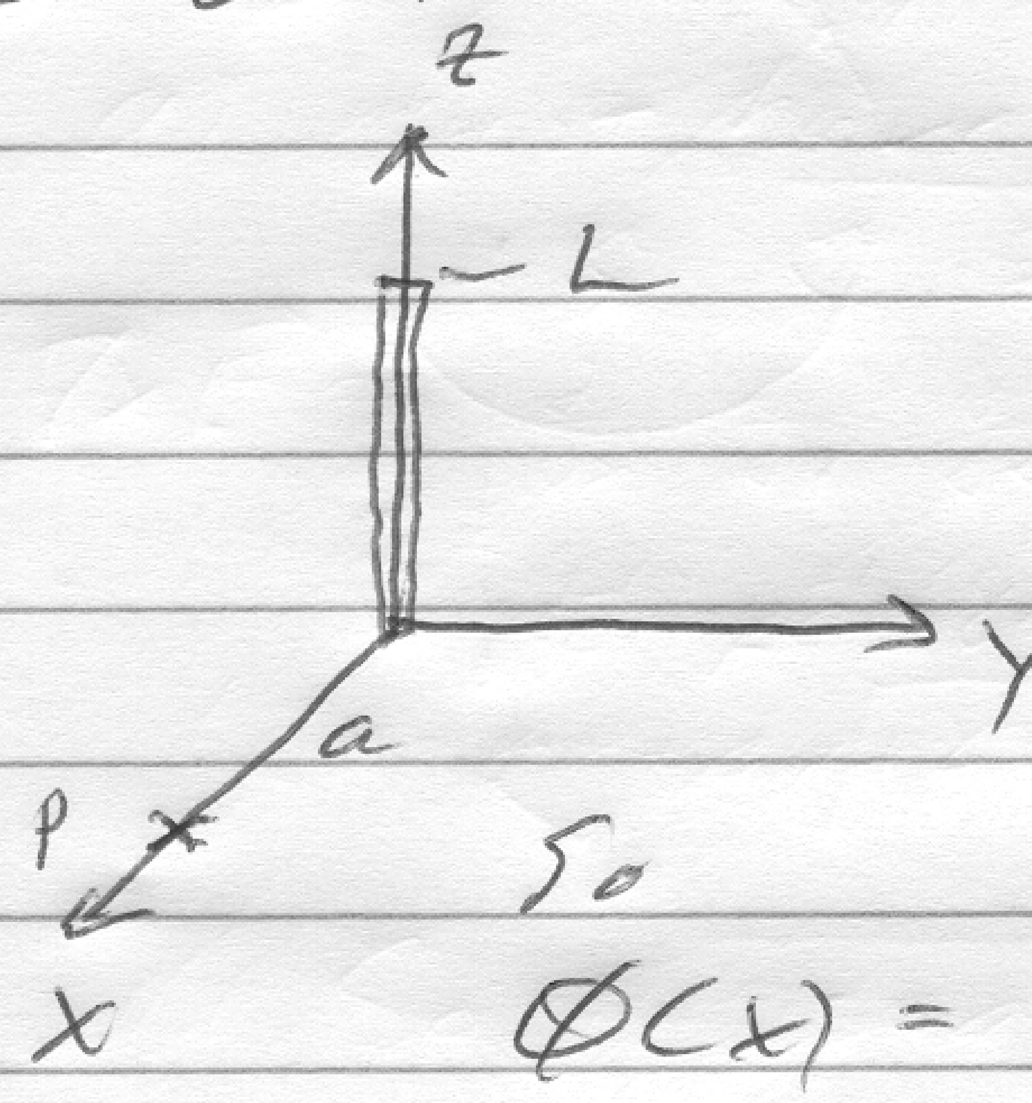
6-19 5-24 $\qquad \lambda(z) = \alpha z^3$; find ϕ at P



$$\phi(x) = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda(z)\,dz}{R}$$

where $R = \sqrt{a^2+z^2} = \sqrt{x^2+R^2}$

So

$$\phi(x) = \frac{\alpha}{4\pi\varepsilon_0}\int_0^L \frac{z^3\,dz}{\sqrt{x^2+z^2}}$$

Tables give $\phi(x) = \frac{\alpha}{4\pi\varepsilon_0}\left[\frac{1}{3}\left(\sqrt{x^2+z^2}\right)^3 - x^2\sqrt{x^2+z^2}\right]\Big|_0^L$

$$= \frac{\alpha}{4\pi\varepsilon_0}\left[\frac{1}{3}\left(\sqrt{L^2+x^2}\right)^3 - \frac{1}{3}x^3 - x^2\sqrt{L^2+x^2} + x^3\right]$$

$$= \frac{\alpha}{4\pi\varepsilon_0}\sqrt{L^2+x^2}\left[\frac{1}{3}L^2 + \frac{1}{3}x^2 + \frac{2/3\,x^3}{\sqrt{L^2+x^2}} - x^2\right]$$

$$\underline{\phi(a) = \frac{\alpha}{4\pi\varepsilon_0}\sqrt{L^2+a^2}\left[\frac{1}{3}L^2 - \frac{2}{3}a^2 + \frac{2}{3}\frac{a^3}{\sqrt{L^2+a^2}}\right]}$$

Actually goes to point charge potential if you expand the $\frac{1}{\sqrt{L^2+a^2}}$ term for $a \gg L$ in 1st 3 terms of binomial expansion!

We can't find E_z because $E_z = -\frac{\partial\phi}{\partial z}$ and we only have $\phi(x)$, not $\phi(x,y,z)$

$W_{ext} = -q\,\phi(a)$

$\hookrightarrow$ since we are moving q from P to ∞, not ∞ to P