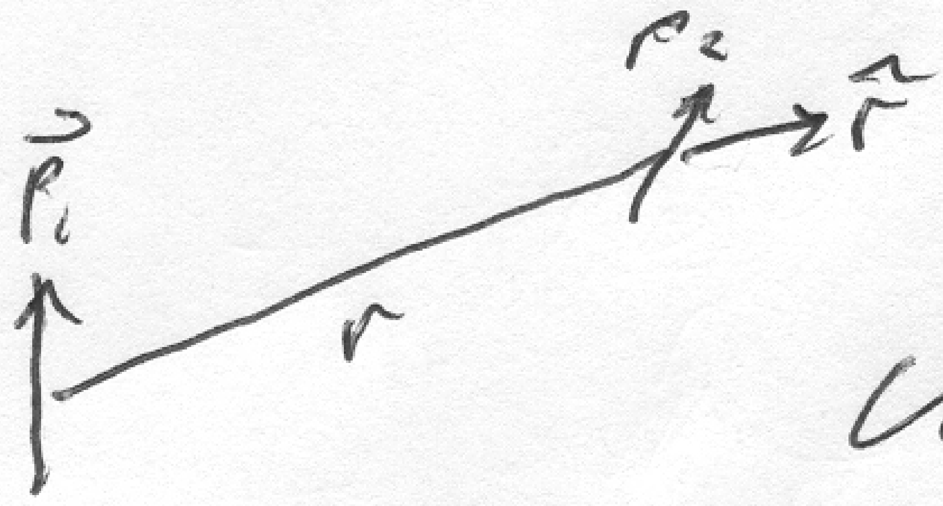
5. 8-19 Set $\vec{p}_1$ at origin in $\hat{z}$ direction



$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}_1\cdot\hat{r})\hat{r} - \vec{p}_1\right]$$

$$U_{int} = -\vec{p}_2\cdot\vec{E}$$

$$U_{int} = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}_1\cdot\vec{p}_2}{r^3} - 3\left(\frac{\vec{p}_1\cdot\hat{r}}{r^3}\right)(\vec{p}_2\cdot\hat{r})\right]$$

We just made up direction of $\hat{r}$, so it is irrelevant.

Now we "merely" take $\vec{F}_2 = -\nabla U_{int}$

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0}\left[3\left[\nabla\left(\frac{\vec{p}_1\cdot\hat{r}}{r^3}\right)\right](\vec{p}_2\cdot\hat{r}) + 3\left(\vec{p}_1\cdot\frac{\hat{r}}{r^3}\right)\nabla(\vec{p}_2\cdot\hat{r}) - \nabla\frac{\vec{p}_1\cdot\vec{p}_2}{r^3}\right]$$

last term easy $= 3\frac{\vec{p}_1\cdot\vec{p}_2}{r^4}\hat{r}$

Now $\nabla(\vec{A}\cdot\vec{B}) = \vec{B}\times(\nabla\times\vec{A}) + \vec{A}\times(\nabla\times\vec{B}) + (\vec{B}\cdot\nabla)\vec{A} + (\vec{A}\cdot\nabla)\vec{B}$

taking $\vec{A} = \vec{p}_1$ and $\vec{B} = \hat{r}/r^3$ for 1st term, all but last term die, leaving $\nabla\left(\frac{\vec{p}_1\cdot\hat{r}}{r^3}\right) = (\vec{p}_1\cdot\nabla)\frac{\hat{r}}{r^3}$

For second term get $(\vec{p}_2\cdot\nabla)\hat{r}$

Now to work these evil bastards out

since $\vec{p}_1 = p_1\hat{z}$

eqn 1-121 $\quad (\vec{p}_1\cdot\nabla)\frac{\hat{r}}{r^3} = \hat{x}\left(p_1\frac{\partial}{\partial z}\left(\frac{\hat{r}\cdot\hat{x}}{r^3}\right)\right) + \hat{y}\left(p_1\frac{\partial}{\partial z}\frac{\hat{r}\cdot\hat{y}}{r^3}\right) + \hat{z}\left(p_1\frac{\partial}{\partial z}\frac{\hat{r}\cdot\hat{z}}{r^3}\right)$

now $\frac{\hat{r}}{r^3} = \frac{\vec{r}}{r^4} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^2}$

components

$\hat{x}$: $p_1\frac{\partial}{\partial z}\frac{x}{(\ \ )^2} = -\frac{4zxp_1}{(x^2+y^2+z^2)^3} = -\frac{4zxp_1}{r^6}$

$\hat{y}$: $p_1\frac{\partial}{\partial z}\frac{y}{(\ \ )^2} = -\frac{4zyp_1}{(x^2+y^2+z^2)^3} = -4\frac{zyp_1}{r^6}$

$\hat{z}$: $p_1\frac{\partial}{\partial z}\frac{z}{(\ \ )^2} = \frac{p_1}{(x^2+y^2+z^2)^2} - \frac{4z^2p_1}{(x^2+y^2+z^2)^3}$

$= \frac{p_1}{r^4} - 4\frac{z^2p_1}{r^6}$

$$(\vec{p}_1\cdot\nabla)\frac{\hat{r}}{r^3} = \frac{p_1}{r^4}\hat{z} - \frac{4zp_1}{r^6}\vec{r}$$

$$= \frac{\vec{p}_1}{r^4} - 4\frac{zp_1\hat{r}}{r^5} \quad \text{and}$$

$p_1 z = \vec{p}_1\cdot\vec{r}$

$$(\vec{p}_2\cdot\hat{r})(\vec{p}_1\cdot\nabla)\frac{\hat{r}}{r^3} = \frac{(\vec{p}_2\cdot\hat{r})\vec{p}_1}{r^4} - 4\frac{zp_1}{r^5}\hat{r}(\vec{p}_2\cdot\hat{r}) = \frac{(\vec{p}_2\cdot\hat{r})\vec{p}_1}{r^4} - 4\frac{(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r})}{r^4}\hat{r}$$