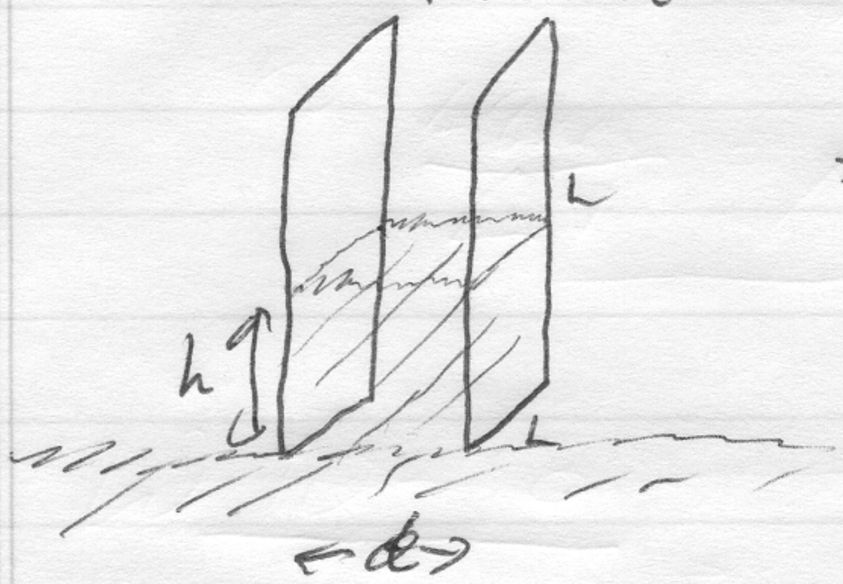
6. W 10-36 a parallel plate capacitor, sides L, separation d is charged to Q, then disconnected. It is allowed to suck up a dielectric fluid with mass density ρ and dielectric constant K_e. Find h



Work done charging to Q
= (Stored Ue with fluid at h)
+ (grav. P.E. of fluid)

$$W_{charge} = \frac{Q^2}{2C_0} = \frac{Q^2 d}{2\varepsilon_0 L^2}$$

C_0 ← empty

filled to h with fluid $C(h) = \frac{\varepsilon_0 L}{d}[L + (K_e - 1)h]$ (problem 10-32)

$$U_e(h) = \frac{Q^2}{2C(h)} = \frac{Q^2 d}{2\varepsilon_0 L} \frac{1}{L + (K_e - 1)h}$$

Gravitational P.E.

$$U_g = mg\left(\frac{h}{2}\right) = \frac{1}{2}\rho(hLd)hg = \frac{1}{2}\rho L d h^2 g$$

$\frac{h}{2}$ ← center of mass

so $$\frac{Q^2 d}{2\varepsilon_0 L^2} = \frac{Q^2 d}{2\varepsilon_0 L} \frac{1}{L + (K_e - 1)h} + \frac{1}{2}\rho L d g h^2$$

Algebrate!

$$\frac{Q^2 d}{2\varepsilon_0 L} + \frac{Q^2 d (K_e - 1)h}{2\varepsilon_0 L^2} = \frac{Q^2 d}{2\varepsilon_0 L} + \frac{1}{2}\rho L d g h^2 (L + (K_e - 1)h)$$

$$\frac{1}{2}\rho L d g (K_e - 1) h^2 + \frac{1}{2}\rho d g L^2 h - \frac{Q^2 d (K_e - 1)}{2\varepsilon_0 L^2} = 0$$

quadratic formula give positive root

$$h = \frac{L}{2(K_e - 1)}\left[\sqrt{1 + \frac{4(K_e - 1)^2 Q^2}{\varepsilon_0 \rho g L^5}} - 1\right]$$