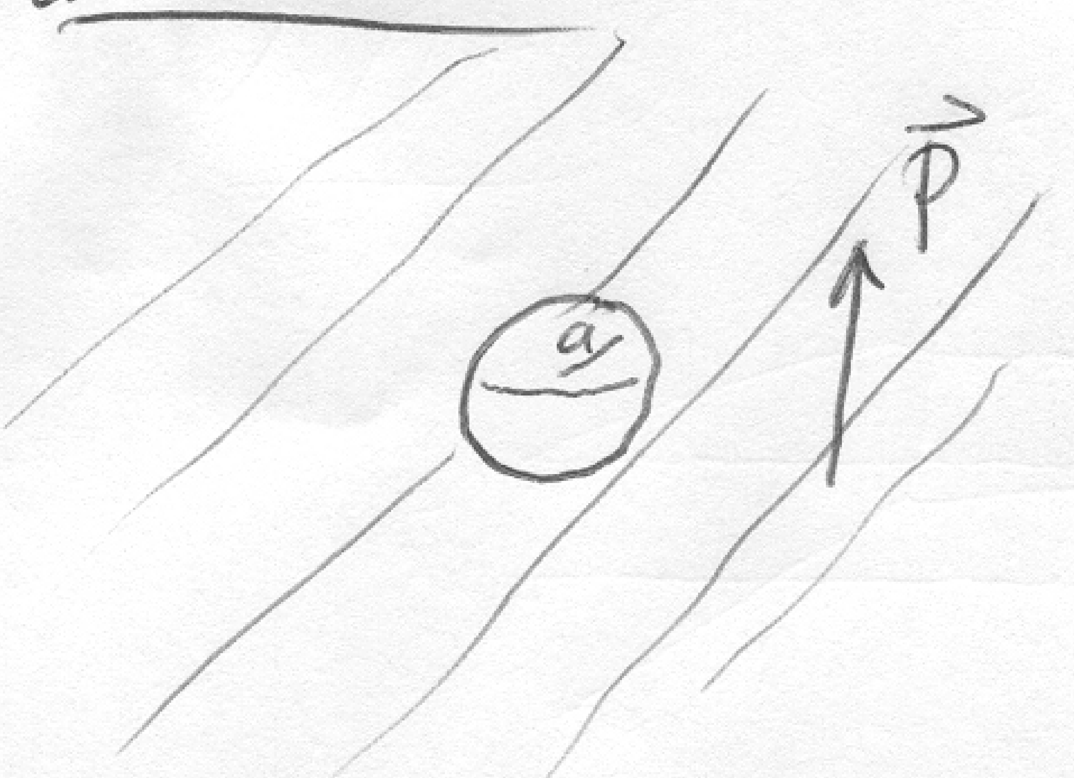
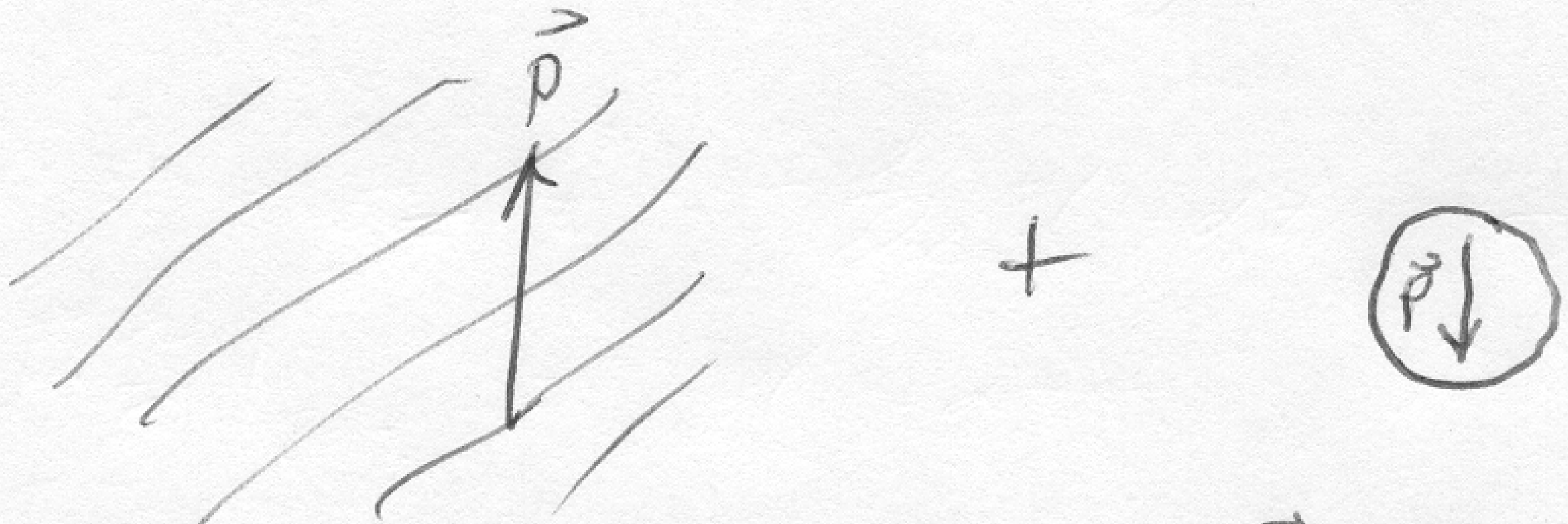
3. W 10-10



Find $\vec{E}$ at center of spherical cavity

Superposition

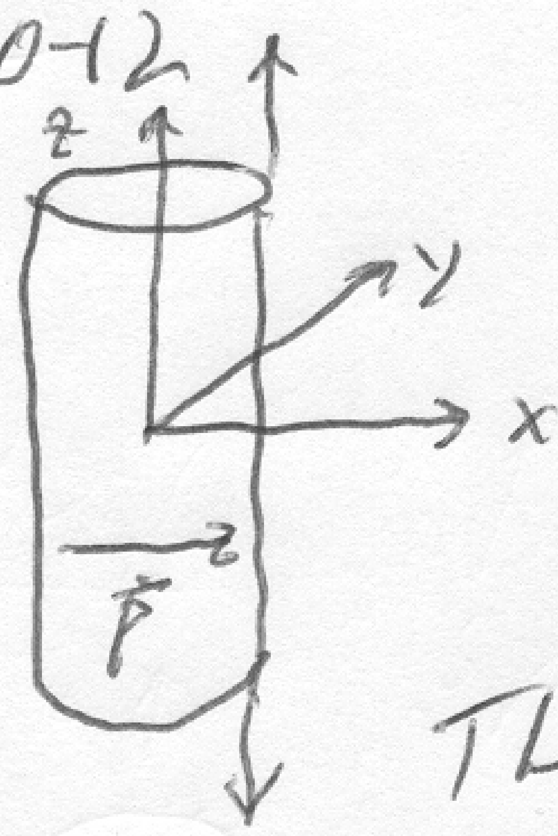


$\vec{P} = P\hat{z}$, $\vec{D} = 0$, so $\vec{E}_p = -\frac{P}{\varepsilon_0}\hat{z}$

$\vec{P} = -P\hat{z}$, $\vec{E}_{sphere} = \frac{P}{3\varepsilon_0}\hat{z}$

$$\vec{E} = \vec{E}_p + \vec{E}_{sphere} = \left(-\frac{P}{\varepsilon_0} + \frac{P}{3\varepsilon_0}\right)\hat{z} = -\frac{2}{3}P/\varepsilon_0\,\hat{z}$$

4. W 10-12



$\vec{P} = P\hat{x}$ $\quad \hat{n} = \hat{\rho}'$ $\quad \sigma_b = \vec{P}\cdot\hat{n} = P\hat{x}\cdot\hat{\rho}' = P\cos\varphi'$

origin $\vec{r} = 0$ $\quad \vec{r}' = a\hat{\rho}' + z'\hat{z} = a\cos\varphi'\hat{x} + a\sin\varphi'\hat{y} + z'\hat{z}$

so $\vec{R} = -a\cos\varphi'\hat{x} - a\sin\varphi'\hat{y} - z'\hat{z}$

and $R = \sqrt{a^2\cos^2\varphi' + a^2\sin^2\varphi' + (z')^2} = \sqrt{a^2 + (z')^2}$

Then

$$\vec{E} = \frac{P}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\int_0^{2\pi}\frac{\left[-a\cos^2\varphi'\hat{x} - a\sin\varphi'\cos\varphi'\hat{y} - z'_{\cos\varphi'}\hat{z}\right]a\,d\varphi'\,dz'}{(a^2 + (z')^2)^{3/2}}$$

The $\hat{y}$ and $\hat{z}$ terms vanish in the φ' integral, and $\int^{2\pi}\cos^2\varphi'\,d\varphi' = \pi$, so

$$\vec{E} = -\frac{Pa^2}{4\varepsilon_0}\int_{-\infty}^{\infty}\frac{\hat{x}}{(a^2+(z')^2)^{3/2}}dz' = -\frac{Pa^2}{4\varepsilon_0}\frac{z'}{\sqrt{a^2+(z')^2}}\Bigg|_{-\infty}^{\infty}\frac{1}{a^2}\hat{x}$$

$$\vec{E} = -\frac{P}{2\varepsilon_0}\hat{x}$$

This turns out to be uniform throughout the cylinder as we will show in Ch. 11.