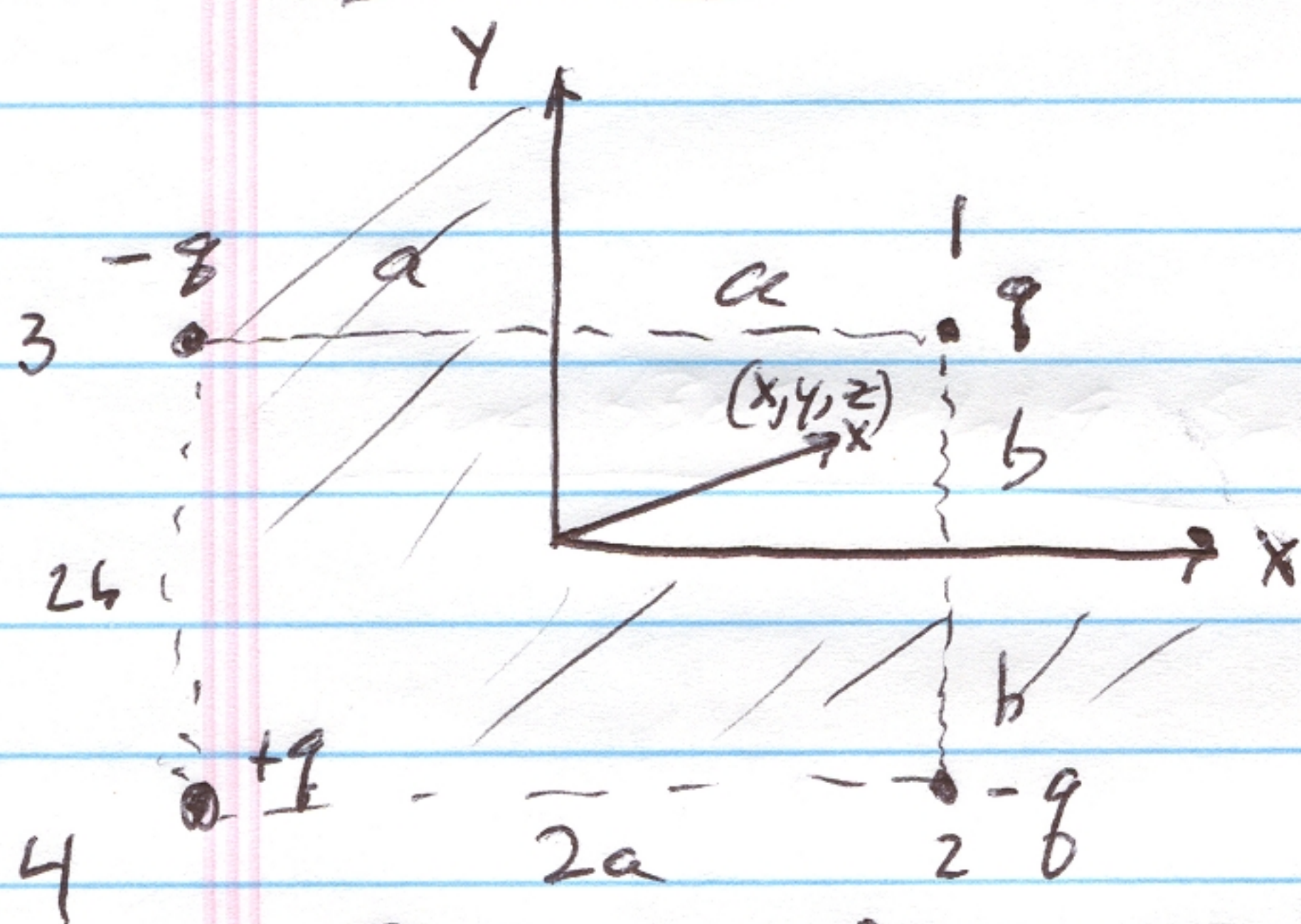


Phys 3305 HW9 Solutions

Q 11-3



Use image charges as shown. Justified if it works!

$$\phi(x, y, z) = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

For ϕ_1 $\vec{r}_1' = a\hat{x} + b\hat{y}$

$$R_1 = |\vec{r} - \vec{r}_1'| = \sqrt{(x-a)^2 + (y-b)^2 + z^2}$$

for ϕ_2 $\vec{r}_2' = a\hat{x} - b\hat{y}$, so $R_2 = \sqrt{(x-a)^2 + (y+b)^2 + z^2}$

Similarly, for 3 $R_3 = \sqrt{(x+a)^2 + (y-b)^2 + z^2}$

and for 4 $R_4 = \sqrt{(x+a)^2 + (y+b)^2 + z^2}$ so

Find ϕ
$$\phi(x, y, z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{(x-a)^2 + (y-b)^2 + z^2}} - \frac{1}{\sqrt{(x-a)^2 + (y+b)^2 + z^2}} - \frac{1}{\sqrt{(x+a)^2 + (y-b)^2 + z^2}} + \frac{1}{\sqrt{(x+a)^2 + (y+b)^2 + z^2}} \right]$$

for $x=0$

$$\phi(0, y, z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{a^2 + (y-b)^2 + z^2}} - \frac{1}{\sqrt{a^2 + (y+b)^2 + z^2}} - \frac{1}{\sqrt{a^2 + (y-b)^2 + z^2}} + \frac{1}{\sqrt{a^2 + (y+b)^2 + z^2}} \right] = 0$$

for $y=0$, 1st + 2nd terms, and 3rd + 4th terms cancel!

so $\phi(x, 0, z) = 0$ and the b.c.s are satisfied.

Find $E_y(x, y, z) = -\frac{\partial}{\partial y} \phi$

$$E_y(x, y, z) = \frac{-q}{4\pi\epsilon_0} \left[\frac{-(y-b)}{R_1^3} + \frac{(y+b)}{R_2^3} + \frac{(y-b)}{R_3^3} - \frac{(y+b)}{R_4^3} \right]$$