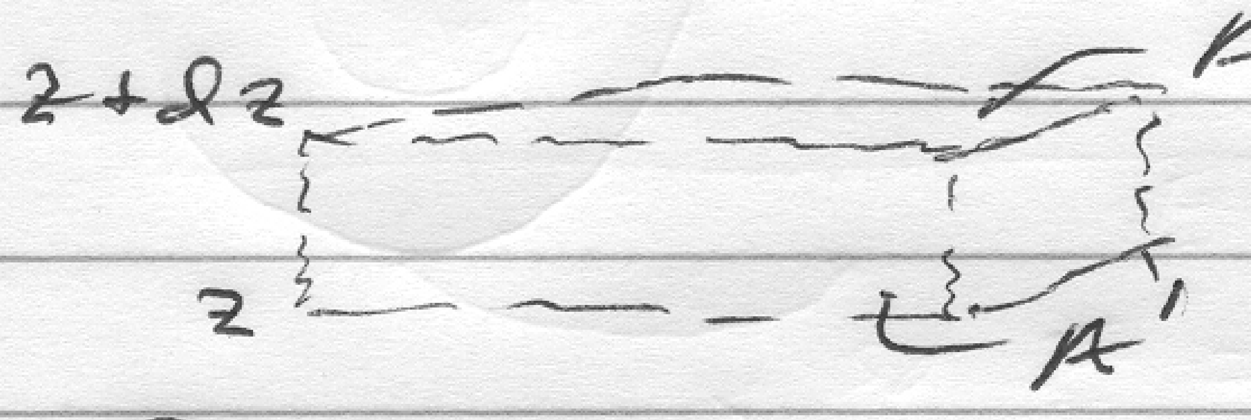
7. W 4-10

$$\vec{E} = -E_0\left(Ae^{-\alpha z} + Be^{-\beta z}\right)\hat{z}$$

Find average charge density versus z




Gaussian box

$$\Phi_{net} = \Phi_{top} + \Phi_{bottom}$$

$$= -E_0 A'\left(Ae^{-\alpha(z+dz)} + Be^{-\beta(z+dz)}\right)$$

$$+E_0 A'\left(Ae^{-\alpha z} + Be^{-\beta z}\right)$$

now $e^{-\alpha(z+dz)} = e^{-\alpha z}e^{-\alpha dz}$ $\qquad \alpha dz \ll 1$

so $e^{-\alpha dz} \simeq 1-\alpha dz$

$$\Phi = E_0 A'\left[-Ae^{-\alpha z}(1-\alpha dz) - Be^{-\beta z}(1-\beta dz)\right.$$

$$\left. + Ae^{-\alpha z} + Be^{-\beta z}\right]$$

4 terms cancel, leaving

$$\Phi = E_0 A'\left[Ae^{-\alpha z}\alpha dz + Be^{-\beta z}\beta dz\right]$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{\rho_{AV}}{\varepsilon_0}A'dz$$

$$\rho_{AV} = \varepsilon_0\left[E_0\left(\alpha Ae^{-\alpha z} + \beta Be^{-\beta z}\right)\right]$$

Looks positive to me.

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Easy way using $\nabla\cdot\vec{E} = \rho(\vec{r})/\varepsilon_0$ !

$\vec{E} = E_z\hat{z}$, with $E_z(z)$ only

$\nabla\cdot\vec{E} = \frac{dE}{dz} = \rho(\vec{r})/\varepsilon_0$

then $\rho = \varepsilon_0\frac{dE}{dz}$ gives same answer