

E_y should vanish on the $x=0$ plane, where it is tangential. For $x=0$, $R_1=R_3$ and $R_2=R_4$

$$E_y(0, y, z) = \frac{q}{4\pi\epsilon_0} \left[\frac{(y-b)}{R_1^3} + \frac{y+b}{R_2^3} - \frac{y-b}{R_1^3} - \frac{y+b}{R_2^3} \right] = 0$$

Find induced σ_f on $y=0$ plane, where E_y is normal.

$$\sigma_f = \epsilon_0 E_y|_{y=0}$$

$$= \frac{-q}{4\pi\epsilon_0} \left[\frac{b}{\sqrt{(x-a)^2 + b^2 + z^2}} + \frac{b}{\sqrt{(x-a)^2 + b^2 + z^2}} - \frac{b}{\sqrt{(x+a)^2 + b^2 + z^2}} - \frac{b}{\sqrt{(x+a)^2 + b^2 + z^2}} \right]$$

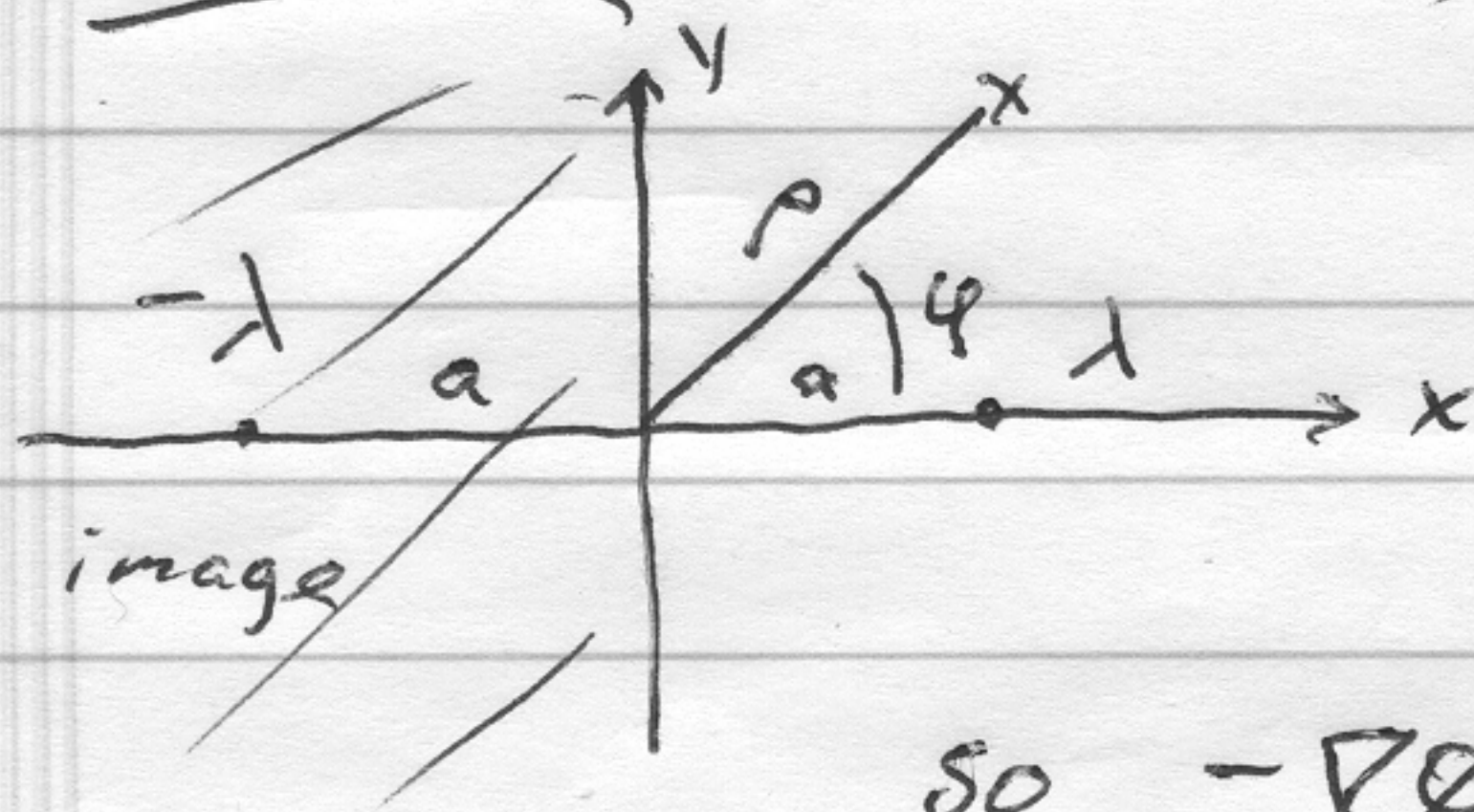
$$\sigma_f = \frac{-2qb}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{(x-a)^2 + b^2 + z^2}} - \frac{1}{\sqrt{(x+a)^2 + b^2 + z^2}} \right]$$

$$\text{or } \sigma_f = \frac{qb}{2\pi\epsilon_0} \left[\frac{1}{\sqrt{(x+a)^2 + b^2 + z^2}} - \frac{1}{\sqrt{(x-a)^2 + b^2 + z^2}} \right]$$

Since $x > 0$, 2nd term is larger than 1st,

$$\text{so } \underline{\sigma_f < 0}$$

Q 11-11 from (5-34), $\phi(r, \varphi) = \frac{\lambda}{4\pi\epsilon_0} \ln \left(\frac{a^2 + r^2 + 2ar \cos \varphi}{a^2 + r^2 - 2ar \cos \varphi} \right)$



$$\sigma(x=0) = \epsilon_0 E_x|_{x=0} = -\epsilon_0 \frac{\partial \phi}{\partial x} \Big|_{x=0}$$

at $x=0$ $\varphi = \pi/2$ and $\hat{x} = -\hat{\varphi}$ for positive y .

$$\text{so } -\nabla \phi \cdot \hat{x} = \nabla \phi \cdot \hat{\varphi} \Big|_{\varphi=\pi/2} = \frac{1}{r} \frac{\partial \phi}{\partial \varphi} \Big|_{\varphi=\pi/2}$$

$$\text{and } \sigma(x=0) = \epsilon_0 \frac{1}{r} \frac{\partial \phi}{\partial \varphi} \Big|_{\pi/2}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left[\frac{-2ar \sin \varphi}{a^2 + r^2 + 2ar \cos \varphi} - \frac{2ar \sin \varphi}{a^2 + r^2 - 2ar \cos \varphi} \right] \Big|_{\pi/2}$$