2. 7-8 $Q_i = \sum_{j=1}^{n} c_{ij}\phi_j$

The energy will be given by

$$U_e = \frac{1}{2}\sum_i Q_i\phi_i$$

$\frac{1}{2}$ prevents double counting

$$U_e = \frac{1}{2}\sum_{ij} c_{ij}\phi_j\phi_i$$

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2 conductors

$$U_e = \tfrac{1}{2}c_{11}\phi_1^2 + \tfrac{1}{2}c_{12}\phi_1\phi_2 + \tfrac{1}{2}c_{21}\phi_2\phi_1 + \tfrac{1}{2}c_{22}\phi_2^2$$

$c_{12} = c_{21}$, so

$$U_e = \tfrac{1}{2}c_{11}\phi_1^2 + c_{12}\phi_1\phi_2 + \tfrac{1}{2}c_{22}\phi_2^2$$

Showing 6-44; $Q = c_{11}\phi_1 + c_{12}\phi_2$

$-Q = c_{21}\phi_1 + c_{22}\phi_2$

Solve for ϕ_2, ϕ_1; take $\phi_2 - \phi_1 = \Delta\phi$

and after a page of algebra $\frac{Q}{\Delta\phi} =$ eq'n 6-44

$U_e = \frac{1}{2}Q\Delta\phi$ $Q_i = \sum c_{ij}\phi_j$ so $Q_1 = Q = c_{11}\phi_1 + c_{12}\phi_2$

$Q_2 = -Q = c_{21}\phi_1 + c_{22}\phi_2$

- want to express ϕ_s in terms of Q_s,

so need coefficients of potential

$$\mathbb{C} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \quad \det = c_{11}c_{22} - c_{12}^2, \quad \text{cof}\begin{pmatrix} c_{22} & -c_{21} \\ -c_{12} & c_{11} \end{pmatrix}$$

$$\mathbb{P} = \mathbb{C}^{-1} = \frac{1}{\det}\,\widetilde{\text{cof}} = \frac{1}{c_{11}c_{22} - c_{12}^2}\begin{pmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{pmatrix}$$

$Q_1 = Q$ $Q_2 = -Q$ so

$$\phi_1 = \frac{1}{c_{11}c_{22} - c_{12}^2}(c_{22}Q_1 - c_{21}Q_2) = \frac{Q}{c_{11}c_{22} - c_{12}^2}(c_{22} + c_{21})$$

$$\phi_2 = \frac{1}{c_{11}c_{22} - c_{12}^2}(-c_{21}Q_1 + c_{11}Q_2) = \frac{Q}{c_{11}c_{22} - c_{12}^2}(-c_{21} - c_{11})$$

$$\Delta\phi = \phi_1 - \phi_2 = \frac{Q}{c_{11}c_{22} - c_{12}^2}(c_{11} + c_{22} + 2c_{12})$$

$$U_e = \frac{1}{2}Q\Delta\phi = \frac{1}{2}\frac{c_{11} + c_{22} + 2c_{12}}{c_{11}c_{22} - c_{12}^2}Q^2 = \frac{1}{2}\frac{Q^2}{C}$$