0 = \cancel{\sinh(\gamma L)\ A} = \sum_{m,n} A_{mn} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) \sinh(\gamma L)$$

oops

(γ depends on m, n)