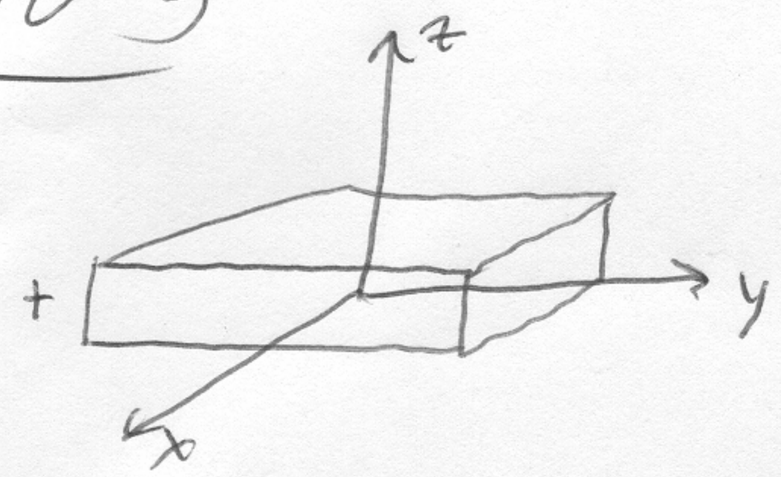
Phys 3305 HW 8 Solutions

1. W 10-3



$$\vec{P} = P(A(1+\alpha z)\hat{z}$$

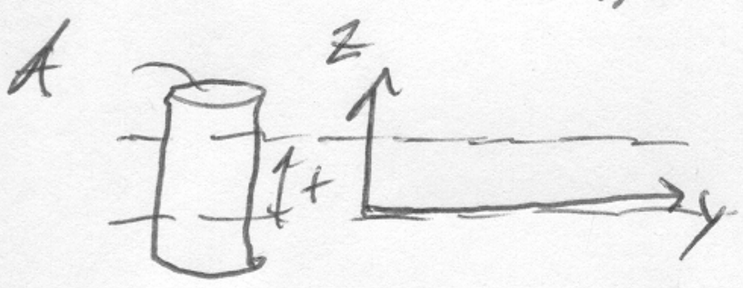
Well then,

$$\rho_b = -\nabla\cdot\vec{P} = -\frac{\partial}{\partial z}P_z = -\frac{d}{dz}P(1+\alpha z) = -\alpha P$$

at $z=+$ $\quad \sigma_b = \sigma_{b+} = \vec{P}\cdot\hat{z}\,|_{z=+} = P(1+\alpha+)$

at $z=0$ $\quad \sigma_{b-} = \vec{P}\cdot(-\hat{z})\,|_{z=0} = -P$

$$\text{Total } Q_b = \int \rho_b \, d\tau + \int \sigma_{b+}\, da + \int \sigma_{b-}\, da$$

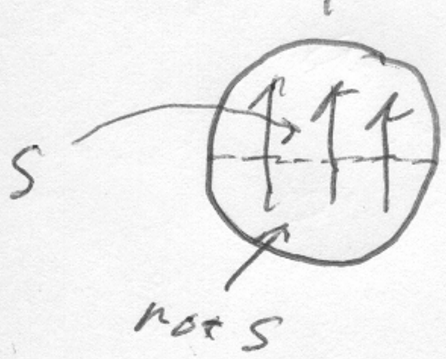


$$Q_b = \rho_b A + + \sigma_{b+} A + \sigma_{b-} A$$

$$= (-\alpha P +)A + (P + P\alpha +)A(-PA)$$

$$Q_b = 0$$

2. W 10-6



Find total positive bound charge of polarized sphere

$$\vec{P} = P\hat{z} \qquad \sigma = \vec{P}\cdot\hat{n} = \vec{P}\cdot\hat{r} = P\cos\theta$$

Positive σ for $0 \le \theta \le \pi/2$.

$$Q_+ = \int_S \sigma\, da = P\int_0^{2\pi}\int_0^{\pi/2} \cos\theta\, a^2 \sin\theta\, d\theta\, d\phi$$

$$= 2\pi a^2 P \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta$$

$$= \pi a^2 P \sin^2\theta\,\Big|_0^{\pi/2} = \underline{\pi a^2 P}$$