Physics 3305 HW #4 Solutions

1. W 5-2

Is $\vec{A} = x^2 y \hat{x} + x y^2 \hat{y} + a^3 e^{-\beta y} \cos \alpha x \, \hat{z}$

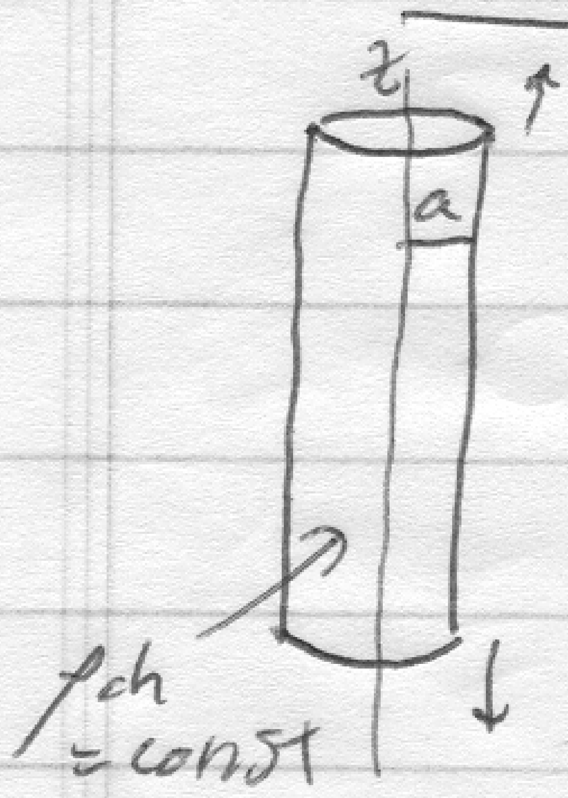
a possible $\vec{E}$ field? Need $\nabla \times \vec{A} = 0$

$$\nabla \times \vec{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \hat{x}\left(-\beta a^3 e^{-\beta y} \cos \alpha x - 0\right) + \hat{y}\left(0 + \alpha a^3 e^{-\beta y} \sin \alpha x\right) + \hat{z}\left(y^2 - x^2\right)$$

$\neq 0$ No - can't find $\phi$

2. W 5-12




Since this is an unphysical charge distribution, we might guess that $\phi$ will diverge, as direct integration will show. Need to define $\phi = 0$ somewhere + find difference

Use Gauss' law results $\quad \vec{E} = \begin{cases} \frac{\rho_{ch} \rho}{2\varepsilon_0} \hat{\rho} & \rho < a \\ \frac{\rho_{ch} a^2}{2\varepsilon_0 \rho} \hat{\rho} & \rho > a \end{cases}$

Set $\phi = 0$ at $\rho = 0$ (on axis)

Then $\phi(\rho) = -\int_0^{\rho} \vec{E} \cdot d\vec{l} = -\int_0^{\rho} E \, d\rho'$ where $E = \vec{E} \cdot \hat{\rho}$

For $\rho < a$ $\quad \phi(\rho) = -\frac{\rho_{ch}}{2\varepsilon_0} \int_0^{\rho} \rho' d\rho' = -\frac{\rho_{ch}}{4\varepsilon_0} \rho^2$

For $\rho > a$ $\quad \phi(\rho) = \phi(a) - \int_a^{\rho} \vec{E} \cdot d\vec{l}$

$$= -\frac{\rho_{ch} a^2}{4\varepsilon_0} - \frac{\rho_{ch} a^2}{2\varepsilon_0} \int_a^{\rho} \frac{1}{\rho'} d\rho'$$

$$\phi(\rho) = -\frac{\rho_{ch} a^2}{2\varepsilon_0}\left[\frac{1}{2} + \ln(\rho/a)\right]$$

diverges as $\rho \to \infty$