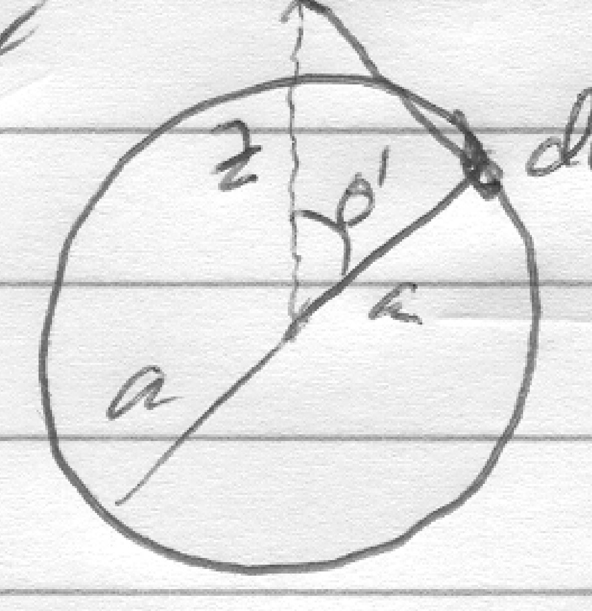
4. 5-14



Uniform σ, no ρ
- can pick z axis for field point with no loss of generality

$$dq = \sigma a^2 \sin\theta' \, d\theta' d\varphi'$$

$$R = \sqrt{z^2 + a^2 - 2az\cos\theta'}$$

$$\Phi(z) = \frac{\sigma a^2}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\pi} \frac{\sin\theta' \, d\theta' d\varphi'}{\sqrt{z^2+a^2-2az\cos\theta'}}$$

$u = \cos\theta'$
$du = -\sin\theta' d\theta'$

$$= -\frac{\sigma a^2}{2\varepsilon_0} \int_1^{-1} \frac{du}{\sqrt{z^2+a^2-2az\,u}}$$

$$\int \frac{dx}{\sqrt{b-cx}} = -\frac{2\sqrt{b-cx}}{c}$$

here $b = z^2+a^2$ and $c = 2az$

$$\Phi(z) = \frac{\sigma a^2}{2\varepsilon_0}\left(-\frac{1}{2az}\right)(2)\sqrt{z^2+a^2-2azu}\,\Big|_{-1}^{1}$$

$$= \frac{\sigma a}{2\varepsilon_0 z}\left[\,|z+a| - |z-a|\,\right]$$

$\underline{z > a}$

$$\Phi(z) = \frac{\sigma a}{2\varepsilon_0 z}\left[z + a - (z-a)\right]$$

$$\Phi_{out}(z) = \frac{\sigma a^2}{\varepsilon_0 z} = \frac{KQ}{z} \quad \text{with } Q = 4\pi a^2 \sigma$$

$\underline{z < a}$

$$\Phi_{in}(z) = \frac{\sigma a}{2\varepsilon_0 z}\left[z + a - (a - z)\right]$$

$$\Phi_{in}(z) = \frac{\sigma a}{2\varepsilon_0} = \text{constant}$$

Note $\Phi_{in} = \Phi_{out}$ at $r = 0$ so $\vec{E}$ is finite there