

5. W 10-18

Find ρ_b, σ_b

In material, from Gauss' Law

for $\vec{D}$, $\vec{D} = \frac{q}{4\pi r^2} \hat{r}$ so $\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{q}{4\pi \epsilon_0 r^2} \hat{r}$

and $\vec{P} = \chi \epsilon_0 \vec{E} = (K-1) \epsilon_0 \vec{E}$
 $= \frac{K-1}{K \epsilon_0} \epsilon_0 \frac{q}{4\pi r^2} \hat{r}$

$$\vec{P} = \frac{K-1}{K} \frac{q}{4\pi} \frac{\hat{r}}{r^2}$$

$$\rho_b = -\nabla \cdot \vec{P} = -\frac{K-1}{K} \frac{q}{4\pi} \nabla \cdot \frac{\hat{r}}{r^2} = 0 \quad \text{from 1-145}$$

$$\sigma_b = \vec{P} \cdot \vec{n} = \vec{P} \cdot (-\hat{r}) = -\frac{K-1}{K} \frac{q}{4\pi a^2}$$

$$Q_b = 4\pi a^2 \sigma_b = -\frac{K-1}{K} q$$

10-13 shows that the total bound charge must vanish. To reconcile our result with this there must be a surface at a with an infinitesimal σ_b with total charge $\frac{K-1}{K} q$.