Phys 3305 Hw #6 Solutions

7-4 $\rho = A r^n$, from 5-9 answers

$$\phi_i = \frac{Q}{4(n+2)\pi\varepsilon_0 a}\left[(n+3) - \left(\frac{r}{a}\right)^{n+2}\right]$$

$\rho = 0$ for $r > a$

$$U_e = \frac{1}{2}\int \rho(\vec{r})\phi(\vec{r})\,d\tau$$

$$= \frac{1}{2}\frac{QA}{4(n+2)\pi\varepsilon_0 a}(4\pi)\int_0^a r^n\left[(n+3) - \left(\frac{r}{a}\right)^{n+2}\right] r^2\,dr$$

$$= \frac{QA}{2(n+2)\varepsilon_0 a}\left[\int_0^a (n+3) r^{n+2}\,dr - \frac{1}{a^{n+2}}\int_0^a r^{2n+4}\,dr\right]$$

$$= \frac{QA}{2(n+2)\varepsilon_0 a}\left[\frac{n+3}{n+3}a^{n+3} - \frac{1}{2n+5}a^{n+3}\right]$$

$$= \frac{QA\,a^{n+2}}{2(n+2)\varepsilon_0}\left[1 - \frac{1}{2n+5}\right]$$

But $Q = A\int_0^a r^{n+2}\,dr\,(4\pi) = \frac{4\pi A}{n+3}a^{n+3}$

so $A = \frac{Q(n+3)}{4\pi a^{n+3}}$

and $U_e = \frac{Q^2(n+3)}{8\pi\varepsilon_0(n+2)a}\left[1 - \frac{1}{2n+5}\right]$

for $n = 0$ should give uniform sphere results

$n=0$ $\quad U_e = \frac{3Q^2}{16\pi\varepsilon_0 a}\left[1 - \frac{1}{5}\right] = \frac{3}{16}\frac{4}{5}\frac{Q^2}{\pi\varepsilon_0 a}$

$$= \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 a}$$ and it does!