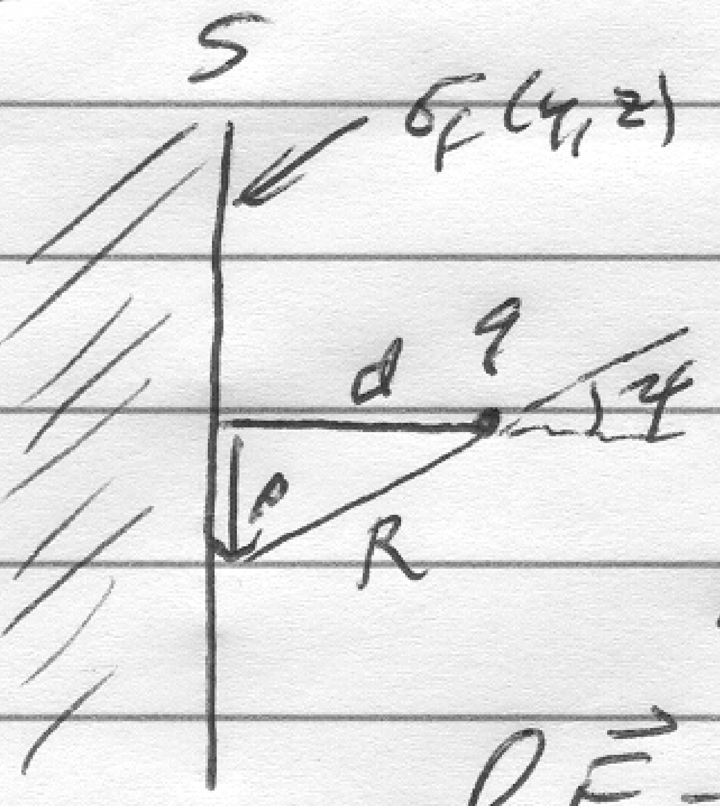
Phys 3305 HW 9 Solutions

11-2



$$\sigma_f(y,z) = \frac{-qd}{2\pi(d^2+y^2+z^2)^{3/2}}$$

$$\rho^2 = y^2+z^2$$

by symmetry $\vec{F}_{tot} = F_{tot}\,\hat{x}$, $F_{tot} = \vec{F}_{tot}\cdot\hat{x}$

$$d\vec{F} = \frac{q}{4\pi\varepsilon_0}\frac{\sigma_f\,da}{R^3}\vec{R} \qquad R = \sqrt{\rho^2+d^2}$$

$$d\vec{F}\cdot\hat{x} = \frac{q}{4\pi\varepsilon_0}\frac{\sigma_f\,da}{R^3}\vec{R}\cdot\hat{x} \qquad \vec{R}\cdot\hat{x} = R\cos\psi = R\frac{d}{\sqrt{\rho^2+d^2}}$$

So
$$F_{tot} = \frac{q}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\infty}\frac{\sigma_f\,\rho\,d\rho\,d\varphi}{R^3}\frac{Rd}{\sqrt{\rho^2+d^2}}$$

$$= -\frac{q^2d^2}{8\pi^2\varepsilon_0}\int_0^{2\pi}\int_0^{\infty}\frac{\rho\,d\rho\,d\varphi}{(\rho^2+d^2)^3} = -\frac{q^2d^2}{4\pi\varepsilon_0}\int_0^{\infty}\frac{\rho\,d\rho}{(\rho^2+d^2)^3}$$

From tables

$$\int\frac{x\,dx}{(x^2+a^2)^3} = -\frac{1}{4}\frac{1}{(x^2+a^2)^2}$$

So
$$F_{tot} = \frac{q^2d^2}{16\pi\varepsilon_0}\frac{1}{(\rho^2+d^2)^2}\Big|_0^{\infty} = \underline{-\frac{q^2}{16\pi\varepsilon_0 d^2}} = \frac{-q^2}{4\pi\varepsilon_0}\frac{1}{(2d)^2}$$