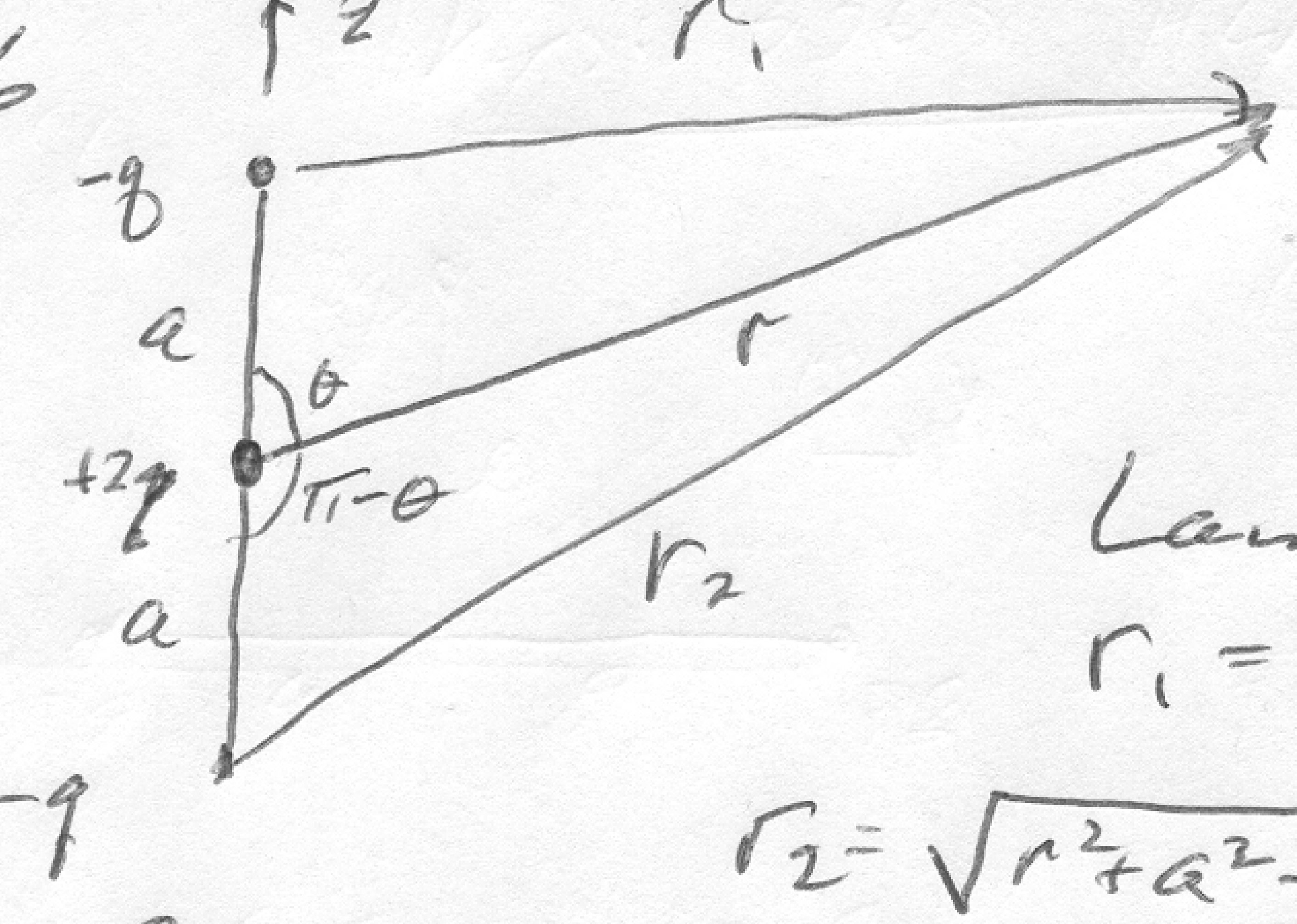
2.8-6



$$\phi(\vec r) = \frac{q}{4\pi\epsilon_0}\left[\frac{2}{r} - \frac{1}{r_1} - \frac{1}{r_2}\right]$$

Law of cosines

$$r_1 = \sqrt{r^2+a^2-2ar\cos\theta}$$

$$r_2 = \sqrt{r^2+a^2-2ar\cos(\pi-\theta)} = \sqrt{r^2+a^2+2ar\cos\theta}$$

So

$$\phi(\vec r) = \frac{q}{4\pi\epsilon_0}\left[\frac{2}{r} - \frac{1}{\sqrt{r^2+a^2-2ar\cos\theta}} - \frac{1}{\sqrt{r^2+a^2+2ar\cos\theta}}\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\left[2 - \frac{1}{\sqrt{1+\frac{a^2}{r^2}-2\frac{a}{r}\cos\theta}} - \frac{1}{\sqrt{1+\frac{a^2}{r^2}+2\frac{a}{r}\cos\theta}}\right]$$

Now $\frac{1}{\sqrt{1+x}} \simeq 1 - \frac{1}{2}x + \frac{3}{8}x^2$ if $x \ll 1$

So

$$\phi(\vec r) \simeq \frac{q}{4\pi\epsilon_0 r}\Big[\cancel{2} - \Big(\cancel{1} - \frac{1}{2}\Big(\frac{a^2}{r^2} - 2\frac{a}{r}\cos\theta\Big) + \frac{3}{8}\Big(\frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\Big)^2\Big)$$
$$- \Big(\cancel{1} - \frac{1}{2}\Big(\frac{a^2}{r^2} + 2\frac{a}{r}\cos\theta\Big) + \frac{3}{8}\Big(\frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\Big)^2\Big)\Big]$$

$$\simeq -\frac{q}{4\pi\epsilon_0 r}\Big[-\frac{1}{2}\frac{a^2}{r^2} + \cancel{2\frac{a}{r}\cos\theta} + \frac{3}{8}\frac{a^4}{r^4} - \cancel{\frac{3}{4}\frac{a^3\cos\theta}{r^3}} + \frac{3}{2}\frac{a^2}{r^2}\cos^2\theta$$
$$-\frac{1}{2}\frac{a^2}{r^2} - \cancel{2\frac{a}{r}\cos\theta} + \frac{3}{8}\frac{a^4}{r^4} + \cancel{\frac{3}{4}\frac{a^3}{r^3}\cos\theta} + \frac{3}{2}\frac{a^2}{r^2}\cos^2\theta\Big]$$

drop terms of power higher than $\frac{a^2}{r^2}$

$$\simeq \frac{q}{4\pi\epsilon_0 r}\left[\frac{a^2}{r^2} - \frac{3}{2}\frac{a^2}{r^2}\cos^2\theta - \frac{3}{2}\frac{a^2}{r^2}\cos^2\theta\right]$$

$$= \frac{(-qa^2)}{4\pi\epsilon_0 r^3}\left[3\cos^2\theta - 1\right]$$

if $Q^a = -4qa^2 = \sum_i (-q)(3z_i^2 - r_i^2)$

$$= \frac{Q^a}{4\pi\epsilon_0 r^3}\frac{\left[3\cos^2\theta - 1\right]}{4}$$

! Yay!