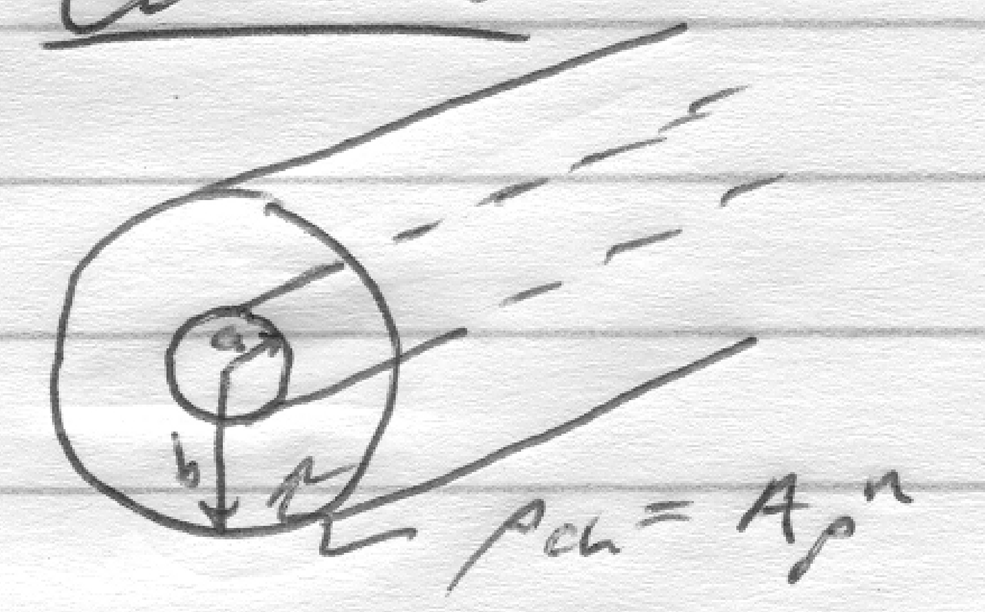
8. W4-8



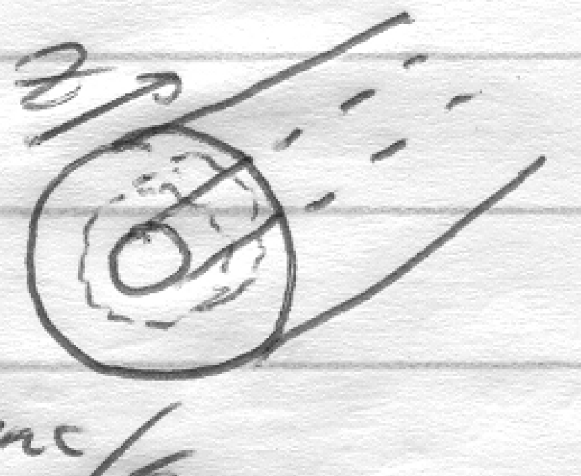
$\rho_{ch} = A\rho^n \quad a<\rho<b$

Find $\vec{E}$ everywhere.

The usual symmetry arguments give $\vec{E} = E(\rho)\hat{\rho}$

$\underline{\rho<a}$ $\quad Q_{enc.} = 0 \Rightarrow \vec{E} = 0$

$\underline{a<\rho<b}$



Cylindrical Gaussian surface

$\int \vec{E}\cdot d\vec{a} = 0$ on endcaps

$$\underset{\text{curved side}}{\int} \vec{E}\cdot d\vec{a} = Q_{enc}/\varepsilon_0$$

$$Q_{enc} = \int_0^\rho \rho_{ch}\, dV = \int_z^{z+l}\int_0^{2\pi}\int_a^\rho A\rho'^n \rho' d\rho' d\varphi' dz'$$

$$Q_{enc} = 2\pi A l \int_a^\rho \rho'^{n+1} d\rho' = \frac{2\pi A l}{n+2}\left[\rho^{n+2} - a^{n+2}\right]$$

so $\int \vec{E}\cdot da = 2\pi\rho l E(\rho) = \frac{1}{\varepsilon_0}\frac{2\pi A l}{n+2}\left[\rho^{n+2} - a^{n+2}\right]$

and $\underline{E(\rho) = \frac{A}{\varepsilon_0(n+2)}\frac{1}{\rho}\left[\rho^{n+2} - a^{n+2}\right]} \quad \underline{a<\rho<b}$

$\underline{\rho>b}$ $\quad Q_{enc} = 2\pi A l \int_a^b \rho'^{n+1} d\rho' = \frac{2\pi A l}{n+2}\left[b^{n+2} - a^{n+2}\right]$

$$E(\rho) = \frac{1}{2\pi\rho l}\frac{1}{\varepsilon_0}\frac{2\pi A l}{n+2}\left[b^{n+2} - a^{n+2}\right]$$

$\underline{E(\rho) = \frac{A}{\varepsilon_0(n+2)}\frac{1}{\rho}\left[b^{n+2} - a^{n+2}\right]} \quad \underline{b<\rho}$

This should give prob. 4-7 results when $a\to 0$ and $n \to 0$

$\underline{a<\rho<b}$ $\quad E(\rho) = \frac{A}{2\varepsilon_0}\left(\frac{\rho^2 - a^2}{\rho}\right) = \frac{A\rho}{2\varepsilon_0}$ ✓

$\underline{\rho>b}$ $\quad E(\rho) = \frac{Ab^2}{2\varepsilon_0}\frac{1}{\rho}$ ✓