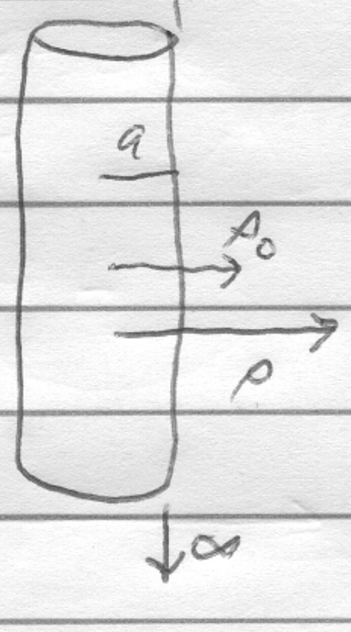
3305 HW5 solutions

1. 6-2



$\lambda = q_\ell$ Conductor

Find Φ outside

From Gauss' Law

$$\vec{E}_{out} = \frac{q_\ell}{2\pi\varepsilon_0}\frac{1}{\rho}\hat{\rho}$$

$$\Phi(\rho) - \Phi(\rho_0) = -\int \vec{E}\cdot d\vec{s} = -\frac{q_\ell}{2\pi\varepsilon_0}\int_{\rho_0}^{\rho}\frac{d\rho}{\rho}$$

(since $d\vec{s} = d\rho\hat{\rho}$)

$$\Phi(\rho) - \Phi(\rho_0) = -\frac{q_\ell}{2\pi\varepsilon_0}\ln(\rho/\rho_0) = \frac{q_\ell}{2\pi\varepsilon_0}\ln\left(\frac{\rho_0}{\rho}\right)$$

taking $\Phi(\rho_0) = 0$

$$\underline{\Phi(\rho) = \frac{q_\ell}{2\pi\varepsilon_0}\ln(\rho_0/\rho)}$$

Inside cylinder $\Phi(\rho<a) = \underline{\Phi(a) = \frac{q_\ell}{2\pi\varepsilon_0}\ln(\rho_0/a)}$

At surface $\vec{E}_s = -\nabla\Phi = ?$

$$\vec{E}_s = -\frac{d}{d\rho}\Phi\hat{\rho}\Big|_a = -\frac{q_\ell}{2\pi\varepsilon_0}\frac{d}{d\rho}\left[\ln(\rho_0) - \ln(\rho)\right]\hat{\rho}\Big|_a$$

$$= +\frac{q_\ell}{2\pi\varepsilon_0\rho}\hat{\rho}\Big|_a \quad \text{or} \quad \underline{\vec{E}_s = \frac{q_\ell}{2\pi\varepsilon_0 a}\hat{\rho}}$$

relate q_ℓ & σ $\quad q_\ell l = \sigma\, 2\pi a l = Q$ $\quad q_\ell = \sigma(2\pi a)$

σ_0 $\quad \underline{\vec{E}_s = \frac{\sigma}{\varepsilon_0}\hat{n}}$ $\quad \hat{\rho} = \hat{n}$

P_{11} $\quad \Phi(a) = P_{1\ell}\, Q$ $\quad Q$ is infinite, and $\Phi(a)$ is arbitrary to within an additive constant, so can't have a unique P_{11}.