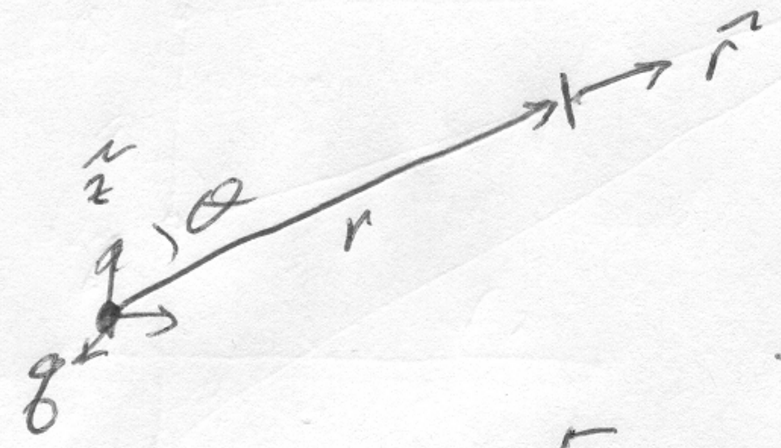
4. 8-18



8-82 + 8-83 depend on E_{0z}, the z component of the Coulomb field

$$E_{0z} = \hat{z} \cdot \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \hat{r} = \frac{q\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{qz}{4\pi\varepsilon_0 r^3} = \frac{qz}{(x^2+y^2+z^2)^{3/2}}$$

now $U_Q^a = \cancel{\text{(illegible)}} - \frac{Q^a}{4} \frac{\partial E_{0z}}{\partial z}$

$$= \frac{-qQ^a}{(4\pi\varepsilon_0)4}\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$= \frac{qQ^a}{(4\pi\varepsilon_0)4}\left[\frac{3z^2}{r^5} - \frac{1}{r^3}\right] = \underline{\frac{qQ^a}{(4\pi\varepsilon_0)4r^3}\left[3\cos^2\theta - 1\right]}$$

$$\vec{F}_Q^a = \frac{Q^a}{4} \nabla\left(\frac{\partial E_{0z}}{\partial z}\right)$$

$$= \frac{Q^a}{4}\left[\hat{x}\frac{\partial}{\partial x}\left(\frac{\partial E_{0z}}{\partial z}\right) + \hat{y}\frac{\partial}{\partial y}\left(\frac{\partial E_{0z}}{\partial z}\right) + \hat{z}\frac{\partial}{\partial z}\left(\frac{\partial E_{0z}}{\partial z}\right)\right]$$

$$= \frac{qQ^a}{(4\pi\varepsilon_0)4}\left[\hat{x}\left(\frac{-3x}{(x^2+y^2+z^2)^{5/2}} + \frac{15xz^2}{(x^2+y^2+z^2)^{7/2}}\right)\right.$$

$$+\hat{y}\left(\frac{-3y}{(x^2+y^2+z^2)^{5/2}} + \frac{15yz^2}{(x^2+y^2+z^2)^{7/2}}\right)$$

$$\left.+\hat{z}\left(\frac{-3z}{(x^2+y^2+z^2)^{5/2}} - \frac{6z}{(x^2+y^2+z^2)^{5/2}} + \frac{15z^3}{(x^2+y^2+z^2)^{7/2}}\right)\right]$$

$$= \frac{qQ^a}{16\pi\varepsilon_0}\left[\frac{(x\hat{x}+y\hat{y}+z\hat{z})(-3)}{r^5} + \frac{15(x\hat{x}+y\hat{y}+z\hat{z})z^2}{r^7} - \frac{6z}{r^5}\hat{z}\right]$$

$$= \frac{qQ^a}{16\pi\varepsilon_0}\left[-\frac{3\hat{r}}{r^4} + 15\frac{z^2}{r^6}\hat{r} - \frac{6z}{r^5}\hat{z}\right]$$

Can rewrite

$$\vec{F}_Q^a = \frac{qQ^a}{16\pi\varepsilon_0 r^4}\left[-3\hat{r} + 15\cos^2\theta\,\hat{r} - 6\cos\theta\,\hat{z}\right]$$