and so $\sigma(x=0) = -\frac{\lambda a}{\pi}\frac{1}{a^2+\rho^2} = -\frac{\lambda a}{\pi}\frac{1}{a^2+y^2}$

along the y axis.

$\lambda_{induced}$?

on surface $dq = \sigma(x=0)\,dy\,dz \qquad d\lambda_{ind} = \frac{dq}{dz} = \sigma\,dy$

so $\lambda_{ind} = \int_{-\infty}^{\infty}\sigma\,dy = 2\int_0^{\infty}\sigma\,dy$ (symmetric)

$$= -\frac{2\lambda a}{\pi}\int_0^{\infty}\frac{dy}{a^2+y^2} = -\frac{2\lambda a}{\pi}\frac{1}{a}\underbrace{\tan^{-1}\left(\frac{y}{a}\right)\Big|_0^{\infty}}_{\pi/2}$$

$\lambda_{ind} = -\lambda$ as expected!

Force/length on λ is same as would be exerted by image line charge

$$d\vec{F} = dq\,\vec{E}_{(image\ at\ \lambda)}$$

$$\frac{d\vec{F}}{dz} = \text{force/length} = \lambda\vec{E} = \lambda\left(\frac{-\lambda}{2\pi\varepsilon_0(2a)}\hat{x}\right)$$

$$= \frac{-\lambda^2}{4\pi\varepsilon_0 a}\hat{x}$$